University of California, Davis
Department of Economics
Microeconomics

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Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **any three** of the following four questions
[If you answer all four questions please indicate which three you want to be graded]

QUESTION 1

Consider the following interaction between a police officer (Player 1) and a motorist (Player 2). At the start of the interaction, the police officer observes the motorist speeding. The officer has three choices: leave the motorist alone (*L*), pull the motorist over to give her a ticket (*T*) or pull her over to arrest her (*A*). If the officer leaves the motorist alone then the game ends and both players get a payoff of 0. If the officer pulls her over, then the motorist decides whether to stay put (*s*) or drive away (*d*). When the motorist makes this decision, she only knows that the officer has pulled her over, but cannot tell whether the officer has decided to give her a ticket or to arrest her. If the motorist drives away then she is equally likely to get caught (*C*) and to escape (*E*). The von Neumann-Morgenstern payoffs are as follows:

- If the officer pulls the motorist over to give her a ticket and she stays put, then the officer gets 3 and the motorist gets -5.
- Arresting the motorist means extra paperwork for the police officer. Therefore, if the officer pulls the motorist over to arrest her and she stays put, then the officer gets a payoff of just 2, while the motorist gets -10.
- If the officer pulls the motorist over and the motorist drives away and is caught, then the officer gets a payoff of 5, while the motorist gets -15.
- If the officer pulls the motorist over and the motorist drives away and escapes, then the motorist gets 0; the officer gets -10 if he was pulling the motorist over to ticket her, and -11 if he was pulling her over to arrest her (since he will still have to do the extra paperwork).

(a) **(a.1)** Represent this situation as an extensive-form game with the officer moving first.
(a.2) Find all the subgame-perfect equilibria, including the mixed-strategy ones.

(b) **(b.1)** Now represent the situation in a slightly different way: the officer is still moving first, but makes his decision in two steps: first he decides between leaving the motorist alone (*L*) and not leaving the motorist alone ($\neg$L); in the latter case he then makes the further decision whether to go for a ticket or for an arrest.
(b.2) Find all the subgame-perfect equilibria, including the mixed-strategy ones.

Now let us change the story a little bit. There are two types of police officers: those (the *T* type) who only give you a ticket (if you were speeding) and those (the *A* type) who arrest you. Each police officer knows his own type, but motorists only know the percentage of police officers who are *T* types, which is *p*. The value of *p* is public information and thus common knowledge. The police officer now only has two choices: leave the motorist alone (*L*) or pull the motorist over (*P*); the further action (ticket or arrest) is automatic according to the type. The payoffs are as described above.

(c) Use states and information partitions to represent this situation of incomplete information. Associate with every state the game that is played in that state.

(d) Use the Harsanyi transformation to convert the situation of incomplete information into an extensive-form game.

(e) Find all the pure-strategy Bayesian equilibria of the game of part (d) for the case where $p = \frac{3}{5}$.

QUESTION 2

Let e denote the level of education. There are three types of potential workers: those (type L) with productivity θ_L, those (type M) with productivity θ_M and those (type H) with productivity θ_H, with $\theta_H > \theta_M > \theta_L > 0$. For each type $i \in \{L, M, H\}$ the fraction of type i in the population is $\frac{1}{3}$. Each potential worker knows her own type, while the potential employer cannot tell the type of any potential worker, although he knows the distribution of types in the population. The employer observes the education level of each potential worker (but not her type) and offers a wage which depends on the applicant's level of education. For every type $i \in \{L, M, H\}$ the cost of acquiring e units of education is $\frac{e}{\theta_i}$. Each worker's utility is given by the difference between the wage she is paid and the cost of education.

(a) [Note: for this part do **not** assume that each worker must be paid a wage equal to her productivity.] Is there an incentive-compatible situation where (1) the employer offers two wages, depending on the education level: wage w^* to those whose education level is e^* and wage $w_M \neq w^*$ to those whose education level is $e_M \neq e^*$ and refuses to hire anybody with education $e \notin \{e^*, e_M\}$, (2) both types θ_L and θ_H choose education level e^*, while type θ_M choose education e_M? [Note that you should make no assumptions about whether $e_M < e^*$ or $e^* > e_M$ and similarly for w_M and w^*.] If there is such an incentive-compatible situation, please describe it in detail. If your claim is that it does not exist, please prove it.

For parts (b) and (c) assume that the employer pays each worker a wage equal to the worker's expected productivity (as computed by the employer, who is risk neutral).

(b) Define and describe in detail a pooling equilibrium, that is, a signaling equilibrium where all three types make the same choice of education level, call it $\bar{e}$. [Assume that the employer believes that anybody who shows up with education level $e \neq \bar{e}$ must be of type L.]

(c) Find all the pooling equilibria when $\theta_L = 1,\ \theta_M = 2,\ \theta_H = 6$.

Now let us change the situation as follows. There are only two types of potential workers: those with productivity θ_L and those with productivity θ_H, with $\theta_H > \theta_L > 0$. The fraction of type θ_L in the population is equal to the fraction of type θ_H. Assume that the cost of education is the *same for both types* and is given by $c(e) = e$. Suppose that the utility of worker of type $\theta \in \{\theta_L, \theta_H\}$ who is paid wage w and chooses education level e is $U(w, e, \theta) = \theta w - e$. Assume also that $e \in [a, b]$ with $0 < a < b$, that is, there is a minimum level of education a that every worker must have (it is mandated by the government) and a maximum level of education b (e.g. corresponding to a PhD). As before, each potential worker knows her own type, while the potential employer cannot tell the type of any potential worker, although he knows the distribution of types in the population. The employer observes the education level of each potential worker (but not her type) and offers a wage which depends on the applicant's level of education.

(d) **(d.1)** Are there separating signaling equilibria (where different types of workers choose different education levels)? If there are, please describe such equilibria (note that you have to specify the wage that the employer offers for every possible level of education and you cannot assume that it is zero). If not, please prove your claim. [Recall that part of the definition of a signaling equilibrium is that each worker is paid a wage equal to her true productivity]

(d.2) Is there a separating equilibrium when $a = 6,\ b = 14,\ \theta_L = 3,\ \theta_H = 5$? If yes, please describe it. If not, please explain why not.

Question 3

Consider an economy with two goods, a private good and a public good subject to congestion. One unit of private good can be transformed into a unit of public good : if z units of private good are used then $y = z$, where y is the quantity of public good produced. There are I agents with initial resources $(\omega^i)_{i=1}^I$ in private good who can use the public good with varying intensity. Let q^i denote the intensity of use of agent i. Think of y as a number which summarizes the characteristics of a freeway (width, length, road quality) and of q^i as the miles that agent i drives on the freeway. When agent i uses the public good with intensity q^i, it costs the agent $c(q^i)$ of private good (think of $c(q^i)$ as the cost of gas). The utility of an agent for the public good depends not only on y but also on the congestion $Q = \sum_{i=1}^I q^i$ of the public good. That is, the agents' preferences depend on $Y = f(y, Q)$, which we can call the quality of public good, where f is increasing in y and decreasing in Q. The preferences of agent i are thus represented by a function $u^i(x^i, q^i, Y)$ where x^i is the consumption of private good, q^i the intensity of use of the public good, Y is the quality of the public good and u^i is an increasing function. All functions considered in this problem are continuously differentiable.

(a) Write a maximum problem whose solutions are the Pareto optimal allocations of this economy. Be careful to write correctly the feasibility constraints.

(b) Write the first-order conditions for an interior solution ($x^i > 0, y > 0, q^i > 0$ for all i) of this maximum problem.

(c) By eliminating the multipliers in (b), show that

(i) the optimality of the provision y^* of the public good requires that a condition, akin to the Samuelson condition, must be satisfied, involving all the agents' valuations $\frac{\partial u^i}{\partial Y} \Big/ \frac{\partial u^i}{\partial x^i}$ of an additional unit of quality of the public good and $\frac{\partial f}{\partial y}$, all evaluated at the Pareto optimum;

(ii) for each $i = 1, \ldots, I$, the optimality of the intensity of use q^{i*} of the public good implies a condition involving the agent's valuation $\frac{\partial u^i}{\partial q^i} \Big/ \frac{\partial u^i}{\partial x^i}$ of intensity of use, the marginal cost $c'(q^i)$, all agents' valuations $\frac{\partial u^j}{\partial Y} \Big/ \frac{\partial u^j}{\partial x^j}, j = 1, \ldots, I$, of the quality of the public good and $\frac{\partial f}{\partial Q}$, all evaluated at the Pareto optimum.

(d) To check that your FOCs are right, derive the formulae in (c) by marginal reasoning, evaluating the marginal cost of, and the agents' propensity to pay for, a marginal increase in the amount y produced, and the agents' marginal willingness to pay or need to be compensated for a marginal change in the intensity q^i of agent i's use of the public good.

(e) Consider an equilibrium $\left((\bar{x}^i, \bar{q}^i)_{i=1}^I, \bar{y}, \overline{Q}, \overline{Y}, \bar{\tau}\right)$ such that: (i) the government taxes the agents' endowments at the rate $\bar{\tau}$ to finance the provision $\bar{y} = \bar{\tau} \sum_{i=1}^I \omega^i$ of the public good; (ii) agents are small and take the level of congestion $\overline{Q}$ as independent of their own intensity of use. Show that, no matter how $\bar{\tau}$ is chosen, the equilibrium is not Pareto optimal. Explain why, and suggest a way of improving on the equilibrium.

Question 4

We were all pretty relieved that Ali from our first prelim got such a simple consumption problem. Yet, we also realized that putting him into the Cobb-Douglas straightjacket missed some features of the story. Since the overall theme of the second prelim should be "We can do better!", let's also improve Ali's consumption model. As before, write $x_a, x_t, x_g \geq 0$ for the amounts of his consumption of almonds, toothpicks, and gifts, respectively. We assume for simplicity that these goods are infinitesimally divisible. Let $x = (x_a, x_t, x_g)$. Instead of assuming a utility function of the Cobb-Douglas form, we assume now that his utility function is given by

$$u(x_a, x_t, x_g) = \alpha \ln(x_a - b_a) + \theta \ln(x_t - b_t) + (1 - \alpha - \theta) \ln(x_g - b_g)$$

with $\alpha, \theta, b_i > 0$, $\alpha + \theta < 1$ and $x_i - b_i > 0$ for $i \in \{a, t, g\}$. As before, his income or wealth is denoted by $w > 0$. Finally, we denote by $p_a, p_t, p_g > 0$ the prices of almonds, toothpicks, and gifts, respectively, and let $p = (p_a, p_t, p_g)$.

a. Use the Kuhn-Tucker approach to derive step-by-step the Walrasian demand function $x(p, w)$. Verify also second-order conditions.

b. Verify that the demand function is homogenous of degree zero and satisfies Walras' Law.

c. Provide a verbal interpretation of this demand system. (It will be helpful to consider the demand system in its "expenditure form" by multiplying both sides of each demand equation by its respective price.)

d. You would expect that the more almonds Ali eats, the more they get stuck in his teeth and the more toothpicks he purchases. In light of such considerations, does it make sense to assume Ali has the utility function above? (Consider changes in the demand for almonds and toothpicks caused by changes in the price of almonds and changes in the parameters b_a and α, respectively.)

e. Consider now a utility function given by

$$\hat{u}(x_a, x_t, x_g) = (x_a - b_a)^{\alpha} (x_t - b_t)^{\theta} (x_g - b_g)^{1-\alpha-\theta},$$

with $\alpha, \theta, b_i > 0$, $\alpha + \theta < 1$, and $x_i - b_i > 0$ for all $i \in \{a, t, g\}$. How is this utility function related to the one given at the beginning of Question 4?

f. Remember that when Professor Schipper interviewed Ali about how exactly he arrives at his optimal consumption bundle, Ali expressed ignorance about maximizing utility subject to his budget constraint. Instead, he seemed to minimize his expenditure on consumption such that he reaches a certain level of utility. A smart undergraduate student walked by and claimed that this is clear evidence against the assumption of utility maximization in economics. Since Professor Schipper likes Linear Expenditure Systems as much as Cobb-Douglas utility functions, he conveniently sent the student to you so that you can show him how expenditure minimization works. Again, use the Kuhn-Tucker approach to derive the Hicksian demand function but use the utility function in part e instead.

Simplification: Let's not write our fingers to the bone. Assume from now on (for all parts g to ℓ) that Ali got rid of his girlfriend. Sure you must be sad about it but there is clearly a tradeoff between having a girlfriend and completing successfully and on time a prelim exam. Most important to Ali: No more gifts! Thus, we can consider now the case of two goods, almonds and toothpicks, only. Set $\theta = 1 - \alpha$ to economize on parameters.

g. Derive the expenditure function.

h. Show that the expenditure function is homogeneous of degree 1 in prices, strictly increasing in $\bar{u}$ as well as nondecreasing and concave in the price of each good taken separately.

i. Derive Ali's indirect utility function using the expenditure function just derived.

j. Verify that Ali satisfies Roy's identity with respect to almonds.

k. Verify the (own price) Slutsky equation for the example of almonds.

ℓ. Because of the drought, the price of almonds changes from p_a^0 to p_a^1. The newly hired Senior Vice Provost for Crocodile Welfare at UC Davis is eager to prove his usefulness by offering Ali to sacrifice part of his income in return to continue offering almonds from campus at the old price p_a^0. Ali is not too excited but likes to find out more about the proposal, in particular about how much income he would be willing to sacrifice in return. He plans to turn to Professor Schipper for advice on the exact income deduction that would make him indifferent between accepting this deduction or the higher price of almonds. Unfortunately, Professor Schipper is away in Europe. Luckily the arboretum is buzzing with first year econ PhD students rehearsing microeconomics. So he approaches you to calculate the amount. Help him!