University of California, Davis | Date: August 24, 2017
Department of Economics | Time: 5 hours
Microeconomics | Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer **any three** of the following four questions
[If you answer all four questions, please indicate which three you want to be graded]

Question 1

Assume that your grandmother pays for your studies. You probably agree that this assumption is more realistic than most assumptions we make in economics given the cost of living and our "generous" financial support at UC Davis. Imagine the following story: You pay a visit to your grandmother to thank her for her truly generous support. She asks what path-breaking insights you have learned in 200A. After mentioning that the central topic of 200A is consumer theory you proudly tell her that you learned that when prices go up then demand may go up or down. She looks at you with some consternation and starts to wonder whether supporting your studies makes any sense. After all, she knew this all along without having studied 200A. Since you rely on her support, her doubts naturally alarm you. You desperately try to search your memory for something less trivial to tell her. You vaguely recall the "compensated law of demand":

For any $(p, w), (p', w')$ with $w' = p' \cdot x(p, w)$ we have

$$(p' - p) \cdot (x(p', w') - x(p, w)) \leq 0$$

with strict inequality if $x(p, w) \neq x(p', w')$.

(With this notation, $p \in \mathbb{R}^L_{++}$ is a price vector, where L is the number of commodities in the economy; $w \in \mathbb{R}_+$ denotes the consumer's wealth; $x(p, w)$ denotes Walrasian demand at prices p and wealth w. Throughout, we assume that $x(p, w)$ is single-valued in $\mathbb{R}^L$ for any p and w.)

a.) In order to discuss the "compensated law of demand" with your grandmother, you need a verbal interpretation of it. Give a verbal interpretation of the "compensated law of demand".

b.) Your grandmother asks in what sense the "compensated law of demand" is a law. You recall the following proposition that we proved in class:

Proposition: Suppose that the Walrasian demand function $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law. Then $x(p, w)$ satisfies the weak axiom of revealed preference if and only if it satisfies the compensated law of demand.

We recall the weak axiom of revealed preference: The Walrasian demand function $x(p, w)$ satisfies the weak axiom of revealed preference if for any (p, w) and (p', w'),

$$\text{if } p \cdot x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w), \text{ then } p' \cdot x(p, w) > w'.$$

Let's see whether you are able to prove the first direction of the proposition: If $x(p, w)$ satisfies the weak axiom of revealed preference then it satisfies the compensated law of demand. I will guide you step-by-step through a proof:

1. Consider the case in which $x(p', w') = x(p, w)$. This should be easy.

2. Consider now the non-trivial case $x(p', w') \neq x(p, w)$. Rewrite

   $$(p'-p)\cdot(x(p', w') - x(p, w)) = p'\cdot(x(p', w') - x(p, w)) - p\cdot(x(p', w') - x(p, w)) \,.$$

   Consider separately each term of the right-hand side. Derive the signs of each of the two terms using (some) assumptions of the proposition and the fact that we assumed $w' = p' \cdot x(p, w)$ for the compensated law of demand. To derive the sign of the second term of the right-hand side you will need to make use of the weak axiom of revealed preference.

3. Put everything together to conclude the proof of the first direction.

c.) Given the characterization of the compensated law of demand by the weak axiom of revealed preference, you need to explain to your grandmother the meaning of the weak axiom of revealed preference. Provide a verbal interpretation of the weak axiom of revealed preference.

d.) The weak axiom of revealed preference may be viewed as a condition on the "consistency of consumer choice". We may be inclined to label a consumer violating this condition as being "irrational" (unless a more complex setting is considered). So the above proposition characterizes the compensated law of demand in terms of consistency of consumer choice. But is it really a characterization in terms of rational consumer choice in the sense of having complete and transitive preferences over consumption bundles? That is, is the following conjecture true?

Conjecture: Suppose that the Walrasian demand function $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law. Then $x(p, w)$ satisfies the compensated law of demand if and only if the consumer has complete and transitive preferences over consumption bundles.

Consider both directions of the conjecture separately. Moreover, consider the special case of $L = 2$ separately.

(You don't have to present a detailed proof. You can make use of results learned in class. The line of arguments should be clear.)

Question 2

In class we saw a stronger version of the first fundamental theorem of welfare economics (FFTWE) for exchange economies than for production economies. We also had to work hard to prove the second fundamental theorem (SFTWE) for production economies.

In this exercise you obtain: (i) a strong version of the FFTWE for production economies; and (ii) a simple proof of the SFTWE for exchange economies.

1. Consider a production economy

$$\{\mathcal{I}, \mathcal{J}, (u^i, w^i)_{i\in\mathcal{I}}, (Y^j)_{j\in\mathcal{J}}, (s^{i,j})_{(i,j)\in\mathcal{I}\times\mathcal{J}}\},$$

   where each u^i is locally-nonsatiated and $Y^j \neq \varnothing$. Suppose that there are as many firms as there are individuals, and that $s^{i,i} = 1$ for all i.

   Say that an allocation (x, y) *is in the core* of this production economy if there do not exist $\mathcal{H} \subseteq \mathcal{I}$ and $(\hat{x}^i, \hat{y}^i)_{i\in\mathcal{H}}$ such that:[1]

   (a) for all $i \in \mathcal{H}$, $\hat{y}^i \in Y^i$;

   (b) $\sum_{i\in\mathcal{H}} \hat{x}^i = \sum_{i\in\mathcal{H}} w^i + \sum_{i\in\mathcal{H}} \hat{y}^i$;

   (c) for all $i \in \mathcal{H}$, $u^i(\hat{x}^i) \geqslant u^i(x^i)$; and

   (d) for some $i \in \mathcal{H}$, $u^i(\hat{x}^i) > u^i(x^i)$.

   Argue the following version of the FFTWE for this economy: *If (p, x, y) is a competitive equilibrium, then allocation (x, y) is in the core of the economy.*

2. Consider a society $\mathcal{I}$, where the preferences of the individuals are represented by the utility functions $(u^i : \mathbb{R}^L_+ \to \mathbb{R})_{i\in\mathcal{I}}$. Assume that all these functions are continuous, strictly quasiconcave and strictly monotone. Recall that, given these assumptions, our existence theorem says that for any profile $(w^i)_{i\in\mathcal{I}}$ of individual endowments, exchange economy $\{\mathcal{I}, (u^i, w^i)_{i\in\mathcal{I}}\}$ possesses a competitive equilibrium. Now, using this existence result:

   (a) Argue that if a profile of individual endowments $(\bar{w}^i)_{i\in\mathcal{I}}$ is a Pareto efficient allocation, then there exist prices p such that the pair $(p, (\bar{w}^i)_{i\in\mathcal{I}})$ is a competitive equilibrium of the exchange economy $\{\mathcal{I}, (u^i, \bar{w}^i)_{i\in\mathcal{I}}\}$.[2]

   (b) Argue the following version of the SFTWE: *Fix initial endowments $(w^i)_{i\in\mathcal{I}}$. If allocation $\hat{x} = (\hat{x}^i)_{i\in\mathcal{I}}$ is Pareto efficient, then there exists $(p, (\hat{w}^i)_{i\in\mathcal{I}})$ such that $\sum_i \hat{w}^i = \sum_i w^i$ and $(p, \hat{x})$ is a competitive equilibrium for economy $(\mathcal{I}, (u^i, \hat{w}^i)_{i\in\mathcal{I}})$.*

[1] Note that this definition is sensible only under our assumption on the ownership structure of the industry!

[2] This simply says that if the initial endowments of the economy constitute an efficient allocation, then the society does not need to trade.

Question 3

This question consists of two independent parts, each of which carries one half of the question's grade.

1. Let $\mathcal{I}_1 = \{1, \ldots, I_1\}$ and $\mathcal{I}_2 = \{I_1 + 1, \ldots, I_1 + I_2\}$ be two (disjoint) local societies, and denote by $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ the global society resulting from their integration. For each individual in these societies, let u^i be continuous, strongly quasiconcave and strictly monotone, and let $w^i \in \mathbb{R}^L_{++}$. In this exercise you are going to argue that each society can protect itself against any damage brought about by globalization.

   For each local society, $k = 1, 2$, let $(\bar{p}^k, (\bar{x}^i)_{i \in \mathcal{I}_k})$ be a competitive equilibrium of exchange economy $\{\mathcal{I}_k, (u^i, w^i)_{i \in \mathcal{I}_k}\}$. Show that there exist $(\hat{w}^i)_{i \in \mathcal{I}}$ such that:

   (a) for each $k = 1, 2$, $\sum_{i \in \mathcal{I}_k} \hat{w}^i = \sum_{i \in \mathcal{I}_k} w^i$; and

   (b) for any competitive equilibrium $(p, (x^i)_{i \in \mathcal{I}})$ of the (global) economy $\{\mathcal{I}, (u^i, \hat{w}^i)_{i \in \mathcal{I}}\}$ one has that $u^i(x^i) \geqslant u^i(\bar{x}^i)$, for all $i \in \mathcal{I}$.

   (*Hint*: The claim is that each society can implement a local fiscal policy that guarantees that any global equilibrium is Pareto superior, for itself, to its own local equilibrium. Now, how would you make sure that nobody can be made worse off, through *voluntary* trade, than at some given allocation?)

2. In a standard exchange economy $\{\mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}}\}$ the distribution of wealth is *biased in favour only of agent* 1 if for any competitive equilibrium (p, x) of the economy the following three properties are true:

   (a) for each $i \geqslant 2$, $u^1(x^1) > u^1(x)$ for every x such that $p \cdot x \leqslant p \cdot w^i$;

   (b) for every $i \geqslant 2$, there exists x such that $p \cdot x \leqslant p \cdot w^1$ and $u^i(x) > u^i(x^i)$; and

   (c) for every $i, j \geqslant 2$, $u^i(x^i) \geqslant u^i(x)$ for every x such that $p \cdot x \leqslant p \cdot w^j$.

   The distribution *permits subsistence* if $w^i \geqslant w_*$ for all individuals, where w_* is some minimal bundle.

   Determine assumptions under which fiscal policy can ensure that the distribution of wealth is biased in favour only of agent 1 and permits subsistence. State your claim formally and prove it.

QUESTION 4

Players 1 and 2 have agreed to play tennis tomorrow. However, this will be possible only if at least one of them shows up early to claim the court. Each player is either a morning person or a night person. Each player knows his own type, and, regardless of his own type, assigns probability $\frac{1}{3}$ to the other player being a **morning** person. Each player chooses between showing up early (E) to claim the court or showing up late (L); the choices are made independently and simultaneously. The basic payoffs are as follows: (1) if either player shows up early, then playing is possible, and each player receives a payoff of 4, (2) if both show up late, then they will not be able to play and each player receives a payoff of 0. **In addition** to those payoffs, **a morning person who shows up early** also receives an extra payoff of 1, whereas **a night person who shows up early** suffers a payoff penalty of −3. Thus, for example, if they are both night persons and Player 1 shows up early, while Player 2 shows up late then Player 1's payoff is $4-3$ = 1 and Player 2's payoff is 4. All these payoffs are von Neumann-Morgenstern payoffs.

(a) Describe this situation using states and information partitions. With every state associate the relevant game.

(b) Find the common prior.

(c) Use the Harsanyi transformation to convert the representation of part (a) into an extensive-form game.

(d) (d.1) How many pure strategies does Player 2 have?

(d.2) Write down one possible **mixed** strategy for Player 1, which is not a pure strategy.

(d.3) Write down one possible **behavior** strategy for Player 1.

(e) Find a weak sequential equilibrium of the game of Part (c) and prove that there are no other (pure- or mixed-strategy) weak sequential equilibria.

(f) Prove that the weak sequential equilibrium found in Part (e) is also a sequential equilibrium.