Oligopsony

Meaning of oligopsony. From Greek: “oligo” = a few, a small number; “psony” comes from Greek opsōniā, purchase of food; in Economics it refers to the purchase of any commodity/factor of production. When a few firms dominate the purchase of a commodity (service, factor of production) the market is characterized by an oligopsony structure.

Examples of oligopsony markets:
1. Tobacco in the world (5 firms: Altria (formerly Philip Morris International), British American Tobacco (Brown and Williams), Imperial Tobacco, Japan Tobacco, Reynolds American-Lorillard).
2. Cocoa (3 firms: Cargill, Archer Daniels Midlands, Callebaut).
3. Meat Packing (4 firm concentration: 83% in 2013)
4. Rubber-Tire industry
5. Diamond industry

As in the oligopoly market, there are various models of oligopsony. We will discuss two types: Cournot-Nash quantity equilibrium and Bertrand-Nash price equilibrium for differentiated products.

Cournot-Nash quantity equilibrium on factor markets
Assumptions: non-cooperative behavior
1. There are \( K \) oligopsony firms in the market.
2. All oligopsony firms face the same market inverse supply function for some input (factor of production) \( p_s = g + Gx^r \) (only one commodity input, \( g \neq 0, G > 0 \)).
3. The purchased commodity is perceived as homogeneous by the oligopsony firms that purchase the input and produce a final commodity to sell, \( x^D_k \), on a competitive market with a technology \( A_k, k = 1, ..., K \).
4. Buyers (Oligopsony firms) are basically unaware of the market strategy of other firms. In other words, each firm maximizes profit with respect to its choice variable (the factor of production) assuming that the choice variables of other firms are unaffected (via input price) by the given firm’s choice. This must be true at equilibrium for the entire industry.
5. Industry entry is blocked because this model is static.
6. Oligopsony firms do not decide the price to buy the commodity input, service, factor of production. Each buyer makes his quantity decision on the basis of the market supply price, \( p_s \), at which the total quantity of input from all firms, \( x^S \), can be bought.
Oligopsony firms may behave in a variety of ways on the demand (for final commodity) market. In order to focus on the oligopsony behavior, we assume that oligopsony firms behave as price takers on the demand market.

Therefore, the behavioral objective of the $k$th oligopsony firm is to maximize profit
\[ \max \pi_k = cx_k^D - p_x x_k^S = cx_k^D - (g + G x^S_k) x_k^S \]
Primal
\[
\begin{align*}
\text{subject to} \\
D &\leq S \\
A_k x_k^D &\leq x_k^S
\end{align*}
\]
with all nonnegative variables.

The relevant KKT conditions of this $k$th firm are:

**Lagrange function**
\[ L_k = cx_k^D - (g + G \sum_{h=1}^{K} x_h^S) x_k^S + y_k (x_k^S - A_k x_k^D) \]

**Relevant KKT conditions**
\[
\begin{align*}
\frac{\partial L_k}{\partial x_k^D} &= c - A_k' y_k \leq 0 \quad \rightarrow \quad \text{output (} MR - MC \leq 0) \quad (1) \\
\frac{\partial L_k}{\partial x_k^S} &= -(g + G \sum_{h=1}^{K} x_h^S) - G x_k^S + y_k \leq 0 \quad \rightarrow \quad \text{input (} -MC + MR \leq 0) \\
\frac{\partial L_k}{\partial x_k^D} &\cdot x_k^D = x_k^D c - x_k^D A_k' y_k = 0 \quad (3) \\
\frac{\partial L_k}{\partial x_k^S} &\cdot x_k^S = -x_k^S (g + G \sum_{h=1}^{K} x_h^S) - x_k^S G x_k^S + x_k^S y_k = 0 \quad (4)
\end{align*}
\]

The term $G x_k^S$ is a measure of the market power per unit of commodity input of the $k$th oligopsony firm. Recall that profit is always defined as total revenue minus the total cost of the physical plant. Therefore, by adding together the two CSC equations (3) and (4) we obtain (using the primal CSC $-x_k^D A_k' y_k + x_k^S y_k = 0$)
\[
\begin{align*}
&cx_k^D - x_k^S A_k' y_k - x_k^S (g + G \sum_{h=1}^{K} x_h^S) - x_k^S G x_k^S + x_k^S y_k = 0 \\
&cx_k^D - x_k^S p_s - x_k^S G x_k^S = 0 \\
&TR_k - TC_ppk = x_k^S g x_k^S = \pi_k
\end{align*}
\]
Hence, the profit of the $k$th oligopsony firm is $\pi_k = x_k^S G x_k^S = G (x_k^S)^2$ since we deal with only one input. The dual specification of the $k$th oligopsony firm begins with the simplification of the Lagrange function using the two CSC equations (3) and (4)
\[
\begin{align*}
L_k &= cx_k^D - (g + G \sum_{h=1}^{K} x_h^S) x_k^S + y_k (x_k^S - A_k x_k^D) \\
&= y_k A_k x_k^D + x_k^S G x_k^S - y_k x_k^S + y_k x_k^S - y_k A_k x_k^D \\
&= x_k^S G x_k^S
\end{align*}
\]
Therefore, the Dual problem of the \( k \)th oligopsony firm is

\[
\begin{align*}
\text{Dual} & \quad \min TCMO_k = x^S_k G x^S_k \\
\text{subject to} & \quad MC \geq MR \\
& \quad A_k' y_k \geq c \\
& \quad (g + G \sum_{h=1}^{K} x^S_h) + G x^S_k \geq y_k \\
\end{align*}
\]

with all nonnegative variables.

The dual objective function says that an outside economic agent (a dual entrepreneur) who would like to buy out the oligopsony firm should minimize the Total Cost of Market Options, \( x^S_k G x^S_k \), that is, she should minimize the cost of purchasing the supply function “owned” (together with the other oligopsony firms) by the primal entrepreneur. The first dual constraint deals with the marginal cost and marginal revenue of the commodity output to be sold on the final commodity market. The second dual constraint deals with the factor of production (input) purchased on the input market. The marginal cost is given by the price \( p_s = (g + G x^S_k) \) that the \( k \)th oligopsony firm (together with all other firms) will pay the suppliers of the input at the quantity \( x^S = \sum_{h=1}^{K} x^S_h \) plus the market power of the \( k \)th oligopsony firm represented by \( G x^S_k \). The “marginal revenue” for the input is the shadow price of it, \( y_k \). In this case, since we are dealing with an input, it is better to use the terminology “marginal valuation” (in microeconomic textbooks it is referred to as “marginal expenditure”) rather than “marginal revenue.” The valuation of the firm’s input quantity should never be overvalued. Certainly not valued above \( (p_s + G x^S_k) \).

The \( k \)th oligopsony firm’s problem specified above cannot be solved in isolation from the other oligopsony firms because the total quantity of input to be purchased on the market – appearing in the primal and dual specifications – is unknown. Analogously to the oligopoly Cournot-Nash equilibrium, therefore, we need to specify a comprehensive model where all the oligopsony firms are represented.

\[
\begin{align*}
\text{Primal} & \quad \max AUX = c x^D - (g + \frac{1}{2} G x^S) x^S - \frac{1}{4} G \sum_{k=1}^{K} (x^S_k)^2 \\
\text{subject to} & \quad A_k x^D \leq x^S_k \\
& \quad \sum_{k=1}^{K} x^D_k = x^D \\
& \quad \sum_{k=1}^{K} x^S_k = x^S \\
\end{align*}
\]

with all nonnegative variables.

The objective function is an auxiliary function formulated as such in order to generate the proper KKT conditions of all the oligopsony firms. The rationale for this type of auxiliary objective function will become clear after the dual model is developed. Recall that any mathematical programming problem is solved via its corresponding KKT conditions. Therefore, we must check that the KKT conditions derived
from the above auxiliary model include the KKT conditions developed for the \( k \)th oligopsony firm. In the overall oligopsony model it is necessary to have this list of KKT conditions:

1. The technology relations for every oligopsony firm (first primal constraint).
2. The overall quantity of commodity produced and sold by these firms (second primal constraint).
3. The overall quantity of commodity input purchased by these firms (third primal constraint).
4. The marginal cost/marginal revenue relation, for every oligopsony firms, relating to the commodity output sold.
5. The marginal cost/marginal valuation, for every oligopsony firm, relating to the input purchased and used in the production of the final commodity.
6. The price formation relation, that is, the inverse supply function.

Let us check whether these KKT relations are generated by the primal AUX model formulated above.

Lagrange function

\[
L = c x^D - (g + \frac{1}{2} Gx^S)x^S - \frac{1}{2} G \sum_{k=1}^{K} (x_k^S)^2 + \sum_{k=1}^{K} y_k [x_k^S - A_k x_k^D] + p^D \left[ \sum_{k=1}^{K} x_k^D - x^D \right] + p_s \left[ x^S - \sum_{k=1}^{K} x_k^S \right]
\]

Relevant KKT conditions

\[
\frac{\partial L}{\partial x^D} = c - p^D \leq 0 \quad (5)
\]
\[
\frac{\partial L}{\partial x^S} = -(g + Gx^S) + p_s \leq 0 \quad (6)
\]
\[
\frac{\partial L}{\partial x_k^D} = -A_k' y_k + p^D \leq 0 \quad (7)
\]
\[
\frac{\partial L}{\partial x_k^S} = -Gx_k^S + y_k - p_s \leq 0 \quad (8)
\]

Complementary slackness conditions

\[
x^D \frac{\partial L}{\partial x^D} = x^D c - x^D p^D = 0 \quad (9)
\]
\[
x^S \frac{\partial L}{\partial x^S} = -x^S (g + Gx^S) + x^S p_s = 0 \quad (10)
\]
\[
x_k^D \frac{\partial L}{\partial x_k^D} = -x_k^D A_k' y_k + x_k^D p^D = 0 \quad (11)
\]
\[
x_k^S \frac{\partial L}{\partial x_k^S} = -x_k^S Gx_k^S + x_k^S y_k - x_k^S p_s = 0 \quad (12)
\]

Note that the addition of KKT conditions (5) and (7) results in the identical KKT condition (1). Furthermore, the addition of KKT conditions (6) and (8) results in the identical KKT condition (2). Therefore, the comprehensive primal AUX model satisfies the conditions for representing an oligopsony industry operating under Cournot-Nash assumptions.

It remains to develop the dual specification of the oligopsony industry as a whole. To achieve that objective it is necessary to follow the usual process of simplifying the Lagrange function using the CSC relations (9)-(12) summed over the \( K \) oligopsony firms, when required. To make the process as transparent as possible I re-state below the CSC conditions as follows:
\[ x^D c = x^D p^D \]  
\[ x^S p_s = x^S g + x^S G x^S \] (9)  
\[ p^D \sum_{k=1}^{K} x^D_k = \sum_{k=1}^{K} x^D_k A_k' y_k \] (10)  
\[ \sum_{k=1}^{K} x^S_k y_k = p_s \sum_{k=1}^{K} x^S_k + \sum_{k=1}^{K} x^S_k G x^S_k \] (11)  
\[ k = 1 \]  
\[ \sum_{k=1}^{K} x^S_k = \sum_{k=1}^{K} x^S_k + \sum_{k=1}^{K} x^S_k G x^S_k \] (12)  
\[ k = 1 \]

The computations require a diligent substitution of the terms on the left-hand side of the equality sign in equations (9)-(12) that are detailed in the following development. The two lines below reproduce the Lagrange function where all the parentheses are eliminated:

\[
L = c x^D - (g + \frac{1}{2} G x^S) x^S - \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k + \sum_{k=1}^{K} y_k [x^S_k - A_k x^D_k] + p^D \left[ \sum_{k=1}^{K} x^D_k - x^D \right] + p_s \left[ x^S - \sum_{k=1}^{K} x^S_k \right]
\]

\[
= c x^D - g x^S - \frac{1}{2} x^S G x^S - \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k + \sum_{k=1}^{K} y_k x^S_k - \sum_{k=1}^{K} y_k A_k x^D_k + p^D \sum_{k=1}^{K} x^D_k - p^D x^D + p_s x^S - p_s \sum_{k=1}^{K} x^S_k
\]

Now we proceed to replace the terms in equations (9)-(12) starting from the third line of the Lagrange function. The replaced terms are indicated by square brackets:

\[
L = c x^D - (g + \frac{1}{2} G x^S) x^S - \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k + \sum_{k=1}^{K} y_k [x^S_k - A_k x^D_k] + p^D \left[ \sum_{k=1}^{K} x^D_k - x^D \right] + p_s \left[ x^S - \sum_{k=1}^{K} x^S_k \right]
\]

\[
= c x^D - g x^S - \frac{1}{2} x^S G x^S - \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k + \sum_{k=1}^{K} y_k x^S_k - \sum_{k=1}^{K} y_k A_k x^D_k + p^D \sum_{k=1}^{K} x^D_k - p^D x^D + p_s x^S - p_s \sum_{k=1}^{K} x^S_k
\]

\[
= [x^D p^D] - g x^S - \frac{1}{2} x^S G x^S - \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k + \left[ p_s \sum_{k=1}^{K} x^S_k + \sum_{k=1}^{K} x^S_k G x^S_k \right] - \sum_{k=1}^{K} y_k A_k x^D_k + \sum_{k=1}^{K} y_k A_k x^D_k
\]

\[
- p^D x^D + \left[ x^S g + x^S G x^S \right] - p_s \sum_{k=1}^{K} x^S_k
\]

\[
= \frac{1}{2} x^S G x^S + \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k
\]

Finally, the dual specification of the comprehensive auxiliary model of the Cournot-Nash oligopsony industry is stated, with all nonnegative variables, as

Dual

\[
\min AUX2 = \frac{1}{2} x^S G x^S + \frac{1}{2} \sum_{k=1}^{K} x^S_k G x^S_k
\]

subject to

\[
MC \geq MR
\]

\[
p^D \geq c
\] (13)  
\[
(g + G x^S) \geq p_s
\] (14)  
\[
A_k' y_k \geq p^D
\] (15)  
\[
G x^S_k + p_s \geq y_k
\] (16)

Dual constraints (13) and (15) require that every oligopsony firm (that is, price takers on the final commodity market) operate its plant up to the equality of marginal cost across the oligopsony industry.
This is seen by assuming that each firm will produce a positive quantity of the final commodity, that is, \( x_k^D > 0 \). It follows (by complementary slackness) that \( A_k' y_k = c, \ k = 1, \ldots, K \). Dual constraints (14) and (16) require that the marginal valuation of the input purchased by each oligopsony firm, and used in the production of the final commodity, be not greater than the supply price plus the market power of each firm. With the assumption that each oligopsony firm will purchase a positive quantity of the single input, \( x_k^S > 0 \), and by complementary slackness, it follows that \( y_k = p_s + Gx_k^S \).

**Meaning of the two auxiliary objective functions.**
At the optimal solution, the primal objective function is equal to the dual objective function, that is

\[
AUX = cx^D - \left[ g + \frac{1}{2} Gx^S \right] x^S - \frac{1}{2} \sum_{k=1}^{K} x_k^S Gx_k^S = \frac{1}{2} x^S Gx^S + \frac{1}{2} \sum_{k=1}^{K} x_k^S Gx_k^S = AUX_2
\]

Reorganizing terms, the following economic expressions are obtained:

\[
cx^D - \left[ g + Gx^S \right] x^S = \sum_{k=1}^{K} x_k^S Gx_k^S
\]

\[
\pi = p^D x^D - p_s x^S = \sum_{k=1}^{K} x_k^S Gx_k^S = \text{Total market power}
\]

Hence, in a Cournot-Nash equilibrium model, while the oligopsony firms operate to maximize the industry profit, there is a countervailing entity whose objective is to minimize the total amount of market power. This entity is Congress and the governmental agencies in charge of applying anti-trust laws.

**Example 1. Duopsony.** Let us consider two oligopsony firms and the corresponding dual constraints (14) and (16) dealing with the valuation of the purchased (and used) input. By combining (constraints (14) and (16) and assuming positive quantity of the purchased input we can write

\[
y_k = p_s + Gx_k^S = (g + Gx^S) + Gx_k^S = g + G(x_k^S + x_k^S) + Gx_k^S.
\]

Therefore, the two explicit valuation relations are

\[
y_1 = g + 2Gx_1^S + Gx_2^S = p_s + Gx_1^S \quad (17)
\]

\[
y_2 = g + Gx_1^S + 2Gx_2^S = p_s + Gx_2^S \quad (18)
\]

with total valuations equal to

\[
x_1^S y_1 = x_1^S g + 2x_1^S Gx_1^S + x_1^S Gx_2^S = x_1^S p_s + x_1^S Gx_1^S \quad (19)
\]

\[
x_2^S y_2 = x_2^S g + x_2^S Gx_1^S + 2x_2^S Gx_2^S = x_2^S p_s + x_2^S Gx_2^S \quad (20)
\]

These relations are reproduced in figure 1 and figure 2.

Figure 1 reproduces the diagrams of two oligopsony firms in an overlapping manner. This shows that firm 1 fulfills the requirement

\[
y_1 = g + 2Gx_1^S + Gx_2^S = p_s + Gx_1^S
\]

and similarly firm 2

\[
y_2 = g + Gx_1^S + 2Gx_2^S = p_s + Gx_2^S
\]

Figure 2 shows the same two firms placed side-by-side. This second figure allows to see clearly the price formation according to the inverse supply function \( p_s = g + Gx^S \). The two diagrams show the...
profit of the individual oligopsony firms and the marginal valuation of the purchased quantity of input that was used in the production of some final commodity.

Figure 1. Duopsony. Firms are overlapping

Figure 2. Duposony. Firms are side-by-side
Example 2. Three oligopsony firms. Oligopsony firms purchase a factor of production and use it to produce outputs to sell on the final commodity markets. We assume that the final commodity markets are competitive. Hence the three firms act as price takers on these markets. We also assume that these three firms produce two final commodities with a linear technology involving the purchased input (say rubber) but also available limiting inputs such as machinery and labor. Here are the data:

\[
A_1 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.4 & 0.7 \end{bmatrix}
\]

\[
A_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 5 \\ 13 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 20 \\ 24 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 10 \\ 33 \end{bmatrix}
\]

Matrices \(A_1\) contain the technical coefficients to produce the final commodities using the purchased input. Matrices \(A_2\) contain the technical coefficients to produce the final commodities using available limiting inputs. The input supply vectors \(b_2\) deal only with the available limiting inputs (machinery and labor). Final commodity prices are \(c' = \begin{bmatrix} 220 & 300 \end{bmatrix}\). The supply function of the factor of production to be purchased by the three oligopsony firms is \(p_s = 10 + 3x^s = g + Gx^s\).

The primal comprehensive auxiliary model of this example is as follows (all nonnegative variables)

Primal \[
\text{max } AUX = c'x^D - (g + \frac{1}{2}Gx^s)x^s - \frac{1}{2}G \sum_{k=1}^{K} (x_k^s)^2
\]

subject to

\[
D \leq S
\]

\[
A_1 x_k^D \leq x_k^s
\]

\[
A_2 x_k^D \leq b_2
\]

\[
\sum x_k^D = x^D
\]

\[
\sum x_k^s = x^S
\]

The dual specification can be patterned from the dual model (13)-(16)

Dual \[
\text{min } AUX = \frac{1}{2} x^S Gx^s + \frac{1}{2} K \sum_{k=1}^{K} x_k^s Gx_k^s + \sum_{k=1}^{K} b_2' y_2
\]

subject to

\[
MC \geq MR
\]

\[
p^D \geq c
\]

\[
(g + Gx^s) \geq p_s
\]

\[
A_1' y_1 + A_2' y_2 \geq p^D
\]

\[
Gx^s + p_s \geq y_1
\]

with all nonnegative variables.

Therefore, the total profit, \(\pi_o\), of the oligopsony industry is

\[
\pi_o = TR - TC_{pp} = c'x^D - \left( p_s x^s + \sum_{k=1}^{K} y_2 b_2 \right) = \sum_{k=1}^{K} x_k^s Gx_k^s.
\]
Oligopsony Cartel (Monopsony)

Oligopsony firms tend to collude and form a cartel, although the success of the cartel is often ephemeral. Firms collude in order to increase oligopsony profit. To do so they act (collectively) as a monopsony and wish to reduce the total quantity of purchased input that determines the price of that factor of production (via the supply function). In turn, they must agree (through their cartel council) to reduce also the quantity of the other limiting inputs allocated to production of the final commodities. Let us assume that the cartel council decided to reduce the input capacity of each oligopsony firm by 20 percent. The Primal cartel model, then, can be specified as

**Primal Cartel-monopsony:**

$$\max O_{\text{cartel}} \pi = c' x^D - (g + G x^S) x^S$$

subject to

1. $$D \leq S$$
2. $$A_1 x^D_k \leq x^S_k$$
3. $$A_2 x^D_k \leq 0.8 b_k$$
4. $$\sum_{k=1}^{3} x^D_k = x^D$$
5. $$\sum_{k=1}^{3} x^S_k = x^S$$

with all nonnegative variables. The dual of the cartel-monopospsy problem is

**Dual**

$$\min TC = 0.8 \sum_{k=1}^{K} y^2_k b_{2,k} + x^S G x^S$$

subject to

1. $$MC \geq MR$$
2. $$p^D \geq c$$
3. $$A_1 y^1_k + A_2 y^2_k \geq p^D$$
4. $$g + G x^S \geq p_r$$
5. $$p_r \geq y^1_k$$

with all nonnegative variables.

Therefore, the cartel-monopsony profit is

$$\pi_{c-M} = c' x^D - (g + G x^S) x^S - 0.8 \sum_{k=1}^{K} y^2_k b_{2,k} = x^S G x^S$$

The empirical results of this numerical Example 2 are as follows:

<table>
<thead>
<tr>
<th>Oligopsony industry</th>
<th>Cartel-monopsony</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry profit:</td>
<td>6,147.00</td>
</tr>
<tr>
<td>Firm 1 profit:</td>
<td>1,875.00</td>
</tr>
<tr>
<td>Firm 2 profit:</td>
<td>3,072.00</td>
</tr>
<tr>
<td>Firm 3 profit:</td>
<td>1,200.00</td>
</tr>
<tr>
<td>Oligopsony price:</td>
<td>241.00</td>
</tr>
<tr>
<td>Cartel profit:</td>
<td>11,383.68</td>
</tr>
<tr>
<td>Firm 1 profit:</td>
<td>3,696.00</td>
</tr>
<tr>
<td>Firm 2 profit:</td>
<td>4,730.88</td>
</tr>
<tr>
<td>Firm 3 profit:</td>
<td>2,956.80</td>
</tr>
<tr>
<td>Cartel price:</td>
<td>194.80</td>
</tr>
</tbody>
</table>

(GAMS file oligopsony.3firms_V2.gms)
As expected, a cartel-monopsony achieves a level of profit that is higher than the profit of the oligopsony industry with non-cooperative firms. The purchased input price and the purchased quantity of that input are higher for the oligopsony industry than for the cartel. Each firm evaluates the use of the purchased input at a higher level when non-cooperative behavior is in place.

Bertrand-Nash equilibrium in an oligopsony industry with differentiated products

The Bertrand-Nash oligopsony model is the mirror image of the Bertrand-Nash oligopoly model. In this market structure, oligopsony firms face supply markets for similar but slightly (or more) differentiated products. Oligopsony firms regard these commodities as inputs for their plants that produce commodity outputs to be sold on markets of various complexity. To keep the analysis as simple as possible, we will assume that output markets (final commodity markets), where Bertrand oligopsony firms sell their outputs, will be competitive markets. That is, Bertrand firms are price takers on their output markets. Bertrand oligopsony firms choose prices of the factor of production to offer on the supply markets. Economic agents who supply the quantity of factor of production will decide how much to offer at those particular prices and to which oligopsony firm. The crucial piece of information for each Bertrand oligopsony firm is an estimated supply function of the factor of production that includes also the prices of all rival Bertrand firms. The crucial difference with respect to the Cournot-Nash quantity model is that in the Bertrand-Nash price model each oligopsony firm develops (“owns”) its supply function for the factor of production to be purchased.

We assume that there are $K$ Bertrand oligopsony firms with the following differentiable supply functions

$$ q^S_i = f_i(p_i, p_j), \quad i = 1, \ldots, K, j = 1, \ldots, K, \quad i \neq j $$

where $q^S_i$ is the quantity of commodity input purchased by the $i$th Bertrand firm; $p_i$ is the price offered by the $i$th Bertrand firm; $p_j, j \neq i$, $j = 1, \ldots, K$ is an estimate made by the $i$th Bertrand firm of the price offered by the other $(K - 1)$ oligopsony firms. Each oligopsony firm uses proprietary technology $A_i$ to
transform the purchased input into an output (or more than one output) to sell on a competitive market (markets) at the given constant price $c$. Each oligopsony firm wishes to maximize profit.

**Decision model of the $i$th Bertrand oligopsony firm**

$$\text{max } \pi_i = TR_i - TC_{pp,i} = cx_i^D - p_iq_i^S$$

subject to

$$A_i x_i^D \leq q_i^S \quad \text{technology relations}$$

$$q_i^S = f_i(p_i, p_j) \quad \text{supply function}$$

with all nonnegative variables. The “solution” of this individual firm model requires the development of the corresponding KKT conditions

Lagrange function

$$L_k = cx_i^D - p_iq_i^S + y_i[q_i^S - A_i x_i^D] + \lambda_i[f_i(p_i, p_j) - q_i^S]$$

Relevant KKT conditions

$$\frac{\partial L_i}{\partial x_i^D} = c - A_i y_i \leq 0$$

$$\frac{\partial L_i}{\partial p_i} = -q_i^S - p_i \frac{\partial q_i^S}{\partial p_i} + y_i \frac{\partial q_i^S}{\partial p_i} + \lambda_i \frac{\partial f_i}{\partial p_i} - \lambda_i \frac{\partial q_i^S}{\partial p_i} \leq 0$$

$$= -q_i^S - p_i \frac{\partial q_i^S}{\partial p_i} + y_i \frac{\partial q_i^S}{\partial p_i} \leq 0 \quad \text{because } \frac{\partial f_i}{\partial p_i} = \frac{\partial q_i^S}{\partial p_i}$$

$$\frac{\partial L_i}{\partial q_i^S} = -p_i + y_i - \lambda_i \leq 0 \quad \rightarrow \quad \lambda_i = y_i - p_i \quad \text{measure of market power}$$

Assuming that $p_i > 0$, the second KKT condition can be re-written as

$$-f_i(p_i, p_j) - p_i \frac{\partial q_i^S}{\partial p_i} + y_i \frac{\partial q_i^S}{\partial p_i} = 0$$

and, by invoking the implicit function theorem (IFT), we can solve it for $p_i$ in terms of $p_j$ and $y_i$, that is, we can obtain what is called the **best-response function** of the $i$th oligopsony firm:

**best-response function** of $i$th firm

$$p_i = g_i(p_j, y_i).$$

The same procedure can be applied to the $j$th oligopsony firm to obtain its

**best-response function** of $j$th firm

$$p_j = g_j(p_i, y_j).$$

The solution (price equilibrium) of the system of best-response functions of all oligopsony firms requires knowledge of the shadow price valuation, $y_i$ and $y_j$, of the purchased input. That is, after the quantity of input was purchased on the supply market, it must be used in the production of commodities to be sold on the final commodity market. It is this (profitable) use of the purchased input that will attribute to it the best possible valuation.

**Example 3. Michelin and Firestone**

To illustrate the process of obtaining the set of equilibrium prices of an oligopsony industry described above we choose to study a duopsony market between Michelin and Firestone, two firms buying
natural rubber to produce tires. The natural rubber market is dominated by several oligopsony firms but to make this example rather manageable we will deal with Michelin (M) and Firestone (F) only. The respective supply functions are

\[ q^s_M = 150 - 1.5 p_F + 5 p_M \]
\[ q^s_F = 180 + 4 p_F - 2 p_M \]

We assume that Michelin and Firestone produce one output that they sell on a competitive market at the given constant price of \( c = 200 \). The technology to produce this output is \( A_M = 0.660343 \) and \( A_F = 0.701094 \), respectively. The best-response function of Michelin is derived from the following primal model

\[ \pi_M = TR_M - TC_{pp,M} = cx^D_M - p_M q^s_M \]
subject to

\[ A_M x^D_M \leq q^s_M \]
\[ q^s_M = 150 - 1.5 p_F + 5 p_M \]

with all nonnegative variables.

Lagrange function

\[ L_M = cx^D_M - p_M q^s_M + y_M [q^s_M - A_M x^s_M] + \lambda_M [150 - 1.5 p_F + 5 p_M - q^s_M] \]

Relevant KKT conditions:

\[ \frac{\partial L_M}{\partial x^D_M} = c - A_M y_M \leq 0 \]
\[ \frac{\partial L_M}{\partial p_M} = -q^s_M - p_M \frac{\partial q^s_M}{\partial p_M} + y_M \frac{\partial q^s_M}{\partial p_M} + \lambda_M 5 - \lambda_M 5 \frac{\partial q^s_M}{\partial p_M} \leq 0 \]
\[ = -[150 - 1.5 p_F + 5 p_M] - p_M 5 + y_M 5 + \lambda_M 5 - \lambda_M 5 \leq 0 \]
\[ = -150 + 1.5 p_F - 10 p_M + 5 y_M \leq 0 \]
\[ \frac{\partial L_M}{\partial q^s_M} = -p_M + y_M - \lambda_M \leq 0 \]

The best-response function for Michelin is obtained from the second KKT condition assuming that the firm will purchase a positive quantity of rubber, \( q^s_M > 0 \). Hence,

Michelin best-response function

\[ p_M = -\frac{150}{10} + 0.15 p_F + 0.5 y_M \]
\[ = -15 + 0.15 p_F + 0.5 y_M \]

By similar process, the Firestone best-response function can be stated as

Firestone best-response function

\[ p_F = -\frac{180}{8} + 0.25 p_M + 0.5 y_F = -22.25 + 0.25 p_M + 0.5 y_F \]

We note again that the best-response function of Bertrand oligopsony firms depend on the marginal valuation of the purchased input, \( y_M \) and \( y_F \). The industry equilibrium prices, therefore, must be computed from a comprehensive model such as

\[ \max \pi = \pi_M + \pi_F = (cx^D_M - p_M q^s_M) + (cx^D_F - p_F q^s_F) \]
subject to

\[ A_M x^D_M \leq q^s_M \]
\[ A_F x^D_F \leq q^s_F \]
\[ p_M = -15 + 0.15 p_F + 0.5 y_M \]
\[ p_F = -22.25 + 0.25 p_M + 0.5 y_F \]
\[ q_M^* = 150 - 1.5 p_F + 5 p_M \]
\[ q_F^* = 180 + 4 p_F - 2 p_M \]
\[ A_M y_M \geq c \]
\[ A_F y_F \geq c \]

with all nonnegative variables.

The empirical results of Example 3 are as follows:
(GAMS file Bertrand_Oligopsony_Michelin_Firestone.gms)

**Michelin**

| Profit: 101,331.20 |
| Price: 160.51 |
| Input quantity: 711.80 |
| Marginal expenditure: 302.87 |
| Market power: 142.36 |
| Output quantity: 1,077.92 |
| Market share: 0.5869 |

**Firestone**

| Profit: 62,505.64 |
| Price: 160.51 |
| Input quantity: 501.02 |
| Marginal expenditure: 285.27 |
| Market power: 124.76 |
| Output quantity: 714.63 |
| Market share: 0.4131 |

Note that equilibrium prices are identical (this is due to a wise selection of technical coefficients). But the different supply functions cause the advantage of Michelin over Firestone.

Firestone wishes to increase its tire market share (and its profit). To achieve this goal the firm invests in new technology that reduces the technical coefficient to produce its output quantity from 0.701094 to 0.65, close to the technical coefficient of Michelin. With this result:

**Michelin**

| Profit: 98,859.03 |
| Price: 162.26 |
| Input quantity: 703.06 |
| Marginal expenditure: 302.87 |
| Market power: 140.61 |
| Output quantity: 1,077.92 |
| Market share: 0.5637 |

**Firestone**

| Profit: 73,745.66 |
| Price: 172.16 |
| Input quantity: 544.12 |
| Marginal expenditure: 307.69 |
| Market power: 135.53 |
| Output quantity: 714.63 |
| Market share: 0.4363 |

The reduction of Firestone’s technical coefficient to a level very close to that of Michelin has caused an increase in Firestone’s market share (for final commodity tires) of 5.3%. Its profit increases by 15% and its market power by 8%. Michelin’s market position deteriorates (slightly).
Bertrand Cartel under oligopsony market

Oligopsony firms facing differentiated products may “tacitly” collude, that is, they tacitly “agree”, say, to let technology and advertising expenses as they are. The Bertrand cartel model, then, takes on the following specification:

Primal

\[
\max \pi_c = \sum_{i=1}^{K} \pi_i = \sum_{i=1}^{K} \left( c x_i p_i - p_i q_i s \right)
\]

subject to

\[
D \leq S \quad \text{dual variables}
\]

\[
A_i x_i p_i \leq q_i s
\]

\[
q_i s \leq f_i(p_i, p_j)
\]

with all nonnegative variables. Notice that this cartel model is similar to the Bertrand oligopsony model discussed above without the best-response function of the individual oligopsony firms.

The dual specification passes, as usual, through the Lagrange function

\[
L = \sum_{i=1}^{K} \left( c x_i p_i - p_i q_i s \right) + \sum_{i=1}^{K} q_i s \left( q_i s - A_i x_i p_i \right) + \sum_{i=1}^{K} \lambda_i \left[ f_i(p_i, p_j) - q_i s \right]
\]

with the following relevant KKT conditions

\[
\frac{\partial L}{\partial x_i} = c - A_i y_i \leq 0
\]

\[
\frac{\partial L}{\partial p_i} = -q_i s - p_i \frac{\partial q_i s}{\partial p_i} + \lambda_i \frac{\partial f_i}{\partial p_i} + \lambda_j \frac{\partial f_j}{\partial p_i} \leq 0
\]

\[
\frac{\partial L}{\partial q_i s} = -p_i + y_i - \lambda_i \leq 0
\]

Relevant complementary slackness conditions are:

\[
x_i s \frac{\partial L}{\partial x_i s} = x_i s c - x_i s A_i y_i = 0 \quad \rightarrow \quad x_i s c = x_i s A_i y_i \quad (1)
\]

\[
q_i s \frac{\partial L}{\partial q_i s} = -q_i s p_i + q_i s y_i - q_i s \lambda_i = 0 \quad \rightarrow \quad q_i s y_i = q_i s p_i + q_i s \lambda_i \quad (2)
\]

Simplification of the Lagrange function using complementary slackness conditions (1) and (2) results in the dual objective function:

\[
L = \sum_{i=1}^{K} \left( c x_i p_i - p_i q_i s \right) + \sum_{i=1}^{K} q_i s \left( q_i s - A_i x_i p_i \right) + \sum_{i=1}^{K} \lambda_i \left[ f_i(p_i, p_j) - q_i s \right]
\]

\[
= \sum_{i=1}^{K} x_i s A_i y_i - \sum_{i=1}^{K} p_i q_i s + \sum_{i=1}^{K} q_i s p_i + \sum_{i=1}^{K} q_i s \lambda_i - \sum_{i=1}^{K} y_i A_i x_i p_i + \sum_{i=1}^{K} \lambda_i f_i(p_i, p_j) - \sum_{i=1}^{K} \lambda_i q_i s
\]

\[
= \sum_{i=1}^{K} \lambda_i f_i(p_i, p_j) = \sum_{i=1}^{K} \lambda_i q_i s
\]

Therefore, the dual model of the Bertrand-Nash cartel is as follows:

Dual

\[
\min \text{TMP} = \sum_{i=1}^{K} \lambda_i f_i(p_i, p_j)
\]

subject to

\[
MC \geq MR \quad \text{dual variables}
\]
\[
\begin{align*}
A_i y_i & \geq c \\
q_i^s + p_i \frac{\partial q_i^s}{\partial p_i} & \geq \sum_{i=1}^{\kappa} \lambda_i \frac{\partial f_i}{\partial p_i} \\
p_i + \lambda_i & \geq y_i
\end{align*}
\]

with all nonnegative variables.

The meaning of the dual problem begins with the objective function. TMP stands for Total Market Power. Assuming that each oligopsony firm purchases a positive quantity of the input, \( q_i^s > 0 \), the third dual constraint will be binding, that is, \( p_i + \lambda_i = y_i \) by complementary slackness condition. And therefore \( \lambda_i = y_i - p_i \) stands for a measure of the oligopsony market power (under tacit collusion) of the \( i \)th firm per unit of input under cartel agreement. In the microeconomics literature, the symbol \( y_i \) is referred to as “marginal expenditure.” Hence, the difference between the marginal expenditure and the supply price measures the market power per unit of purchased input. Total market power, therefore, is the product of the quantity of purchased input, \( f_i(p_i, p_j) \), times the market power per unit of input summed over all oligopsony firms (under collusion).

Who wants to minimize total market power? We have already mentioned that in 1890 Congress passed the Sherman Antitrust Act with the intention of “protecting trade and commerce against unlawful restraints and monopolies.” The US government created agencies in charge of administering antitrust laws. Whether the Sherman Act worked (works) or not is another story. What is of extreme interest (to me) is that the dual of maximizing profit of an oligopsony industry is minimizing its market power. This conclusion has no political overtones.

An alternative interpretation of the dual problem may be told as in previous dual interpretations. Suppose that a group of billionaires (Bezos, Baffett, JP Morgan, BBM) decides to buy ALL the oligopsony firms who deal in cocoa. They will have to reimburse those firms of the profit they could make by not selling. Hence, BBM will want to minimize the expenditure to buy the cocoa markets which is equivalent to minimize the total market power of the existing oligopsony firms. The dual constraints place a lower limit on the bidding prices of the dual BBM group. In similar fashion, the reduction of market power by the government is subject to lower limits of profitability.

**Example 4. Michelin and Firestone under cartel**

We assume that Michelin and Firestone produce one output that they sell on a competitive market at the given constant price of \( c = 200 \). The technology to produce this output is the original technology, namely \( A_M = 0.660343 \) and \( A_F = 0.701094 \), respectively. The primal model of the Bertrand cartel is

Primal

\[
\begin{align*}
\text{max} \pi_C = & \pi_M + \pi_F = (200 x_M^D - p_M q_M^s) + (200 x_F^D - p_F q_F^s) \\
\text{subject to} & \quad D \leq S \\
A_M x_M^D & \leq q_M^s \\
A_F x_F^D & \leq q_F^s \\
q_M^s & \leq 150 - 1.5 p_F + 5 p_M \\
q_F^s & \leq 180 + 4 p_F - 2 p_M \\
\text{and all nonnegative variables.}
\end{align*}
\]

The dual specification is as follows:
Dual

\[ \min TMP = \lambda_M (150 + 5 p_M - 1.5 p_F) + \lambda_F (180 - 0.2 p_M + 4 p_F) \]

subject to

\[ MC \geq MR \]

\[ A_M y_M \geq 200 \]

\[ A_F y_F \geq 200 \]

\[ q_M^s + 5 p_M \geq 5 \lambda_M - 2 \lambda_F \]

\[ q_F^s + 4 p_F \geq -1.5 \lambda_M + 4 \lambda_F \]

\[ p_M + \lambda_M \geq y_M \]

\[ p_F + \lambda_F \geq y_F \]

and all nonnegative variables.

The empirical results of Example 4 (Bertrand cartel) are as follows:

(GAMS file Bertrand_Oligopoly_Michelin_Firestone.gms)

**Michelin**
- Profit: 173,719.40
- Price: 119.92
- Input quantity: 575.87
- Marginal expenditure: 302.87
- Market power: 182.96
- Output quantity: 872.07
- Market share: 0.5881

**Firestone**
- Profit: 68,360.61
- Price: 115.81
- Input quantity: 403.40
- Marginal expenditure: 285.27
- Market power: 169.46
- Output quantity: 575.39
- Market share: 0.4119

A comparison with the results of the oligopoly example 3 (with original technical coefficients) indicates that a “tacit” collusion substantially improves the profit of each firm. Supply prices are lower under cartel and market power increases. Market shares (of the final commodity) remain practically the same.