University of California, Davis
Department of Economics
Microeconomics

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Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

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Please answer **any three** of the following four questions
[If you answer all four questions please indicate which three you want to be graded]

QUESTION 1

Your entire wealth consists of the balance of your bank account, which is $\$W_0$. You also own a painting which is worthless to you. You have a chance to sell the painting to a potential buyer. All you know about the buyer is that she has a reservation price (i.e. a maximum price she is willing to pay) $r \in \{L, M, H\}$ with $L < M < H$ (L for low, M for medium and H for high). You can make a take-it-or-leave-it offer of a price $P \in [0, \infty)$. The buyer strictly prefers to buy the painting at price P to not buying if and only if $P < r$ (if $P = r$ the buyer is **indifferent** between buying and not buying but **you can assume that she will buy**). You are selfish and greedy, that is, you care only about your wealth and prefer more money to less.

(a) For this question assume that (1) your von Neumann-Morgenstern utility-of-money function is $U(m) = \sqrt{m}$ and (2) you assign equal probability to each of L, M and H.

    (a.1) What price will you offer? [Clearly, your answer must be a function of the parameters L, M and H; for simplicity you can ignore values that make you indifferent between two or more choices.]

    (a.2) What price will you offer if $W_0 = 16$, $L = 9$, $M = 20$ and $H = 33$?

Now let us change the situation as follows (anything that is not explicitly mentioned remains the same as above). Firstly, from now on assume that you are **risk neutral**. Secondly, assume that, instead of one potential buyer, you face **three potential buyers** and you know that one buyer has reservation price L, another buyer has reservation price M and the third buyer has reservation price H (with $L < M < H$), but you do not know which person has which reservation price. Of course, you also know that each buyer knows her own reservation price. A mediator (who is **not** to be treated as a player) picks one potential buyer randomly, with equal probability, and sends her to you. You then make a take-it-or-leave-it offer of a price. If she accepts, then the transaction takes place at that price and the game ends; if she refuses to buy at that price, then she has to leave and ceases to be a player (that is, **she is no longer a potential buyer**). In the latter case the mediator picks one of the two remaining potential buyers randomly, with equal probability, and sends her to you. You then make a take-it-or-leave-it offer of a price (possibly different from the earlier offer) to the second potential buyer; if she accepts, the transaction takes place at that price and the game ends; if she refuses to buy at that price, then she has to leave and ceases to be a player. In the latter case the mediator sends you the remaining potential buyer. You then make her a take-it-or-leave-it offer of a price (possibly different from the earlier offers) to the third potential buyer; if she accepts, the transaction takes place at that price and the game ends; if she refuses to buy at that price, then the game ends and you are stuck with your painting. The buyers have a discount factor of 1, while you have a per-period discount factor of δ with $0 < \delta < 1$ (you make the first offer in period 1, the second offer – if any – in period 2 and the third offer – if any – in period 3). The value of δ is common knowledge. [Once again, assume that – if indifferent between buying and not buying – a buyer chooses to buy.]

(b) Draw the extensive-form game-frame for the simpler case where there are **only two** potential buyers, one with $r = M$ and the other with $r = H$, and – at every stage – you can only offer one of two prices: P_1 or P_2 [no need to write the payoffs].

(c) For the general case described above [three potential buyers, $r \in \{L, M, H\}$] find a weak sequential equilibrium of this game. [You can assume that $W_0 = 0$. Briefly explain why you would only consider offering prices in the set $\{L, M, H\}$.]

(d) Maintaining the same assumptions as in part (c), describe a weak sequential equilibrium for the case where $H = 140$, $M = 90$, $L = 70$ and $\delta = 0.75$.

Question 2

Consider a finite society $\mathcal{I} \equiv \{1, \ldots, I\}$, where the preferences of the individuals are represented by the utility functions $(u^i : \mathbb{R}^L_+ \to \mathbb{R})_{i \in \mathcal{I}}$. Assume that all of these functions are continuous, strictly quasiconcave and strictly monotone. Unlike in class, we are going to treat individual endowments as variables. For individual i, her endowment is denoted by $w^i \in \mathbb{R}^L_+$.

Define the functions $x^i : \mathbb{R}^L_{++} \times \mathbb{R}^L_+ \to \mathbb{R}^L_+$ by

$$x^i(p, w^i) \equiv \operatorname{argmax}_{x \in \mathbb{R}^L_+} \left\{ u^i(x) \mid p \cdot x \leqslant p \cdot w^i \right\}, \tag{1}$$

for each i. Define also the function $Z : \mathbb{R}^L_{++} \times \mathbb{R}^{LI}_+ \to \mathbb{R}^L$ by

$$Z(p, w) \equiv \sum_i [x^i(p, w^i) - w^i] \tag{2}$$

and the set

$$\mathcal{M} \equiv \left\{ (p, w) \in \mathbb{R}^L_{++} \times \mathbb{R}^{LI}_+ \mid Z(p, w) = 0 \right\}. \tag{3}$$

Here, x^i is individual i's demand function and Z is the aggregate excess demand function. Obviously, p is a vector of competitive equilibrium prices of exchange economy $\{\mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}}\}$ if, and only if, $(p, w) \in \mathcal{M}$. Set $\mathcal{M}$ is hence called the *equilibrium set*, or *equilibrium manifold*, of preference profile $(u^i)_{i \in \mathcal{I}}$.

In this exercise you are going to prove that an observer loses no information when working with the equilibrium set, in comparison to the profile of individual demands. In technical language, you will argue that the equilibrium manifold *identifies* both the excess demand function and all the individual demand functions: given set $\mathcal{M}$, there is one and only one function Z that generates $\mathcal{M}$ by Eq. (3); and there is one and only one profile $(x^i)_{i \in \mathcal{I}}$ that generates that Z by Eq. (2).[1]

1. Suppose that p, w and $\hat{w}$ are such that: (a) (p, w) and $(p, \hat{w})$ are both in $\mathcal{M}$; and (b) for all i, $p \cdot w^i = p \cdot \hat{w}^i$. Argue that $\sum_i w^i = \sum_i \hat{w}^i$.

2. Fix p and w, and suppose that $\hat{w}$ is such that: (a) $(p, \hat{w})$ is in $\mathcal{M}$; and (b) for all i, $p \cdot \hat{w}^i = p \cdot w^i$. Argue that $Z(p, w) = \sum_i (\hat{w}^i - w^i)$.

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[1] One can go further and show that, in fact, the set identifies all the individual preferences, in the sense that there is one and only one profile $(u^i)_{i \in \mathcal{I}}$ that generates $(x^i)_{i \in \mathcal{I}}$ by Eq. (1). Asking this further step in a prelim exam would probably qualify as a human rights violation, though.

3. Argue that for any p and any w, there exists $\hat{w} \in \mathbb{R}_+^{LI}$ such that (a) $(p, \hat{w})$ is in $\mathcal{M}$; and (b) for all i, $p \cdot \hat{w}^i \quad p \cdot w^i$. (*Hint:* Think of a profile of endowments such that at prices p each individual i demands $x^i(p, w^i)$ and which guarantees that markets clear.)

4. Use the previous steps to explain how an analyst who *only* observes $\mathcal{M}$ can construct function Z in a unique manner.

5. Argue that there exists a sub-profile of individual endowments for all agents other than $i \quad 1$, say $(\hat{w}^2, \ldots, \hat{w}^I)$, such that for all p and all w^1,

$$x^1(p, w^1) \quad Z(p, w^1, \hat{w}^2, \ldots, \hat{w}^I) + w^1.$$

(*Hint:* Think of a way of kicking all agents but $i \quad 1$ out of the market.)

6. Explain how, once the analyst of part 4 has constructed function Z, she can construct function x^1 in a unique manner.

7. Conclude.

Question 3

Emma is a first-year Ph.D. student of economics. She is very lucky because she got a place to stay in the Aggie Village, the nicest part of Davis. She lives in a beautiful cottage with a little garden. Her modest wealth from being a TA is w. She spends it on coffee and gardening. Let x_1 denote the amount of coffee and x_2 the amount of gardening and let p_1 and p_2 denote the corresponding unit prices. Her budget constraint is given by

$$p_1 x_1 + p_2 x_2 \leq w. \tag{1}$$

Coffee is really a private good in the sense that she is the sole beneficiary of caffeine in her coffee (unless she calls up in panic her fellow student in the middle of the night because she cannot solve her ECN200A homework problem). In contrast, gardening creates a positive externality on others. But so does the gardening of others create a positive externality on her. There is plenty of gardening in the Aggie Village. Denote by e the total externality or public good created from gardening in the community. Her utility function $u(x_1, x_2, e)$ is concave and continuously differentiable with a strictly positive gradient on the interior of its domain. The total externality depends in part on Emma's gardening x_2 and on the externality created by the gardening of others, denoted by e_{-i}. It is assumed to satisfy

$$e \leq e_{-i} + \alpha x_2 \tag{2}$$

for some parameter α satisfying $\alpha > 0$. Emma does not think that she can affect the level of externalities provided by others. For instance, Professor Schipper, who also lives in the Aggie Village, is so busy writing prelim exam questions that talking to him about keeping up his gardening is no use. Thus, we can safely assume that Emma takes e_{-i} as well as p_1, p_2, and w as given.

Since Emma diligently studies microeconomic theory for the prelims, she is eager to maximize her utility function subject to constraints (1) and (2). This yields demand functions $x_1(p_1, p_2, w, e_{-i})$ and $x_2(p_1, p_2, w, e_{-i})$ as well as her optimal desired amount of public good $e(p_1, p_2, w, e_{-i})$.

a.) Write down her Kuhn-Tucker-Lagrangian (ignore non-negativity constraints).

b.) Derive the Kuhn-Tucker first-order conditions (ignore non-negativity constraints).

c.) Use the Kuhn-Tucker conditions and the assumptions that the solution is interior, that it is unique, and that constraints (1) and (2) are satisfied with equality to derive a system of three equations and three unknowns that does not involve multipliers and whose solution defines $x_1(p_1, p_2, w, e_{-i})$, $x_2(p_1, p_2, w, e_{-i})$ and $e(p_1, p_2, w, e_{-i})$. (No need to solve it.)

d.) If you think that Professor Schipper acts in a strange way sometimes, this is not just due to being an economic theorist. The secret is he comes from another

solar system. One feature of these aliens is that they can read immediately the utility function of others. (Although this sounds quite useful, it is rather a curse.)[1] Anyway, as a proof of this claim we print here Emma's utility function:

$$u(x_1, x_2, e) = x_1 + (a_2, a_e) \begin{pmatrix} x_2 \\ e \end{pmatrix} - \frac{1}{2}(x_2, e)B \begin{pmatrix} x_2 \\ e \end{pmatrix}, \tag{3}$$

where $a_2, a_e > 0$ and $B = \begin{pmatrix} b_{22} & b_{2e} \\ b_{e2} & b_{ee} \end{pmatrix}$ is symmetric positive definite. I know, you surely must think "Wow" but let's focus again on the prelim exam. Assume that solutions are interior and that constraints are satisfied with equality. Write out the system of equations from problem c.) for Emma's utility function.

e.) Provide an interpretation of the partial derivatives $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and their signs.

f.) Compute $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$.

g.) Assume $b_{2e} \geq 0$. Derive the signs of $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$.

h.) Assume now $b_{2e} < 0$. Show that without additional assumptions, the signs of $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ remain ambiguous in this case. Find additional assumptions on matrix B and α that allow you to determine the signs of $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$.

i.) We want to get a better understanding of whether good 2 and the externality are complements or substitutes. In our particular context, do we need to distinguish between *gross* complements/substitutes and (net) complements/substitutes?

j.) To figure out whether good 2 and the externality are complements or substitutes, we can use the Slutsky substitution matrix. To this end, compute first Walrasian demand functions as if the externality has a market price p_e. That is, compute Walrasian demand functions $x_2(p_1, p_2, p_e, w)$ and $e(p_1, p_2, p_e, w)$.

k.) How does the fact that good 2 and the externality are substitutes or complements depend on the sign of b_{2e}?

ℓ.) How is the fact that good 2 and the externality are substitutes or complements related to the sign of the derivatives $\frac{\partial x_2(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ and $\frac{\partial e(p_1,p_2,w,e_{-i})}{\partial e_{-i}}$ discussed earlier?

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[1] Any similarity with the novel "The Three-Body Problem" by Chinese writer Liu Cixin is just coincidental.

Question 4

In ECN200A we assumed that consumers have preferences over consumption bundles. In many contexts though it is more natural to think that consumers have preferences over characteristics of consumption bundles like "lots of vitamins", "gluten free", "lots of horse power", "no tail pipe emissions", "many mega pixels" etc. In the following I outline an alternative model of consumer theory, in which preferences are defined over characteristics rather than consumption bundles. You are right, we have never discussed it in class. But there is no need to freak out since we know all the tools that are required to think about such a model.

Goods are indexed by $\ell = 1, ..., L$ and characteristics are indexed by $i = 1, ..., I$. We denote by $a_{i,\ell} > 0$ the quantity of characteristics i possessed by one unit of good ℓ. x_ℓ denotes the quantity of good ℓ. z_i is the quantity of characteristic i. We let $z = (z_1, ..., z_I)$ and assume that $z \in Z \subseteq \mathbb{R}^I_+$. Moreover, as usual we let $x = (x_1, ..., x_L) \in X \subseteq \mathbb{R}^L_+$.

We assume $z_i = \sum_{\ell=1}^{L} a_{i,\ell} x_\ell$ for $i = 1, ..., I$. This is the amount of characteristic z_i derived from a bundle of goods $x = (x_1, ..., x_L)$. We arrange $A = (a_{i,\ell})_{i=1,...,I, \ell=1,...,L}$ into a matrix, in which rows refer to characteristics and columns to goods.

a.) Consider a binary relation $\succsim$ on the space of characteristics, Z. State conditions on $\succsim$ that are sufficient for the existence of a utility function over characteristics $u : Z \longrightarrow \mathbb{R}$ that represents $\succsim$.

b.) Given prices of goods $p = (p_1, ..., p_L) >> 0$ and wealth $w \geq 0$, define the budget set on the characteristics space by

$$K_{p,w,A} := \{z \in Z : \text{ there exist } x \in X \text{ s.t. } z = Ax, p \cdot x \leq w\}.$$

Show that for any $p >> 0$ and $w \geq 0$, the budget set $K_{p,w,A}$ is convex.

c.) Assume that u is monotone and continuously differentiable. Consider the consumer problem

$$\max_{z \in Z} u(z) \text{ s.t. } z \in K_{p,w,A}.$$

Show that necessary conditions for a utility maximum are

$$p_\ell \geq \frac{1}{\lambda} \left(\sum_{i=1}^{I} a_{i,\ell} \frac{\partial u(z)}{\partial z_i} \right) \text{ for } \ell = 1, ..., L.$$

where λ is the Lagrange multiplier w.r.t. to the budget constraint $p \cdot x \leq w$.

d.) Provide an economic interpretation of the term

$$\frac{1}{\lambda} \frac{\partial u(z)}{\partial z_i}.$$

e.) Suppose you are a firm that introduces a new good into the market, let's call it good $L + 1$. Suppose the characteristics of your novel good are described by the vector $(a_{1,L+1}, ..., a_{I,L+1})$, for which each component is strictly positive. What price do you want to charge the consumer per unit of the new good?