Date: August 10, 2020
Time: 3 hours
Reading Time: 15 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer **two** of the following three questions

QUESTION 1

The board of directors of a company consists of five members. One member is the founding president who is looking for a way to keep the company intact. Her first preference is to initiate a poison-pill provision into the company charter. The **poison pill** would be designed to prevent any outside party from attaining control without board approval.

The two young members of the board feel that the situation is more desperate. They believe that a takeover is inevitable and are concentrating on finding a way to make the present transaction more acceptable. Their preferred action is to look for a **white knight**, a buyer who is acceptable to management and the board.

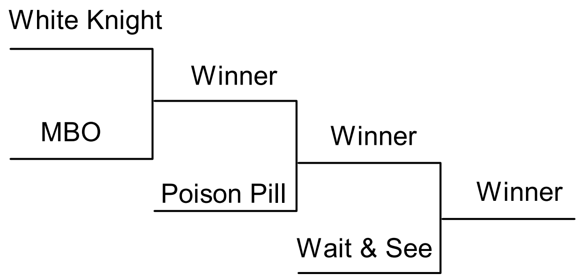
The fourth member, who represents the management, suggests a third possibility. The present managers would like the opportunity to buy the company through a **management buyout**, an MBO.

The fifth member of the board is an outside director. He is cautiously optimistic about the present raider and argues that there is time to see how the offer develops.

After these four options have been discussed at length, everyone ends up with a clear picture of where the others stand on the four proposals. The complete set of preferences is presented below.

	Founder's ranking	Two Young Directors' ranking	Management's ranking	Outside Director's ranking
1st (best)	Poison Pill	White Knight	MBO	Wait & See
2nd	MBO	Poison Pill	Poison Pill	White Knight
3rd	White Knight	Wait & See	Wait & See	Poison Pill
4th (worst)	Wait & See	MBO	White Knight	MBO

They decide that there is a natural order to the decision-making process: begin by comparing an MBO with a White Knight; the chosen alternative is then compared with the Poison Pill option. Having found the best active response, they decide whether this is worth doing by comparing it to Wait & See. The voting procedure is represented by the tree below.



Each vote is by simultaneous, secret ballot and the majority decides the outcome. After each vote the outcome is made known to every member and the voting proceeds to the next stage.

It is common knowledge that they are all **sophisticated thinkers** and **strategic voters** (that is, when faced with a vote between alternative *x* and alternative *y*, a member might vote for *x* even though he/she ranks *y* above *x*, if he/she anticipates that such a vote will affect the final outcome in a way that is favorable according to his/her true preferences).

(a) Describe a pure strategy for a board member in this game. [No need to draw the extensive-form game.]

(b) Using, whenever possible, the notion of weak dominance (pertaining to the choices in any stage of voting) find the subgame-perfect equilibrium of this game. Explain your answer in detail.

(c) Once the founder recognizes what will happen, there is a ploy she can resort to in order to get her most preferred option, namely the Poison Pill. Suppose the founder simply gives her vote to somebody else (who will then use it to double his own vote, which he casts according to his own preferences) and leaves the meeting. To whom should the Founder give the vote? Explain your answer in detail.

Question 2

Consider a monetary lottery L described by the cumulative distribution function $F_L : \mathbb{R} \longrightarrow [0, 1]$. We require that some of the probability mass is on negative outcomes but the expected value is positive.

We say that a decision maker with Bernoulli utility function $u : \mathbb{R} \longrightarrow \mathbb{R}$ and wealth level w is indifferent between accepting and rejecting lottery L if and only if

$$\int u(x + w) dF_L(x) = u(w).$$

Now I introduce a concept that we did not discuss in ECN200A. The goal of this part of the preliminary exam is to relate this new concept to concepts that we do know from ECN200A. Aumann and Serrano (Journal of Political Economy, 2008) axiomatized an index of risk that is defined independently from any utility function. The Aumann-Serrano index of risk of lottery L is defined as the unique number $R(L)$ such that

$$\int e^{-\frac{x}{R(L)}} dF_L(x) = 1.$$

(a) Consider the Bernoulli utility function

$$u(x) = -e^{-ax} \text{ for } a > 0.$$

Show that the Arrow-Pratt coefficient of risk aversion is a for all x.

(b) Consider the class of CARA Bernoulli utility functions (i.e., constant absolute risk aversion). Show that the Aumann-Serrano index of risk of lottery L is the reciprocal of the Arrow-Pratt coefficient of risk aversion of a decision maker with CARA Bernoulli utility who is indifferent between accepting and rejection lottery L. So even though the Aumann-Serrano index of risk is defined independently of any utility function, we can use CARA utility to provide an interpretation.

(c) Require now additionally that lotteries L satisfy $F_L(c) = 0$ for some $c < 0$ and $F_L(d) = 1$ for some $d > 0$. Show that if lottery L' first-order stochastically dominates lottery L, then $R(L') \leq R(L)$.

(d) Again, require additionally that lotteries L satisfy $F_L(c) = 0$ for some $c < 0$ and $F_L(d) = 1$ for some $d > 0$. Show that if a lottery L' second-order stochastically dominates lottery L, then $R(L') \leq R(L)$.

(e) Provide a simple argument for why the converse of (c) or (d) cannot hold.

Question 3

Fix an exchange economy $\{\mathcal{I}, (u^i, w^i)_{i\in\mathcal{I}}\}$.

Two individuals i and i' are said to be *equals* if $w^i = w^{i'}$ and

$$u^i(x) \geqslant u^i(x') \Leftrightarrow u^{i'}(x) \geqslant u^{i'}(x')$$

for all consumption bundles x and x'.

An allocation $\mathbf{x} = (x^i)_{i\in\mathcal{I}}$ satisfies *equal treatment of equals* if $x^i = x^{i'}$ for any two individuals i and i' who are equals. It displays *indifference between equals* if $u^i(x^i) = u^i(x^{i'})$ whenever i and i' are equals.

1. Argue that if $\mathbf{x}$ satisfies equal treatment of equals then it displays indifference between equals, but the opposite implication is not true.

2. Argue that if $\mathbf{x}$ is a competitive equilibrium allocation, then it displays indifference between equals.

3. Argue that if all individuals in the economy have strictly quasi-concave utility functions and $\mathbf{x}$ is a competitive equilibrium allocation, then it satisfies equal treatment of equals.

4. Argue that even if all individuals in the economy have strictly quasi-concave utility functions, there are Pareto efficient allocations that fail to display indifference between equals.

5. Prove that

$$\max_{x_1,x_2}\{(1-x_1)(2-x_2) : 0 \leqslant x_1 \leqslant 1, 0 \leqslant x_2 \leqslant 2 \text{ and } x_1x_2 \geqslant 1\} = 3 - 2\sqrt{2}.$$

6. Use the previous result to argue that the allocation

$$\mathbf{x} = ((9/20, 9/20), (11/20, 11/20), (1, 1))$$

is in the core of the following three-person exchange economy: preferences are represented by

$$u^1(x_1, x_2) = u^2(x_1, x_2) = u^3(x_1, x_2) = x_1x_2,$$

and endowments are $w^1 = w^2 = (1, 0)$ and $w^3 = (0, 2)$.

7. Argue that even if all individuals in the economy have strictly quasi-concave utility functions, there may be allocations in the core of an economy that fail to display indifference between equals.