Mining and quasi-option value*

Christopher Costello† and Charles D. Kolstad‡

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Abstract

We study the timing-of-extraction problem facing a decentralized mine owner when extraction entails environmental damage. As expected, when the environmental damage from mining is known, the socially optimal timing will depend on the magnitude of the damage relative to these costs in the rest of the world. But when environmental damage is uncertain, and these costs are revealed over time, a quasi-option value arises. We show that even if *expected* environmental costs are identical to those in the rest of the world, *any* uncertainty over these costs will cause the social planner to optimally delay mining until better information arrives. We show conditions under which it is optimal to postpone the mining decision indefinitely, and conditions when it is optimal to postpone only for a finite duration. The analysis leverages a crucial observation that distinguishes the non-renewable resource problem from the traditional quasi-option value framework. In the traditional framework, the presence of an irreversible investment and uncertainty can help *nudge* the decision maker to preserve an option, but it by no means implies the decision maker should always preserve the option. In contrast, for a non-renewable resource model, the arbitrage condition underpinning the Hotelling rule suggests that in the absence of uncertainty, the marginal mine owner is completely indifferent between mining immediately and at any point in the future. Thus, for our problem, *any* uncertainty will convince her to defer the mining decision.

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1 Introduction

A seminal result in environmental economics from the last half century is the identification of quasi-option value as a potentially important driver to perserve natural environments (Arrow and Fisher 1974; Henry 1974). The term 'quasi-option value' was used by Arrow and Fisher

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†4410 Bren Hall, UC Santa Barbara and NBER, costello@bren.ucsb.edu
‡Stanford University, UCSB, RFF and NBER, ckolstad@stanford.edu

(1974) to distinguish it from 'option value,' introduced by Weisbrod (1964) to capture the effect of uncertain demand for natural environments (an apparently closely related concept). The quasi-option value concept arose in part out of a proposal to dam the Snake River in Idaho, irreversibly destroying many aspects of the natural environment. Cost-benefit analysis, even taking into account uncertainty, suggested damming the river yielded positive expected net benefits. Yet the fact that once dammed, this wild and scenic river would be lost for centuries seemed to make no difference to the analysis. There seemed to be a cost associated with taking irreversible actions that was missing.

This paper concerns exhaustible resources and quasi-option value, an application that has received scant attention (one exception is Hoel (1978)). What makes the case of an exhaustible resource interesting and different is that the resource in-situ is expected to increase in value over time, according to the Hotelling rule; this is not generally true for canonical applications such as environmental values threatened by a dam or other development. Although the Hotelling rule has many variants, it is fundamentally an intertemporal arbitrage condition which, at the market level, implies indifference between development now or in the future for the marginal mine. This intertemporal indifference suggests that even a small value for the quasi-option value may be enough to tip the optimal development decision of a marginal mine towards delay. The interaction between this principle and quasi-option value can generate fundamentally different results on the development of an exhaustible resource with uncertain, but ultimately knowable, benefits of preservation.

As with many theoretical inquiries in economics, our problem is motivated by a real-world policy dilemma – the potential development of the Pebble Gold Mine in Alaska.[1] Although we do not take a position on that development, this paper does concern decisions by public officials about whether to proceed with immediate development of an exhaustible resource or to postpone development until uncertain environmental costs are better known - the situation Alaskan officials have found themselves in for the case of the Pebble Mine. Other equally-compelling contemporary examples include whether to proceed with hydraulic fracturing ("fracking") for natural gas, or whether to engage in deep seabed mining for copper, gold, or rare earth metals. Each of these examples concern real-world policy questions that pit the benefits of extraction against the uncertain environmental costs of mining. One contribution of this paper is to show that a simple cost benefit analysis, which compares expected benefits to expected costs, will often get the wrong answer. After all, deciding to postpone mining until better information becomes available forfeits none of the resource under ground.

For the remainder of this paper, we remain agnostic about the particular application. We consider the case of a generic mine owner making a decision on when to develop and exhaust

[1] A copper, gold and molybdenum mine has been proposed for the Bristol Bay region of Southwest Alaska - the "Pebble Project" (Parker et al. 2008; Wardropp Engineering 2011). The deposit is one of the largest and richest of its kind in the world. The area is also home to an important salmon fishery and significant ecological resource. Although there is significant uncertainty regarding the risk to the fishery and ecosystems from the mine, the risk was considered significant enough for the US EPA to invoke the Clean Water Act to significantly restrict the development of the mine (Environmental Protection Agency 2014). As of the writing of this article, the project now seems to be in serious if not fatal trouble. Nevertheless, the project is still an excellent example of a mine that appears to be privately profitable but with potentially significant and highly uncertain social (environmental) costs. See also Narula (2014) for a perspective on the politics of the project.

a mine in a context where environmental costs are uncertain,[2] though the uncertainty can be resolved at some point in the future. We contrast the case of a naïve mine owner, who ignores the fact that information will be obtained in the future, with a sophisticated mine owner who utilizes all available information. Since environmental costs are usually external costs, there are two equivalent ways of viewing this problem. One is that the environmental costs have been internalized by the mine owner and thus she bases her decisions on both private and external costs. Alternatively, the decision maker could be viewed as a social planner making a decision about when to permit a mine to open. Mathematically and economically, these conceptual approaches are identical. If all of the uncertainty is with regard to external environmental costs, then a "naïve decision" would correspond to the efficient private decision, excluding social costs. The "sophisticated decision" would correspond to the socially efficient decision, including uncertain external costs. This maps well to the Pebble Mine example that motivated this work.

Our analysis relies fundamentally on the concept of quasi-option value which has its roots in capital theory with irreversible investment (Arrow 1968; Arrow and Kurz 1970) – if there is uncertainty about the value of the natural environment and time will help reduce that uncertainty, then there is a value associated with postponing irreversible development, a value that should be reflected in decision-making about whether to develop immediately or not. Over the years there has been some confusion in the literature about the distinction between option value and quasi-option value. Grappling with the theoretical distinction was muddied further by the fact that financial economics has yet another concept of option value, developed for application to natural resource economics by Dixit and Pindyck (1994). Fortunately, a number of authors, particularly Hanemann (1989), Mensink and Requate (2005), and Traeger (2014), have provided unifying syntheses of and clarity among these three distinct concepts. Furthermore, terminology appears to have evolved to distinguish between the 'Arrow-Fisher-Henry' quasi-option value and the 'Dixit-Pindyck' option value. For clarity in this paper, we will use the term 'Arrow-Fisher-Henry quasi-option value' interchangeably with 'quasi-option value.'

Uncertainty is of fundamental importance to our results. But uncertainty alone is insufficient; there also must be learning about the uncertainty – information must be acquired. As clearly pointed out by Hanemann (1989), quasi-option value is the value of the information ultimately received, conditional on postponing irreversible development. Thus quasi-option value is related to the value of information, but does not require risk aversion.[3]

[2]While we have framed the problem as a decision maker who is uncertain about environmental cost, the story applies equally to any uncertainty over extraction costs that will be revealed over time.

[3]Traeger (2014) provides a clear synthesis of option and quasi-option value. He sets up a simple two period model with learning and distinguishes three types of decision-makers regarding irreversible development: (1) a sophisticated decision-maker (s) who anticipates learning and may postpone part or all of her decision until after information is acquired; (2) a less sophisticated decision-maker (p) who makes all decisions ex ante, before uncertainty is resolved, but allows the possibility to develop in either period; and (3) a pure naïve decision maker (n) who makes an all or nothing decision in the first period based on expectations (depending on how the problem is set up, n and p may be equivalent). The difference between the value of development for s vs. p is the quasi-option value; the difference for p vs n is the simple option value (unrelated to learning); and the difference for s vs. n is the full value of sophistication (which may be greater than the quasi-option value, depending on whether the simple option value is non-zero, which depends on the nature of payoffs and uncertainty).

In the next section we develop a simple model of the timing of extraction of an exhaustible resource with uncertain costs where uncertainty can be resolved over time. We assume external costs have been internalized and contrast naïve decision-making (where we assume that no information will be acquired at a later date) with sophisticated (i.e. socially optimal) decision-making, which takes into account the full structure of the problem. In Section 3 we introduce a mechanism (a tax on extraction) that induces a naïve decision-maker to behave as a sophisticated decision-maker. We discuss alternative market specifications in Section 4 and conclude in Section 5.

2 Model of Social Decisions

The problem we consider is that of a single mine owned by a small producer, denoted with subscript A, operating as a price-taker within a global market for the non-renewable resource produced by the mine. To keep things simple, assume producer A owns a single unit of the resource and her problem is to determine when to deplete her mine (extraction/depletion occurs all at once when it does occur). Producer A faces extraction cost c_A, which includes all external costs. Thus any environmental costs are included in costs faced by the producer (though these costs may be uncertain). The world price at time t is $P(t)$, the interest rate is r, the current period is $t = 0$, and extraction of a unit of the resource in the rest of the world entails cost c_w (this is the cost the market sees, which may be the private cost of extraction or a social cost with internalized external costs). Assuming a competitive global market with fixed reserves, the price path follows Hotelling's rule over time:

$$P(t) = e^{rt}P(0) - c_w(e^{rt} - 1) \tag{1}$$

If $c_w = 0$, then the global price of the resource rises at the rate of interest, r. But if $c_w > 0$, then rent rises at the rate of interest and thus the global price of the resource rises more slowly than the rate of interest.

Equation 1 together with total global reserves R and global demand (assumed to have a choke price $\bar{p} > c_w$) will determine the initial price, $P(0)$, the price path, $P(t)$, and the exhaustion date T, all of which are exogenous to owner A.

2.1 Known cost, c_A

We begin with the simplest case in which A's cost, c_A, is known and ask: When will owner A extract her resource? Since the price path $P(t)$ is exogenous to owner A, she can take it as given. If she decides to mine at date s, her present value profit is:

$$\pi_A(s) = e^{-rs}\left[P(s) - c_A\right] = P(0) - c_w + e^{-rs}(c_w - c_A) \tag{2}$$

This function is increasing in s if $c_A > c_w$ and decreasing in s if $c_A < c_w$. So the optimal mining-time for owner A, t_A^*, is given by:

$$t_A^* = \begin{cases} 0 & \text{if } c_A < c_w \\ \text{any } t \in [0, T] & \text{if } c_A = c_w \\ T & \text{if } c_w < c_A \leq \bar{p}) \\ \text{Never} & \text{if } c_A > \bar{p} \end{cases} \tag{3}$$

When A's cost is known, the intuition behind her timing decision is as follows: When the cost in region A is small relative to the rest of the world, it pays to extract immediately because the rent in region A is rising slower than the rate of interest. When the cost in region A is large but less than the choke price, the opposite holds: although extraction would be optimal, rent is rising faster than the rate of interest so it is optimal to defer extraction until the last moment, which is T. And when the cost is sufficiently large, so that even the highest price possible ($\bar{p}$) would not justify the cost, the resource should be left in the ground indefinitely. We next analyze how uncertainty over A's cost affects her timing decisions.

2.2 Uncertain cost, c_A

Suppose instead that c_A is uncertain and its probability distribution is given by $f(c_A)$. Denote the expected value of c_A by $\bar{c}_A \equiv E[c_A] = \int_{-\infty}^{\infty} c_A f(c_A) dc_A$ (the support of f could be finite or infinite). Following the quasi option value literature, we assume information revealing the true value of c_A will become available at some point in the future. We consider two types of decision makers: A naïve decision maker makes her timing decision ignoring the fact that information will be acquired in the future, and a sophisticated decision maker takes into account the knowledge that new information will arrive in the future. Another interpretation is that the naïve owner ignores environmental externalities (and so, even though new information on environmental costs will arrive, she will not make use of it) and a sophisticated owner fully accounts for all information regarding environmental externalities. We calculate the optimal time to mine (and associated expected payoff) for a naïve and a sophisticated owner A.

When owner A is naïve she ignores new information so the distribution over A will remain $f(c_A)$ forever.[4] In this case, owner A's expected payoff from a decision to mine at date s is given by:

$$E[\pi(s)] = \int_{-\infty}^{\infty} \left(P(0) - c_w + (c_w - c_A)e^{-rs} \right) f(c_A) dc_A \tag{4}$$

$$= P(0) - c_w + e^{-rs}(c_w - \bar{c}_A) \tag{5}$$

Equation 5 bears a striking resemblance to the deterministic payoff in Equation 2. In this case, uncertainty plays only a trivial role - the optimal mining decision depends on the expected value of c_A, but not on any other features of the distribution of c_A. In this case,

[4]Equivalently, we could assume that owner A learns the true value of A only after mining has occurred.

the optimal mining-time for owner A is given by:

$$t_A^* = \begin{cases} 0 & \text{if } \bar{c}_A < c_w \\ \text{any } t \in [0, T] & \text{if } \bar{c}_A = c_w \\ T & \text{if } c_w < \bar{c}_A \leq \bar{p}) \\ \text{Never} & \text{if } \bar{c}_A > \bar{p} \end{cases} \tag{6}$$

So the optimal timing decision under uncertainty (but without learning) is identical to the optimal timing decision under certainty. One simply replaces c_A (in the certainty case) with its expectation $\bar{c}_A$ (in the uncertainty case). Indeed, this seems to be how many real-world policy makers think about mining decisions on public lands: Do the expected benefits outweigh the expected costs?

But the naïve approach above is incomplete because such an owner ignores potentially useful information. We now add learning to the optimal mining timing under uncertainty. While many forms of learning are possible, we adopt a simple version where owner A learns her true cost parameter either upon mining or at some future date $\tau < T$, whichever occurs first. That she would learn her costs upon mining is not too far-fetched. For example, if the uncertain component of costs are the environmental costs, then those costs would be revealed after mining has taken place.[5] If she chooses to defer mining until after date τ, we assume that her costs will be revealed, with certainty, at date τ. This is meant as a heuristic that captures the idea that over time, exogenous information may be revealed that would help owner A identify her true costs. Again, following the environmental example, scientific information may accrue over time that narrows owner A's uncertainty over the true external cost.

The prospect of learning the true cost c_A prior to mining is an enticing one. If deferring the mining decision is not too costly, and provided that A might learn something useful (i.e. something that might change her mine-time), then she may wish to defer, at least until date τ. Whether this is the case depends on an interplay among (1) the *expected* cost, $\bar{c}_A$, (2) others' costs, c_w, (3) the date of information revelation τ, and (4) the shape of distribution $f(c_A)$. In this section we solve for this relationship and highlight its key features that affect the optimal timing decision. The optimal timing decision turns out to hinge on how A's expected cost compares to the actual cost of others ($\bar{c}_A \lesseqgtr c_w$) and the choke price. We examine each case in turn.

2.2.1 "Typical" expected cost: $\bar{c}_A = c_w$

We begin with the most natural starting point in which A's expected cost is equal to others' costs, so $\bar{c}_A = c_w$. Inspecting the result in Equation 6, it is tempting to think that if A's cost is expected to be the same as everyone else's costs, then A would be indifferent about when to mine. This initial logic is incorrect, which leads to our main result:

Proposition 1. *For uncertain cost, c_A, if $\bar{c}_A = c_w$, then it is always optimal to postpone mining until at least date τ.*

[5]For example, tailings from the Pebble Mine either do, or do not, compromise salmon populations and fracking either does, or does not, pollute drinking water.

Proof. Formal proofs reside in the Appendix. □

While the formal proof requires detailed analysis, the intuition behind this result is straightforward. The basic idea is to compare the expected payoff from mining prior to τ (which we call V_1) with the expected payoff from postponing until at least date τ (which we call V_2). If we commit to mining prior to τ, we obtain the expected payoff of the naïve mine owner (because no new information becomes available prior to τ). But the naïve mine owner obtains the same expected payoff regardless of when she mines (see Equation 6). So V_1 is identical to the expected payoff obtained when the mine owner *commits to mining at date* τ. And V_2 is the expected payoff from waiting until date τ and then deciding when (and whether) to mine. Naturally, when viewed this way, it is obviously better to maintain flexibility; i.e. to delay mining until at least date τ.

The flexibility is valuable for two separate reasons. First, if c_A turns out to be large, but not too large (this is revealed at date τ), then it will be optimal to mine at $T > \tau$, in which case $V_2 > V_1$. Second, if c_A turns out to be very large, then it will be optimal to never mine, in which case $V_2 > V_1$. The third possibility is that it is revealed, at date τ, that costs are low (in which case it would have been optimal to mine at date 0). But ex ante, committing to mine at date 0 has a payoff identical to committing to mine at date τ, so the two payoffs are equal in that case. Taken together, when $\bar{c}_A = c_w$ it is always optimal to postpone the mining decision until at least date τ. This simple, yet dramatic result seems to suggest that when A's expected cost is "typical," uncertainty over cost will always persuade her to delay the mining decision.

2.2.2 "High" expected cost: $\bar{c}_A > c_w$

When expected cost is higher than c_w, the result is strengthened further. As expected cost is raised, there is an incentive to push those costs farther into the future, which implies mining at a later date. Indeed, even a naïve mine owner chooses to postpone mining (until at least date τ) in this case (see Equation 6). The result is summarized as follows:

Proposition 2. *For uncertain cost, c_A, if $\bar{c}_A > c_w$, then it is* always *optimal to postpone mining until at least date τ.*

The logic underlying the proof is similar to that in Proposition 1. Here, the ex ante expected payoff from mining prior to τ is lower than the expected payoff from committing to mine at date τ. And the expected payoff from postponing the decision until date τ is strictly greater than the expected payoff of committing to mine at date τ. So postponing the decision until date τ is always preferred to mining prior to τ.

2.2.3 "Low" expected cost: $\bar{c}_A < c_w$

Propositions 1 and 2 reveal that provided A's expected cost is equal to or greater than the costs experienced by other mine owners, it is always optimal to postpone the mining decision until better information arrives. But what if A's expected cost is lower than others' costs? In that case, an interesting tension arises. When $\bar{c}_A$ is low, there is an incentive for the mine owner to extract immediately (see Equation 6); this implies that delaying until τ would require her to sacrifice some expected returns. Whether this force is sufficient to overcome the benefit of preserving flexibility turns out to depend in interesting ways on features of the problem. Our main result here is that even though a simple naïve cost benefit analysis suggests that it is optimal to mine immediately, it is often optimal, instead, to postpone mining until at least date τ. This result is summarized below:

Proposition 3. *For uncertain cost, c_A, if $\bar{c}_A < c_w$, then it may be optimal to postpone mining until at least date τ.*

The proof relies on carefully disentangling the tension between two factors and leveraging the observation that the mine owner would *never* want to mine in the open interval $(0, \tau)$. On the one hand, the fact that expected cost is low suggests that mining should commence immediately because the rent generated is more valuable as a standard investment (earning rate r) than it is in the ground. But on the other hand, there is a benefit to postponing the decision until at least date τ when the true cost c_A is revealed. Under this setup there is no benefit to postponing to some date $s < \tau$ because nothing can be learned, and the ex ante expected payoff is lower than it would be if mining took place at time 0. The benefit of postponing mining until at least τ should be clear: the true cost c_A might be relatively high, in which case it would be optimal to mine at date T, or it might be very high, in which case it will be optimal to never mine the resource. Neither of these options is available if A mines at date 0.

While Proposition 3 shows that it *might* be optimal to postpone mining until at least date τ, we would like to shed light on the factors that make this result more likely. This result is summarized as follows:

Corollary 1. *Under the conditions of Proposition 3, the payoff from postponing the mining decision (until τ) is increasing: (a) as τ becomes smaller and (b) with increasing uncertainty over $f(c_A)$.*[6]

These both accord with straightforward economic intuition. As the learning time (τ) is smaller, so is the cost of delay, so the relative payoff from postponing gets larger. And as the distribution over environmental cost ($f(c_A)$) becomes more spread, the possibility of learning something very useful is increased, so this also increases the value of delay.

3 A Mining Tax to Correct Naive Behavior

The analysis thus far uses the no arbitrage result of Hotelling to show that it may typically be the case that a social planner would find it optimal to delay mining a non-renewable resource until the environmental consequences of doing so have been revealed. Yet a myopic owner who either ignores environmental costs altogether, or at least ignores the possibility of learning about those costs, may wish to mine immediately. Here we examine a mechanism to induce efficient mining by a naïve owner. Hanemann (1989) re-formulates the quasi option value model in an intriguing manner. He shows that the quasi-option value can be thought

[6] We induce "increasing uncertainty" by following the definition of "increasing risk" (though not necessarily "increasing variance") by Rothschild and Stiglitz (1970).

of as an additional "tax" on development - if this tax were included as a cost in the first period, then a simple naïve cost benefit analysis would yield the efficient solution. Borrowing from Hanemann's insights, the purpose of this section is to derive the "mining tax," which we will denote by $\mathcal{Q}$, imposed on extraction in the current period, that would give rise to efficient (socially optimal) mining activity, even from a naïve owner.

It is important to underscore that this is not a tax in the conventional sense. If all of the costs are internalized to the firm then we cannot know if the firm will be myopic or sophisticated. Perhaps the more appropriate situation is where there are external social costs (such as environmental damages) and the goal is to internalize those costs to the firm. If only the expected damages are internalized (i.e., a naïve approach is taken to the external costs), then there is a need for an additional tax to take into account the quasi-option value. And that additional tax – necessary to correct naïve behavior – is what is developed here.

Suppose the naïve small mine owner analyzed above faces a tax, $\mathcal{Q}$, that must be paid to mine prior to date τ. The goal is to derive the value of $\mathcal{Q}$ that, if included as a cost of mining prior to τ, would give rise to the optimal mining time. Thus, we must carefully attend to the incentives of the naïve mine owner. In order to nudge such an owner to delay mining, when she would otherwise would find it desirable to mine, we must impose a sufficiently high penalty on premature mining. On the other hand, this penalty cannot be too large, or she will delay even when it is socially optimal to mine immediately. Let $V^*(s)$ be the expected present value (i.e., at date 0) of the deposit, to a naïve mine owner, who commits to *postponing the mining decision until at least date s*; define $V^*(0)$ as the expected value to a naïve mine owner of making the mining decision at date 0. We are interested in whether a naïve owner would prefer to mine at date 0 or delay until at least date τ. This decision simply depends on $V^*(0) \lesseqgtr V^*(\tau)$. Thus, if we *always* wanted to encourage such an owner to delay mining, imposing a tax on extraction prior to τ of at least $V^*(0) - V^*(\tau)$ would do the job.

However, we do not *always* want her to delay mining. We only want her to delay mining in cases in which it is efficient to delay mining. Let $\hat{V}(s)$ be the expected present value return, to a sophisticated mine owner, who commits to *postponing until at least date s*; define $\hat{V}(0)$ as the expected value of mining at date 0. Then, the efficient decision about whether to mine at date 0 or delay until at least τ will depend on $\hat{V}(0) \lesseqgtr \hat{V}(\tau)$. We only want the naïve mine owner to delay mining in cases in which $\hat{V}(0) < \hat{V}(\tau)$. Making use of the insights from Hanemann (1989), define the following tax:

$$\mathcal{Q} = (V^*(0) - V^*(\tau)) - \left(\hat{V}(0) - \hat{V}(\tau)\right) \tag{7}$$

If this tax is imposed on the naïve mine owner should she mine prior to τ, then such an owner would compare $V^*(0) - \mathcal{Q} \lesseqgtr V^*(\tau)$, and would postpone to date τ if and only if $V^*(0) - \mathcal{Q} < V^*(\tau)$, which is equivalent to

$$V^*(\tau) + \hat{V}(0) - \hat{V}(\tau) < V^*(\tau) \tag{8}$$

Simplifying reveals that the decision boils down to postponing mining if $\hat{V}(0) < \hat{V}(\tau)$, precisely the condition we had hoped to replicate. Thus the tax in Equation (7) "corrects" the naïvete of the non-learning decision maker. Imposing the tax $\mathcal{Q}$ on the naïve mine owner if she mines before τ will precisely align incentives - after incorporating the tax, she mines immediately if and only if it is socially efficient to do so. Note also that $V^*(0) = \hat{V}(0)$ because whether one learns or not, the decision to mine immediately returns the same expected payoff. Thus, we can represent the optimal mining tax explicitly as a function of the underlying parameters of the problem specified above, as is summarized by the following proposition:

Proposition 4. *Define a "mining tax" as*

$$\mathcal{Q} = \hat{V}(\tau) - V^*(\tau) \tag{9}$$

Where:

$$V^*(\tau) = P(0) - c_w + e^{-r\tau}(c_w - \bar{c}_A)$$
$$\hat{V}(\tau) = \int_{-\infty}^{c_w} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A + \int_{c_w}^{\bar{p}} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A)dc_A$$

Such a mining tax, imposed on early extraction by a naïve mine owner, induces the economically efficient mine timing.

This core result defines an "information hurdle" that must be cleared in order to proceed with mining immediately. For instance, if a mine is currently viable based on private costs, but there are uncertain external costs to be learned in the future (e.g., environmental costs), then Equation (9) provides a way of determining the extra payoff necessary for a privately desirable mine to be socially desirable.

While useful for correcting behavior, the tax, $\mathcal{Q}$, is only one possible tax from a family of taxes that would all convert a naïve decision-maker into a sophisticated decision-maker. To see this, consider the simple case in which the sophisticated decision-maker is barely in favor of mining now rather than delay, with $\hat{V}(0) = \hat{V}(\tau) + \varepsilon$, for small ε. It follows that $V^*(0) > V^*(\tau)$.[7] In that case, the naïve decision-maker will make the sophisticated decision absent the tax, yet the tax, $\mathcal{Q}$, can be substantial.[8] In this case, the naïve mine owner is made to pay a large mining tax even though the tax has no desired, or actual, behavioral effects.

To get around this problem, we derive an alternative tax, which we call the "minimum corrective tax," that is the smallest possible tax that just corrects the naïve mining behavior. The minimum corrective tax is given by:

$$\mathcal{Z} = \begin{cases} V^*(0) - V^*(\tau) & \text{if} \quad \hat{V}(\tau) > \hat{V}(0) > V^*(\tau) \\ 0 \quad \text{otherwise} \end{cases} \tag{10}$$

In the example provided above (where $\hat{V}(0) = \hat{V}(\tau) + \varepsilon$), no tax would be levied. In the numerical example that follows, we will calculate and compare the mining taxes $\mathcal{Q}$ and $\mathcal{Z}$.

[7] If a sophisticated decision maker is in favor of mining immediately, then a naïve decision maker is even more strongly in favor of mining immediately.

[8] The tax would be $\mathcal{Q} = V^*(0) - V^*(\tau)$.

3.1 Illustrative example

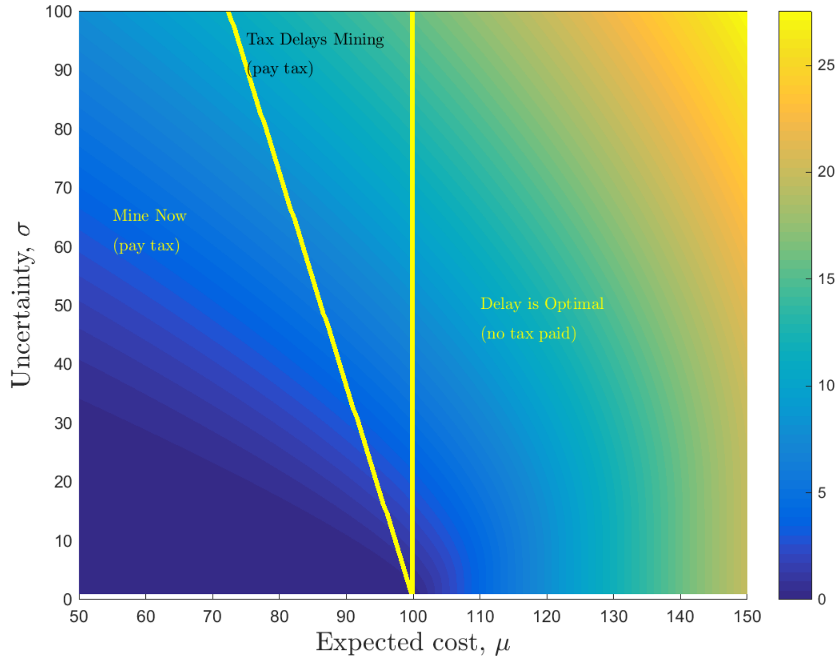
We now illustrate the results of this analysis and provide a concrete example of the corrective mining taxes derived above. Let the initial market reserves be given by R_0, and let market demand be a linear function of quantity extracted: $p(q) = \alpha - \beta q$. Global marginal extraction cost is $c_w = 100$. We selected a set of parameters loosely chosen to reflect the global market for gold.[9] Using these parameters, backward induction reveals a time to exhaustion of $T = 31$ years and a resulting initial price of $p(0) =$\$1,234. Now consider a deposit with an uncertain cost of extraction, c_A. In particular, let the probability density function over true deposit extraction cost c_A be given by $f(c_A) \sim N(\mu, \sigma^2)$, and assume that the true cost will be revealed in $\tau = 10$ years. We compare naïve and sophisticated decisions in this context parametrically, letting μ and σ vary.

In this setting, if $\mu \geq 100$ (i.e. if expected cost in A is greater than or equal to the cost in the rest of the world), then it is socially optimal to postpone mining, even for a naïve mine owner. But if $\mu < 100$, the naïve mine owner will mine immediately. In some such cases, it would, rather, be optimal to postpone mining until date $\tau = 10$ when the true environmental costs will be revealed. Implementing either mining tax $\mathcal{Q}$ or $\mathcal{Z}$ induces the optimal behavior from such an owner. This is illustrated in Figure 1, where the horizontal axis shows expected cost (μ) and the vertical axis shows standard deviation of cost (σ). Three regions are shown in the figure, delineated by the yellow lines. Note first that it is optimal for both naïve and sophisticated owners to delay mining for all points to the right of the vertical yellow line (i.e. for $\mu > 100$); and it is optimal for both naïve and sophisticated owners to mine immediately for all points to the left of the slanted yellow line (i.e. for low values of μ). The information externality arises in the wedge between these two lines where it is optimal for a sophisticated owner to delay but the naïve owner finds it optimal to mine immediately. The various colors in the figure show the magnitude of the mining tax $\mathcal{Q}$, which is sufficient to correct behavior of the naïve mine owner. Clearly $\mathcal{Q}$ is increasing in both μ and σ. Note that on the far left (when expected cost is very low), it is optimal for both owners to mine immediately, so $\mathcal{Q}$ is paid by the naïve owner, but doing so does not change her behavior. On the far right (when expected cost is very high), it is optimal for both owners to delay, so again $\mathcal{Q}$ does not change behavior. But for a band in the middle, $\mathcal{Q}$ causes the naïve mine owner, who would have found it optimal to mine immediately, to delay mining until at least date τ. Notice that for any level of μ, the size of this region is increasing in σ (Corollary 1b).

We have already argued that $\mathcal{Q}$, based on the quasi-option value, may be unnecessarily large to induce desired behavior. This is illustrated in Figure 1, where there are positive values of $\mathcal{Q}$ in all three regions of the figure, yet only in the middle wedge does there need to be a positive $\mathcal{Q}$ since it is only in this region that there is a divergence between naïve and sophisticated decision-making. But even if we only charged $\mathcal{Q}$ in the wedge between the vertical and slanted lines, it would still be excessive. This was the logic that underpinned our derivation of $\mathcal{Z}$ in Section 3. Figure 2 shows the minimum corrective tax, $\mathcal{Z}$, necessary to correct naïve behavior. It is clearly non-zero only in the wedge, the only place where behavior needs correcting.

[9] The parameters are: $\alpha = 5e3$, $\beta = 5e3/2.5e8$, $c_w = 100$, $r = .05$, $R_0 = 3.5e9$.

Figure 1: Mining tax, $\mathcal{Q}$, calculated from Equation 9.

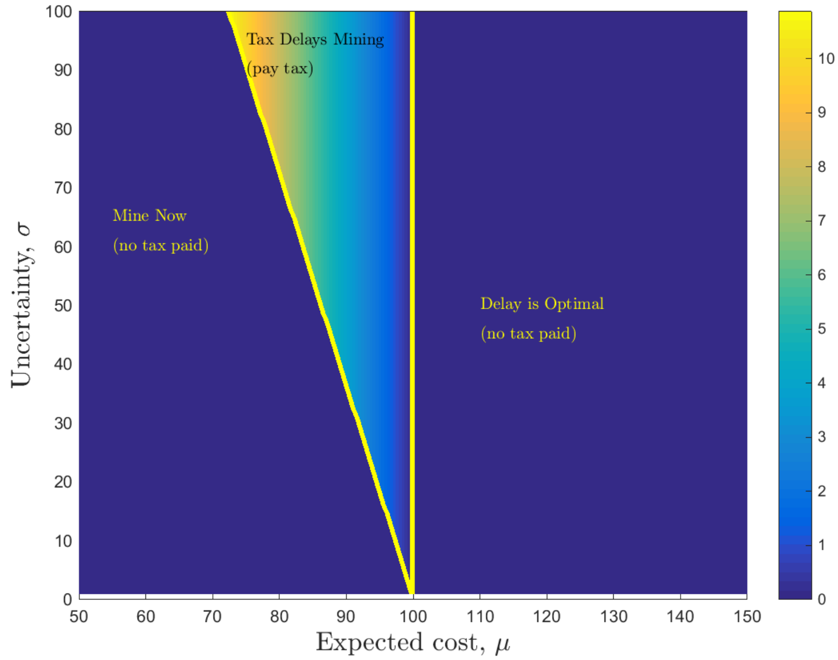


4 Alternative Market Specifications

To sharpen our contribution, we have assumed a relatively simple market in which all mine owners (with the possible exception of owner A) share the same marginal cost of extraction. There are other competitive assumptions which could be adopted regarding the overall resource market: existence of a backstop technology; non-existence of a choke price; heterogeneous world market consisting of mines with constant but differing marginal extraction costs and reserves; and production with uncertain aggregate reserves and reserve additions occurring simultaneously with production; production and exploration with uncertainty about future reserves and/or future demand (Pindyck 1980).

Our results are qualitatively robust to these different competitive market specifications. All of these models result in a global price path, typically with rents rising at the rate of interest. Now introduce a small mine owner A who holds a single unit of the resource and can produce it for cost c_A. If c_A is known, A can perfectly "slot" herself into the queue - there is an optimal time t_A^* at which A should mine in order to maximize return. Suppose instead that c_A is uncertain, has the same mean as the cost that gave rise to t_A^*, and will be revealed at date $\tau > t_A^*$. It is straightforward to show in this model (in a manner similar to the analysis of Proposition 3 and Corollary 1 above) that conditions exist under which A will want to postpone the decision until date τ. It is also straightforward to show that owner A will never postpone to a date in the open interval: (t_A^*, τ), nor will she ever mine at a date prior to t_A^*. Thus, in a manner similar to the dynamics above, adding uncertainty

Figure 2: Minimum Corrective Mining tax, $\mathcal{Z}$, calculated from Equation 10.



to c_A may cause owner A to defer mining to a later date, or possibly defer it indefinitely.

The existence of a backstop technology is equivalent to demand having a choke price, at which demand drops to zero. Without a choke price, the terminal time T may become infinite but that does not change the analysis of the quasi-option value.

5 Discussion

The main result of this paper is that *any* uncertainty about the environmental cost of mining can optimally cause the sophisticated mine owner to postpone extraction, even when the *expected* environmental costs suggest that it is optimal to mine immediately. This does not necessarily imply that the mine will *never* be exploited. As information on environmental costs are revealed, they give rise to new optimal decisions. We showed that if the cost is learned to be relatively low (or even as "expected") then it will be optimal to mine once that information is revealed. On the other hand, if cost is revealed to be large (but not too large), then it will be optimal to mine at a later date. In extreme cases in which the revealed cost turns out to be very large, it will be optimal to postpone indefinitely.

A typical application is when all uncertainty regards external costs, such as environmental costs. Furthermore, internalization of those costs is manifest in a regulatory authority granting or denying permission to mine at any particular point. If the regulatory authority does a standard cost-benefit analysis, based on expected external costs, then it is acting naïvely. We showed that an additional cost, equal to the mining tax, must be added in order for the naïve regulatory authority to behave efficiently.

An extension of this model, which we do not analyze here, occurs when the regulatory authority is subject to political pressure. In that case it may be in the mine's interest to lobby the regulatory authority to allow mining early before uncertainty is resolved. The framework introduced in this paper would allow us to compute how much it would be worth to the mine to obtain early approval rather than wait until uncertainty is resolved and run the risk that external costs end up being high, resulting in a denial of permission to mine.

Our simple theoretical treatment sheds light on a rich array of empirically and policy relevant contemporary problems involving the extraction of non-renewable resources with uncertainty about the associated environmental costs. In such real world applications, environmental costs are often pitted against extraction benefits. This analysis reveals that this is a false tradeoff: The question is not whether we should *ever* mine, but rather whether we should mine today or postpone the decision until better information on environmental costs is revealed. After all, a decision to delay mining forfeits none of the natural resource - it just saves it for the future when a more insightful decision can be made.

Appendix of Proofs

Proof to Proposition 1

Proof. Let V_1 be the expected payoff from mining prior to τ and let V_2 be the expected value of waiting until τ, learning the true value of c_A, and then deciding when to mine. To obtain V_1, because $\bar{c}_A = c_w$, Equation 6 reveals that the non-learning mine owner is indifferent between mining at any date $t \in [0, T]$; in other words, she obtains the same expected payoff from extracting at any date in that closed interval. Sice the mining date is irrelevant, we will use date τ for convenience.

$$V_1 = \int_{-\infty}^{\infty} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A \tag{11}$$

which we split into the relevant regions of the realization of c_A:

$$\begin{aligned} V_1 = &\int_{-\infty}^{c_w} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A \\ &+ \int_{c_w}^{\bar{p}} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A \\ &+ \int_{\bar{p}}^{\infty} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A \end{aligned} \tag{12}$$

V_2 can be decomposed in a similar manner, though the timing of mining will depend on the realized value of c_A. We have

$$\begin{aligned} V_2 = &\int_{-\infty}^{c_w} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A)dc_A \\ &+ \int_{c_w}^{\bar{p}} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A)dc_A \\ &+ 0 \end{aligned} \tag{13}$$

where the third term is 0 because no mining takes place in the event that $c_A > \bar{p}$.

We would like to prove that $V_2 > V_1$. Taking the difference, we see:

$$V_2 - V_1 = \underbrace{\int_{c_w}^{\bar{p}} (c_w - c_A)(e^{-rT} - e^{-r\tau}) f(c_A) dc_A}_{Term1} - \underbrace{\int_{\bar{p}}^{\infty} \left[P(0) - c_w + (c_w - c_A)e^{-r\tau}\right] f(c_A) dc_A}_{Term2} \tag{14}$$

The first term on the RHS is unambiguously positive (this is the benefit of mining at T rather than τ when the true cost is in $[c_w, \bar{p}]$). The second term is unambiguously negative (it is the cost of mining at τ when the cost is extremely high ($c_A > \bar{p}$). Thus, $V_2 > V_1$, which concludes the proof. □

Proof to Proposition 2

Proof. Let V_1 be the expected payoff of committing to mine at some date $s < \tau$ and let V_2 be the expected payoff from postponing the decision until at least date τ. Finally, let V_T be the expected value of committing to mine at date T. By Equation 6, $V_T > V_1$ (when $c_w < \bar{c}_A < \bar{p}$ the non-learning mine operator maximizes payoff by mining at date T). We will show that $V_2 > V_T$, which implies that $V_2 > V_1$. We have:

$$\begin{aligned} V_1 < V_T = &\int_{-\infty}^{c_w} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A) dc_A \\ &+ \int_{c_w}^{\bar{p}} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A) dc_A \\ &+ \int_{\bar{p}}^{\infty} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A) dc_A \end{aligned} \tag{15}$$

And V_2 is given in Equation 13. The difference is given by:

$$V_2 - V_T = \underbrace{\int_{-\infty}^{c_w} (c_w - c_A)(e^{-r\tau} - e^{-rT}) f(c_A) dc_A}_{\text{Term 1}} - \underbrace{\int_{\bar{p}}^{\infty} \left[P(0) - c_w + (c_w - c_A)e^{-rT}\right] f(c_A) dc_A}_{\text{Term 2}} \tag{16}$$

Term 1 is clearly positive and Term 2 is clearly negative, so $V_2 > V_T > V_1$, and $V_2 > V_1$ which proves the result. □

Proof to Proposition 3

Proof. If $\bar{c}_A < c_w$ and A commits to mining prior to τ, she should mine at date 0 (see Equation 6). Let V_1 be the expected payoff from doing so. Let V_2 be the expected payoff from delaying the decision until date τ. We have:

$$V_1 = \int_{-\infty}^{c_w} [P(0) - c_A] f(c_A) dc_A + \int_{c_w}^{\bar{p}} [P(0) - c_A] f(c_A) dc_A + \int_{\bar{p}}^{\infty} [P(0) - c_A] f(c_A) dc_A \tag{17}$$

And V_2 is given in Equation 13. The difference is given by:

$$
\begin{aligned}
V_2 - V_1 = & \underbrace{\int_{-\infty}^{c_w} (c_w - c_A)(e^{-r\tau} - 1) f(c_A) dc_A}_{\text{Term 1}} \\
& + \underbrace{\int_{c_w}^{\bar{p}} (c_w - c_A)(e^{-rT} - 1) f(c_A) dc_A}_{\text{Term 2}} \\
& + \underbrace{\int_{\bar{p}}^{\infty} \left[c_A - P(0)\right] f(c_A) dc_A}_{\text{Term 3}}
\end{aligned} \tag{18}
$$

Term 1 is negative, and Terms 2 and 3 are positive. Proving that conditions exist under which it is optimal to postpone the decision until τ requires showing conditions under which $V_2 > V_1$; we use the following sufficient condition. Take the limit of 18 as $\tau \to 0$. Doing so only affects Term 1, and has no effect on Terms 2 or 3. Clearly $\lim_{\tau \to 0}$ Term $1 = 0$, while Terms 2 and 3 are strictly positive. Thus, small τ provides a sufficient condition for the result to hold. □

Proof to Corollary 1

Proof. Corollary 1(a): Let $\Delta \equiv V_2 - V_1$ from Equation 18. Taking the derivative gives:

$$
\frac{d\Delta}{d\tau} = \int_{-\infty}^{c_w} -r(c_w - c_A) e^{-r\tau} f(c_A) dc_A < 0 \tag{19}
$$

which concludes the proof.
Corollary 1(b): Define $\Phi(c_A)$ as the difference in payoffs between postponing the decision until τ and mining prior to τ, when the true value of A's cost parameter is c_A. First note that $\Delta \equiv V_2 - V_1$ is the integral of $\Phi(c_A)$, weighted by the probability density, $f(c_A)$, as follows.

$$
\Delta = V_2 - V_1 = \int_{-\infty}^{\infty} \Phi(c_A) f(c_A) dc_A \tag{20}
$$

To prove that Δ is increasing in the "risk" of $f(c_A)$ (colloquially, in the "spread" of $f(c_A)$), we rely on the main result of Rothschild and Stiglitz (1970), implying it is sufficient to show that $\Phi(c_A)$ is a convex function of c_A. The function $\Phi(c_A)$ is given as follows:

$$
\Phi(c_A) = \begin{cases} (c_w - c_A)(e^{-r\tau} - 1) & \text{if } c_A < c_w \\ (c_w - c_A)(e^{-rT} - 1) & \text{if } c_w \le c_A \le \bar{p} \\ c_A - P(0) & \text{if } c_A > \bar{p} \end{cases} \tag{21}
$$

Thus, $\Phi(c_A)$ is a continuous, piecewise linear, increasing function of c_A, where each line segment has a higher slope, thus, $\Phi(c_A)$ is convex and the result is proven. □

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