M.S. COMPREHENSIVE EXAMINATION

June 2022

You have four hours, after a 20 minute reading period. You do not need to use the whole time period. You need to answer all three questions, which will be weighted equally.
Watch the time carefully. The logic used to answer each question is important, so be sure to specify your reasoning with full sentences. You may support your answers using diagrams or mathematical derivations where appropriate. If you use graphs, make sure that they are large enough. We are expecting precise and concise answers. Also make sure your writing is legible: if we cannot read it, your answer will be assumed wrong.

I. The welfare costs of U.S. dairy policy (5 points)

In this problem, we will demonstrate how simple economic information can be used to compute the welfare costs of farm policy. Specifically, we consider the U.S. federal milk marketing orders (FMMOs), which are market institutions governing the transformation milk at the regional level. To simplify the analysis, we consider a single region where dairy products are produced and consumed, and we ignore trade.

The key prerogative of a FMMO is to implement a price premium for farm milk to be used in the manufacturing of fluid milk relative to other dairy products such as cheese or yogurt. That is, manufacturers purchasing farm milk for the purpose of producing fluid milk must pay a higher price than manufacturers of other dairy products. The FMMO makes sure that there is no resale of farm milk from manufacturers of other dairy products to manufacturers of fluid milk.

[0.1] 1. Briefly explain why there is an incentive for manufacturers of fluid milk to try and purchase farm milk from manufacturers of other dairy products.

> **Solution:** Because fluid milk manufacturers purchase farm milk at a premium, they have an incentive to purchase it from manufacturers of other dairy products rather than directly from dairy farmers. The manufacturers of other dairy products would find it profitable to sell to fluid milk manufacturers, as they would be able to charge a price higher than the price of farm milk (but lower than the price inclusive of the mandated premium).

Assume that there is a single consumer whose preferences over milk (x_1), other dairy products (x_2), and all other goods (x_0) are representable by the following utility function:

$$U(x_0, x_1, x_2) = x_0 + \beta \ln \left\{ \left[(\beta_1)^{\frac{1}{\sigma}} (x_1)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} (x_2)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

where $\beta > 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$, $\beta_1 + \beta_2 = 1$, and $\sigma > 0$.

We normalize the price of x_0 to one and denote p_i the price of x_i, for $i = 1, 2$.

[0.2] 2. What is the consumer's income elasticity of demand for x_1 and x_2? There is no need to do any derivation.

> **Solution:** Both income elasticities are zero because the consumer is quasi-linear, so there are no income effects. Demand for x_1 and x_2 only depends on prices p_1 and p_2.

[0.2] 3. Interpret the parameter σ in words. Again, no need to do any derivation.

> **Solution:** It is the elasticity of substitution between fluid milk and other dairy products. It reflects the ease of substitution between these goods, that is, how fast the ratio of demands would respond to a change in the ratio of prices.

[0.2] 4. If we had more time, using the Lagrange method we would show that the Walrasian demand for x_1 is equal to

$$x_1 = \frac{\beta \beta_1 p_1^{-\sigma}}{\beta_1 p_1^{1-\sigma} + \beta_2 p_2^{1-\sigma}} .$$

Instead, I will just ask you to derive the value of the Lagrange multiplier on the budget constraint.

> **Solution:** The value of the Lagrange multiplier is one, the price of the numeraire, since the consumer is quasilinear. This can be seen by writing the first-order condition with respect to x_0.

[0.3] 5. Using the result in part 4, argue that the expenditure on fluid milk and other dairy products, $p_1 x_1 + p_2 x_2$, is a constant, and state what that constant is.

> **Solution:** The demand for x_2 can be deduced from that for x_1 by symmetry. A simple computation then shows that $p_1 x_1 + p_2 x_2 = \beta$. Therefore, the total dairy expenditure is constant and equal to β, even if the individual demands change with prices.

[0.4] 6. Now show that x_1 can also be written as

$$x_1 = \beta \beta_1 \left[\mathcal{P}(p_1, p_2)\right]^{\sigma-1} p_1^{-\sigma}$$

for some price index $\mathcal{P}$ that you will make explicit. Verify that $\mathcal{P}$ satisfies the properties of a price index, and name these properties. (Hint: There are three of them.)

> **Solution:** It is clear given the expressions in parts 4 and 6 that $\mathcal{P}(p_1, p_2) = \left[\beta_1 (p_1)^{1-\sigma} + \beta_2 (p_2)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Indeed, this price index in increasing in p_1 and p_2, homogenous of degree one, and it satisfies $\mathcal{P}(1, 1) = 1$.

To simplify the problem, we assume that the supply of farm milk is infinitely inelastic, and there are no explicit costs to produce it. We denote by M the fixed quantity of farm milk and by m the price of farm milk paid by manufacturers of other dairy products.

Fluid milk is produced from farm milk according to fixed proportions, one unit of farm milk giving one unit of fluid milk. The per-unit processing cost for fluid milk is denoted $c_1 > 0$. All other dairy products are also produced from farm milk using fixed proportions, and by choice of units we can assume that one unit of farm milk gives one unit of other dairy products, with per-unit processing cost $c_2 > 0$.

[0.2] 7. Write the relationship between the quantity of farm milk (M) and the quantities of fluid milk (x_1) and other dairy products (x_2).

> **Solution:** Since farm milk is used to produce either fluid milk or other dairy products, and each of these products is produced in fixed proportions, with one unit of farm milk giving either one unit of fluid milk or one unit of other dairy products, we must have that $M = x_1 + x_2$.

Since there are constant returns to scale in the production of fluid milk and other dairy products, in equilibrium dairy product manufacturers make zero profit.

[0.2] 8. Write the relationship between the price of farm milk paid by manufacturers of other dairy products and the price of other dairy products in equilibrium.

> **Solution:** Given the fixed proportions assumption, the price of other dairy products must exhaust the price of farm milk and the processing costs, that is, $p_2 = m + c_2$.

Assume that the FMMO mandates that manufacturers of fluid milk pay a relative premium $\tau > 1$ over the price of milk paid by manufacturers of other dairy products. That is, the price paid by fluid milk manufacturers is $\tau m > m$.

[0.2] 9. Write the relationship between the price of farm milk and the price of fluid milk in equilibrium.

> **Solution:** The price of fluid milk must exhaust the price of farm milk as paid by manufacturers and the processing cost, that is, $p_1 = \tau m + c_1$.

We now consider the comparative statics of the equilibrium with respect to the milk price premium, that is, how the equilibrium responds to policy-induced changes in τ. For instance, one may be interested in knowing what would happen if the FMMO were eliminated. We denote with "primes" the values of variables after the change, and with "hats" the *relative changes* in these variables, meant here as the *ratio of the final value to the initial value*. For instance, $\hat{\tau} \equiv \frac{\tau'}{\tau}$ denotes the ratio of the final value of τ to its initial value, $\hat{m} = \frac{m'}{m}$ denotes the relative change in the farm milk price, $\hat{x}_1 \equiv \frac{x_1'}{x_1}$ denotes the relative change in the consumption of fluid milk, and so on.

[0.1] 10. What would be the value of $\hat{\tau}$ corresponding to the elimination of the FMMO?

> **Solution:** It would be $\hat{\tau} = \frac{1}{\tau}$, since in the absence of policy there would be no milk price premium, i.e., $\tau' = 1$.

[0.2] 11. Show that the relative change in x_1 can be written as a function of the relative changes in p_1 and $\mathcal{P}$ as

$$\hat{x}_1 = \hat{\mathcal{P}}^{\sigma-1}\hat{p}_1^{-\sigma} .$$

(<u>Hint</u>: Use part 6.)

Solution: By definition,

$$\hat{x}_1 = \frac{x_1'}{x_1} = \frac{\beta\beta_1 \left[\mathcal{P}(p_1,p_2)'\right]^{\sigma-1}\left(p_1'\right)^{-\sigma}}{\beta\beta_1\left[\mathcal{P}(p_1,p_2)\right]^{\sigma-1} p_1^{-\sigma}} = \left[\frac{\mathcal{P}(p_1,p_2)'}{\mathcal{P}(p_1,p_2)}\right]^{\sigma-1}\left(\frac{p_1'}{p_1}\right)^{-\sigma} = \hat{\mathcal{P}}^{\sigma-1}\hat{p}_1^{-\sigma}.$$

[0.5] 12. Show that

$$\hat{\mathcal{P}} = \left[b_1\hat{p}_1^{1-\sigma} + b_2\hat{p}_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

for parameters b_i that you will make explicit and interpret. (Hint: Do not overspend time on this part.)

Solution: By definition, we have

$$\begin{aligned}\hat{\mathcal{P}} &= \frac{\mathcal{P}'}{\mathcal{P}} \\ &= \frac{\left[\beta_1\left(p_1'\right)^{1-\sigma}+\beta_2\left(p_2'\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\beta_1\left(p_1\right)^{1-\sigma}+\beta_2\left(p_2\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \left[\frac{\beta_1\left(p_1'\right)^{1-\sigma}+\beta_2\left(p_2'\right)^{1-\sigma}}{\beta_1\left(p_1\right)^{1-\sigma}+\beta_2\left(p_2\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \\ &= \left[\frac{\beta_1\left(p_1\right)^{1-\sigma}\left(\hat{p}_1\right)^{1-\sigma}+\beta_2\left(p_2\right)^{1-\sigma}\left(\hat{p}_2\right)^{1-\sigma}}{\beta_1\left(p_1\right)^{1-\sigma}+\beta_2\left(p_2\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}\end{aligned}$$

which has the required form for $b_i \equiv \frac{\beta_i(p_i)^{1-\sigma}}{\beta_1(p_1)^{1-\sigma}+\beta_2(p_2)^{1-\sigma}} = \frac{\beta_i(p_i)^{1-\sigma}}{\mathcal{P}^{1-\sigma}}$. Given the expression for the demand x_1 in part 4, it is clear that $b_i = \frac{p_i x_i}{p_1x_1+p_2x_2}$, that is, b_i can be interpreted as the share of dairy product x_i in total dairy expenditure. Note that this is not exactly the budget share, because there is a third good, the numeraire, and here the share is only computed across dairy products.

[0.3] 13. Show that

$$s_1\hat{x}_1 + s_2\hat{x}_2 = 1$$

for parameters s_i that you will make explicit and interpret. (Hint: Use part 7.)

Solution: The quantity of fluid milk plus the quantity of other dairy products exhausts the quantity of farm milk, that is, $M = x_1 + x_2 = x'_1 + x'_2$. Since $x'_i = x_i \hat{x}_i$, we thus have $x_1\hat{x}_1 + x_2\hat{x}_2 = x_1 + x_2$, that is, $s_1\hat{x}_1 + s_2\hat{x}_2 = 1$, for $s_i = \frac{x_i}{x_1+x_2}$. The share s_i represents the share of fluid milk used in the production of dairy product x_i (in volume).

[0.5] 14. Using parts 8 and 9, show that

$$\hat{p}_1 = \psi_1 \hat{m}\hat{\tau} + 1 - \psi_1$$

and

$$\hat{p}_2 = \psi_2 \hat{m} + 1 - \psi_2$$

for parameters ψ_i that you will make explicit and interpret. (Hint: Start with the second equality.)

Solution: Let us start with the second equality. We know that $p'_2 = m' + c_2$ and that $p_2 = m + c_2$. Therefore,

$$\begin{aligned}\hat{p}_2 &= \frac{p'_2}{p_2} \\ &= \frac{m' + c_2}{m + c_2} \\ &= \frac{m\hat{m} + c_2}{m + c_2} \\ &= \frac{m}{m + c_2}\hat{m} + \frac{c_2}{m + c_2} \\ &= \psi_2\hat{m} + 1 - \psi_2\end{aligned}$$

where $\psi_2 \equiv \frac{m}{m+c_2}$ represents the milk share of the food dollar for other dairy products in the baseline, that is, the share of milk in the total price of other dairy products.

Similarly, we have

$$\begin{aligned}\hat{p}_1 &= \frac{p'_1}{p_1} \\ &= \frac{\tau' m' + c_1}{\tau m + c_1} \\ &= \frac{\tau m \hat{\tau}\hat{m} + c_1}{\tau m + c_1} \\ &= \frac{\tau m}{\tau m + c_1}\hat{\tau}\hat{m} + \frac{c_1}{\tau m + c_1} \\ &= \psi_1\hat{m} + 1 - \psi_1\end{aligned}$$

where $\psi_1 \equiv \frac{\tau m}{\tau m + c_1}$ represents the milk share of the fluid milk dollar in the baseline.

[0.5] 15. Now summarize the set of equalities that determine the equilibrium in relative changes. Check that there are as many equations as unknowns. State explicitly what the unknowns are.

Summarize the calibrating information needed to solve this equilibrium, for a given value of $\hat{\tau}$. (Hint: I am not asking you to solve the equilibrium; there are no new derivations involved.)

Solution:
The equalities that determine the equilibrium in relative changes are: $\hat{x}_i = \hat{\mathcal{P}}^{\sigma-1}\hat{p}_i^{-\sigma}$, for $i = 1,2$, $\hat{\mathcal{P}} = \left[b_1\hat{p}_1^{1-\sigma} + b_2\hat{p}_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$, $s_1\hat{x}_1 + s_2\hat{x}_2 = 1$, $\hat{p}_1 = \psi_1\hat{m}\hat{\tau} + 1 - \psi_1$, and $\hat{p}_2 = \psi_2\hat{m} + 1 - \psi_2$. There are thus 6 equations, and there are 6 unknowns: $\hat{x}_i$ $(i = 1,2)$, $\hat{p}_i$ $(i = 1,2)$, $\hat{\mathcal{P}}$, and $\hat{m}$. (Remember that the quantity of farm milk is assumed to be fixed.) The information needed to solve for the equilibrium in relative changes is that needed to set the share parameters b_i, ψ_i, s_i, as well as the substitution elasticity σ.

To wrap up, we will show how to compute the change in farm profit $\Delta\Pi \equiv \Pi' - \Pi$, as well as the change in consumer surplus, which in this case is also equal to the change in utility $\Delta U \equiv U' - U$. We assume that the consumer's income is unaffected by the change.

[0.2] 16. Show that the change in farm profit relative to the value of milk is

$$\frac{\Delta\Pi}{mM} = \hat{m} - 1 \ .$$

Solution: Since we assume that there is no costs to produce farm milk, the farm profit is simply equal to the milk revenue, mM. Therefore, $\Delta\Pi = (m' - m)\,M = mM\,(\hat{m} - 1)$, which gives the result. This question was actually not very clear because mM is the value of milk based on the price paid by manufacturers of other dairy products. It is equal to the farm profit only if the FMMO does not redistribute the premium paid by manufacturers of fluid milk to dairy farmers, which in practice they do. So in practice farm profits would be larger since the average price received would exceed m. One can compute the change in farm profit under that assumption, but the expression is more complicated. Most students interpreted the question as I originally intended. I gave full credit to every student on that part given the possible confusion.

[0.5] 17. Show that

$$\Delta U = \beta \ln\left[b_1\left(\hat{x}_1\right)^{\frac{\sigma-1}{\sigma}} + b_2\left(\hat{x}_2\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \ .$$

(If we had more time, we could also show an even cooler result, namely that $\Delta U = -\beta\ln\hat{\mathcal{P}}$!)

Solution: By definition,

$$\Delta U = U' - U = x_0' - x_0 + \beta\left\{\ln\left[(\beta_1)^{\frac{1}{\sigma}}\left(x_1'\right)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}}\left(x_2'\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \ln\left[(\beta_1)^{\frac{1}{\sigma}}\left(x_1\right)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}}\left(x_2\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right\} .$$

First note that since the dairy expenditure is constant and equal to β, the expenditure on the numeraire does not change, therefore $x_0' = x_0$. We will then use the rules of logarithms to simplify the expression:

$$\begin{aligned}
\Delta U &= \beta \ln \left\{ \frac{\left[(\beta_1)^{\frac{1}{\sigma}} \left(x_1'\right)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} \left(x_2'\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[(\beta_1)^{\frac{1}{\sigma}} (x_1)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} (x_2)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}} \right\} \\
&= \beta \ln \left\{ \left[\frac{(\beta_1)^{\frac{1}{\sigma}} \left(x_1'\right)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} \left(x_2'\right)^{\frac{\sigma-1}{\sigma}}}{(\beta_1)^{\frac{1}{\sigma}} (x_1)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} (x_2)^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \\
&= \beta \ln \left\{ \left[\frac{(\beta_1)^{\frac{1}{\sigma}} (x_1)^{\frac{\sigma-1}{\sigma}} (\hat{x}_1)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} (x_2)^{\frac{\sigma-1}{\sigma}} (\hat{x}_2)^{\frac{\sigma-1}{\sigma}}}{(\beta_1)^{\frac{1}{\sigma}} (x_1)^{\frac{\sigma-1}{\sigma}} + (\beta_2)^{\frac{1}{\sigma}} (x_2)^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \\
&= \beta \ln \left\{ \left[\frac{\beta_1 (p_1)^{1-\sigma} (\hat{x}_1)^{\frac{\sigma-1}{\sigma}} + \beta_2 (p_2)^{1-\sigma} (\hat{x}_2)^{\frac{\sigma-1}{\sigma}}}{\beta_1 (p_1)^{1-\sigma} + \beta_2 (p_2)^{1-\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \\
&= \beta \ln \left\{ \left[b_1 (\hat{x}_1)^{\frac{\sigma-1}{\sigma}} + b_2 (\hat{x}_2)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}
\end{aligned}$$

where we have used the expression in part 6 in the second-to-last equality to replace the expressions in x_1 and x_2 by expressions in p_1 and p_2.

[0.2] 18. Deduce the change in consumer surplus relative to the expenditure on dairy products, that is, the value of $\frac{\Delta U}{p_1 x_1 + p_2 x_2}$. (Hint: Use part 5.)

Solution: using part 5, we get that $\frac{\Delta U}{p_1 x_1 + p_2 x_2} = \frac{\Delta U}{\beta} = -\ln \hat{\mathcal{P}}$.

II. Choice of mode of transportation (5 points)

Suppose that you would like to investigate the choice between modes of commuting to work. Assume that an individual makes a choice between driving a car and taking public transportation to work. The main variables that determine their decision are the gas price G_i (in dollars per gallon), the public transportation fare F_i (in dollars), and the average waiting time for public transportation W_i (in minutes). There are two outcomes, using public transportation ($Y_i = 1$) or driving a car ($Y_i = 0$).

1. The first model specification that you try is the following:

$$\Pr\{Y_i = 1\} = \beta_0 + \beta_1 G_i + \beta_2 F_i + \beta_3 W_i + \beta_4 W_i^2. \quad (1)$$

(a) Provide an interpretation for coefficients β_1 and β_2. What are the expected signs of these coefficients? Explain.

> **Solution:** β_1 and β_2 capture the average effects of unit change in gasoline price G_i and F_i on the probability of taking the public transport. β_1 is positive, since more expensive gas makes driving less attractive. β_2 is negative, since more expensive fares make public transport less attractive.

(b) What does the sign of β_4 tell us about the readiness to wait for a bus/train an extra minute? Give an example that would imply a specific sign for β_4.

> **Solution:** Parameter β_4 determines how sensitivity to additional minute of waiting time changes with the overall waiting time. For example, longer waiting time would make public transport less attractive, so β_3 is negative. At the same time, intuitively, difference between waiting 1 and 5 minutes is much more prominent than between 30 and 34 minutes. So β_4 is likely to be positive.

(c) Write down a linear regression model corresponding to (1). What condition would the corresponding regression residual satisfy?

> **Solution:**
> $$Y_i = \beta_0 + \beta_1 G_i + \beta_2 F_i + \beta_3 W_i + \beta_4 W_i^2 + U_i,$$
> where $E(U_i | S_i, A_i) = 0$.

(d) Discuss drawbacks and limitations of the linear model specification (1) for estimation of the choice model.

> **Solution:** This question can have multiple answers. Some possibilities include: 1) Predicted probability of choosing a bus may be outside of the interval $[0, 1]$; 2) Marginal effects do not depend of the regressors; 3) Model is likely misspecified, as linear functional form is too artificial.

2. Then you come up with the following latent index model:

$$Y_i^* = \beta_0 + \beta_1 G_i + \beta_2 F_i + \beta_3 W_i + \beta_4 W_i^2 + \varepsilon_i, \quad (2)$$

where Y_i^* is an unobservable index that measures an individual's utility difference between the two options and ε_i represents all unobserved factors affecting their choice. The individual prefers public transportation if $Y_i^* \geq 0$.

Suppose that $\varepsilon_i \sim \mathcal{N}(0,1)$. The CDF and PDF of $\mathcal{N}(0,1)$ are denoted $\Phi(z)$ and $\phi(z)$, respectively. You do not need to use the explicit forms of $\Phi(z)$ and $\phi(z)$ to answer the questions below.

(a) What is the name of this model? You don't need to elaborate on the properties here.

> **Solution:** It is a random utility representation of the Probit model.

(b) Suppose that you know the coefficients in the model. Write down the share of people who prefer driving a car in a city with gas price G^*, public transport fare F^*, and average waiting time W^*.

> **Solution:** For simplicity of notation, lets use vector notation β, $X_i = (1, G_i, F_i, W_i, W_i^2)$. In a city with a sufficiently large population, the share of the people choosing driving will be equal to $P\{Y_i = 0|X_i\}$.
> Using the model, $P\{Y_i = 0|X_i\} = P\{\beta' X_i + \varepsilon_i < 0|X_i\} = P\{\varepsilon_i < -\beta' X_i|X_i\} = \Phi(-\beta' X_i) = 1 - \Phi(\beta' X_i)$.

(c) What is the marginal effect of a reduction in the waiting time on the share of public transportation users?

> **Solution:** By definition, the marginal effect of W_i is $\frac{\partial E(Y_i|X_i=X_i^*)}{\partial W_i} = \frac{\partial P(Y_i=1|X_i=X_i^*)}{\partial W_i}$. Then the answer is
>
> $$\phi(\beta_0 + \beta_1 G^* + \beta_2 F^* + \beta_3 W^* + \beta_4 (W^*)^2)(\beta_3 + 2\beta_4 W^*).$$

(d) Now suppose that you observe a random sample of N individuals from different cities in a country. Write down the corresponding log-likelihood function.

> **Solution:** We observe i.i.d. observations $\{Y_i, X_i\}_{i=1}^N$.
> The log likelihood of the probit model is
>
> $$L(Y, X, \beta) = \sum_{i=1}^{N} Y_i \log \Phi(\beta' X_i) + (1 - Y_i) \log(1 - \Phi(\beta' X_i)) \quad (3)$$