Question 1

Consider an economy with I consumers and L goods. For each consumer $i \in \{1, ..., I\}$, the consumption set is $\mathbb{R}^L_+$. Her utility function is given by

$$u^i(x^i) = -\sum_{k=1}^{L} \alpha_k e^{-\beta^i_k x^i_k},$$

where $\alpha_k, \beta^i_k > 0$ for $k \in \{1, ..., L\}$. As usual we denote $x^i = (x^i_1, ..., x^i_L)$. Consider price vectors $(p_1, ..., p_L)$ and wealth levels $(w^1, ..., w^I)$ for which the solution to the utility maximization problem is interior for every consumer $i \in \{1, ..., I\}$.

a.) Derive the Walrasian demand function for good j by consumer i. (Be careful with your calculations. Double-check! It is easy to make mistakes.)

For each consumer $i \in \{1, ..., I\}$, the utility maximization problem is

$$\max_{x^i \in \mathbb{R}^L_+} u^i(x^i) \text{ s.t. } \quad p \cdot x^i \leq w^i.$$

Form the Lagrangian

$$L^i(x^i, \lambda^i) = u^i(x^i) - \lambda(p \cdot x^i - w^i).$$

The first-order conditions are given by

$$\begin{aligned} \alpha_j e^{\beta^i_j x^i_j} \beta^i_j &= \lambda^i p_j, \text{ for } j \in \{1, ..., L\} \\ p \cdot x^i &= w^i \end{aligned}$$

Dividing the condition for good j by the condition for let's say good L, we obtain

$$\frac{\alpha_j e^{-\beta^i_j x^i_j} \beta^i_j}{\alpha_L e^{-\beta^i_L x^i_L} \beta^i_L} = \frac{p_j}{p_L}, \text{ for } j \in \{1, ..., L-1\}.$$

Rearranging yields subsequently

$$\begin{aligned} e^{-\beta^i_j x^i_j + \beta^i_L x^i_L} &= \frac{p_j \alpha_L \beta^i_L}{p_L \alpha_j \beta^i_j} \text{ for } j \in \{1, ..., L-1\} \\ -\beta^i_j x^i_j + \beta^i_L x^i_L &= \ln\left(\frac{p_j \alpha_L \beta^i_L}{p_L \alpha_j \beta^i_j}\right) \text{ for } j \in \{1, ..., L-1\} \qquad (1) \\ \beta^i_j x^i_j &= -\ln\left(\frac{p_j \alpha_L \beta^i_L}{p_L \alpha_j \beta^i_j}\right) + \beta^i_L x^i_L \text{ for } j \in \{1, ..., L-1\} \end{aligned}$$

and finally

$$x^i_j = \frac{1}{\beta^i_j}\left(-\ln\left(\frac{p_j \alpha_L \beta^i_L}{p_L \alpha_j \beta^i_j}\right) + \beta^i_L x^i_L\right) \text{ for } j \in \{1, ..., L-1\}. \qquad (2)$$

Substituting this equation into consumer i's budget constraint yields

$$\begin{aligned}
\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\left(-\ln\left(\frac{p_k\alpha_L\beta_L^i}{p_L\alpha_k\beta_k^i}\right)+\beta_L^i x_L^i\right)+p_L x_L^i &= w^i\\
x_L^i\left(\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\beta_L^i+p_L\right)-\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k\alpha_L\beta_L^i}{p_L\alpha_k\beta_k^i}\right) &= w^i\\
x_L^i\left(\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\beta_L^i+p_L\frac{\beta_L^i}{\beta_L^i}\right) &= w^i+\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k\alpha_L\beta_L^i}{p_L\alpha_k\beta_k^i}\right)
\end{aligned}$$

Rearranging further yields subsequently

$$\begin{aligned}
\beta_L^i x_L^i\left(\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\right) &= w^i+\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k\alpha_L\beta_L^i}{p_L\alpha_k\beta_k^i}\right)\\
&= w^i+\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k}{\alpha_k\beta_k^i}\right)-\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)\\
&= w^i+\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k}{\alpha_k\beta_k^i}\right)-\sum_{k=1}^{L-1}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)\\
&\quad+\frac{p_L}{\beta_L^i}\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)-\frac{p_L}{\beta_L^i}\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)\\
&= w^i+\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k}{\alpha_k\beta_k^i}\right)-\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)
\end{aligned}$$

and finally

$$x_L^i=\frac{1}{\beta_L^i\left(\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\right)}\left(w^i+\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k}{\alpha_k\beta_k^i}\right)-\ln\left(\frac{p_L}{\alpha_L\beta_L^i}\right)\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\right)$$

Substituting it back into equation (2) yields

$$x_j^i=\frac{1}{\beta_j^i\left(\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\right)}\left(w^i+\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\ln\left(\frac{p_k}{\alpha_k\beta_k^i}\right)-\ln\left(\frac{p_j}{\alpha_j\beta_j^i}\right)\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}\right)\quad\texttt{for } j\in\{1,...,L\}\quad(3)$$

b.) What is the slope of consumer i's Engel curve for good j at (p, w^i)?

We differentiate equation (3) w.r.t. w^i to obtain

$$\frac{\partial x_j^i}{\partial w^i}(p,w^i) = \frac{1}{\beta_j^i\sum_{k=1}^{L}\frac{p_k}{\beta_k^i}}. \tag{4}$$

Engel curves are linear.

c.) Find a condition as general as possible on parameters α_k, β_k^i, $i \in \{1,...,I\}$, $k \in \{1,...,L\}$ guaranteeing the existence of a positive representative consumer. Do we need restrictions on parameters α_k, $k \in \{1,...,L\}$?

A necessary and sufficient condition for the existence of a representative consumer is that indirect utilities are of the Gorman form. It requires that for each good and any two consumers i and h, the slope of the Engel curve of consumer i must be equal to the slope of the Engel curve for consumer h. We observe from equation (4) that this is the case when for any two consumers i and h and any two goods j and k, we have

$$\frac{\beta_j^i}{\beta_k^i} = \frac{\beta_j^h}{\beta_k^h}.$$

That is, parameters $(\beta_1^i,...,\beta_L^i)$ must be a scalar multiple of $(\beta_1^h,...,\beta_L^h)$. No restrictions on parameters α_k, $k \in \{1,...,L\}$ are required.

d.) Consider now the special case with just a single consumer and two goods. The consumer's utility function is given by

$$u(x_1,x_2) = -\alpha_1 e^{-\beta_1 x_1} - \alpha_2 e^{-\beta_2 x_2}.$$

Derive the wealth-expansion path for a given price vector (p_1,p_2).

From equation (1), we get in this case

$$\begin{aligned} -\beta_1 x_1 + \beta_2 x_2 &= \ln\left(\frac{p_1\alpha_2\beta_2}{p_2\alpha_1\beta_1}\right) \\ x_2 &= \frac{1}{\beta_2}\ln\left(\frac{p_1\alpha_2\beta_2}{p_2\alpha_1\beta_1}\right) + \frac{\beta_1}{\beta_2}x_1. \end{aligned}$$

Note that last equation is the equation for the wealth-expansion path. It is a line with slope $\frac{\beta_1}{\beta_2}$.

e.) In problem d.), when does the wealth-expansion path intersect the x_1-axis and when does it intersect the x_2-axis?

It intersects the x_2-axis if $\ln\left(\frac{p_1\alpha_2\beta_2}{p_2\alpha_1\beta_1}\right) > 0$ and the x_1-axis if $\ln\left(\frac{p_1\alpha_2\beta_2}{p_2\alpha_1\beta_1}\right) < 0$.

General Equilibrium Theory

Sketched answers to prelim exam, August 2018

Question 2

Fix a standard, two-person exchange economy $\mathcal{E} = \{(u^1, w^1), (u^2, w^2)\}$. Define its *replica* as the four-person exchange economy

$$\mathcal{E}^2 = \{(u^1, w^1), (u^2, w^2), (u^3, w^3), (u^4, w^4)\},$$

where $(u^3, w^3) = (u^1, w^1)$ and $(u^4, w^4) = (u^2, w^2)$.

1. Argue that if (p, x^1, x^2) is a competitive equilibrium for $\mathcal{E}$, then (p, x^1, x^2, x^3, x^4) with $x^3 = x^1$ and $x^4 = x^2$, is an equilibrium for $\mathcal{E}^2$.

   Answer: Since (p, x^1, x^2) is a competitive equilibrium for $\mathcal{E}$, by definition

   $$x^1 \in \operatorname{argmax}_x\{u^1(x) : p \cdot x \leqslant p \cdot w^1\} \text{ and } x^2 \in \operatorname{argmax}_x\{u^2(x) : p \cdot x \leqslant p \cdot w^2\},$$

   while $x^1 + x^2 = w^1 + w^2$. But then, since $(u^3, w^3, x^3) = (u^1, w^1, x^1)$ and $(u^4, w^4, x^4) = (u^2, w^2, w^2)$, we further have that

   $$x^3 \in \operatorname{argmax}_x\{u^3(x) : p \cdot x \leqslant p \cdot w^3\} \text{ and } x^4 \in \operatorname{argmax}_x\{u^4(x) : p \cdot x \leqslant p \cdot w^4\},$$

   and $x^1 + x^2 + x^3 + x^4 = 2(x^1 + x^2) = 2(w^1 + w^2) = w^1 + w^2 + w^3 + w^4$.

2. Argue that if both utility functions are strictly quasi-concave, and (p, x^1, x^2, x^3, x^4) is a competitive equilibrium for $\mathcal{E}^2$, then, $x^1 = x^3$ and $x^2 = x^4$.

   Answer: Since (p, x^1, x^2, x^3, x^4) is a competitive equilibrium for $\mathcal{E}^2$,

   $$x^1 \in \operatorname{argmax}_x\{u^1(x) : p \cdot x \leqslant p \cdot w^1\} \text{ and } x^3 \in \operatorname{argmax}_x\{u^3(x) : p \cdot x \leqslant p \cdot w^3\}. \qquad (*)$$

   Since $(u^3, w^3) = (u^1, w^1)$, it further follows that

   $$x^3 \in \operatorname{argmax}_x\{u^1(x) : p \cdot x \leqslant p \cdot w^1\}.$$

   But, then, by strict quasiconcavity of u^1, $x^1 = x^3$, by Eq. $(*)$.

   A similar argument shows that $x^2 = x^4$.

3. Argue that if both utility functions are strictly quasi-concave, and (x^1, x^2, x^3, x^4) is in the core of $\mathcal{E}^2$, then, $x^1 = x^3$ and $x^2 = x^4$.

   Answer: Suppose not, and say that $x^1 \neq x^3$. With no loss of generality, assume that $u^1(x^1) \leqslant u^3(x^3)$ and $u^2(x^2) \leqslant u^4(x^4)$. Define $\bar{x}^1 = \frac{1}{2}(x^1 + x^3)$ and $\bar{x}^2 = \frac{1}{2}(x^2 + x^4)$. By strict quasiconcavity,

   $$u^1(\bar{x}^1) > \tfrac{1}{2}[u^1(x^1) + u^1(x^3)] \geqslant u^1(x^1), \text{ and } u^2(\bar{x}^2) \geqslant \tfrac{1}{2}[u^2(x^2) + u^2(x^4)] \geqslant u^2(x^2),$$

   while

   $$\bar{x}^1 + \bar{x}^2 = \tfrac{1}{2}(x^1 + x^2 + x^3 + x^4) = \tfrac{1}{2}(w^1 + w^2 + w^3 + w^4) = w^1 + w^2.$$

   It follows that $\{1, 2\}$ blocks (x^1, x^2, x^3, x^4).

4. Argue that if both utility functions are monotone and strictly quasi-concave, and (p, x^1, x^2) is a competitive equilibrium for $\mathcal{E}$, then (x^1, x^2, x^1, x^2) is in the core of $\mathcal{E}^2$.

   Answer: By part 1, (p, x^1, x^2, x^1, x^2) is a competitive equilibrium for $\mathcal{E}^2$. Since all preferences are locally non-satiated, (x^1, x^2, x^1, x^2) is in the core of $\mathcal{E}^2$, by the FFTWE.

5. Suppose that

$$u^1(x) = u^2(x) = x^1 x^2,$$

   $w^1 = (1, 0)$ and $w^2 = (0, 1)$. Argue that allocation $((0,0),(1,1))$ is in the core of $\mathcal{E}$, yet allocation

$$((0,0),(1,1),(0,0),(1,1))$$

   is *not* in the core of $\mathcal{E}^2$.

   Answer: To see that $((0,0),(1,1))$ is in the core of $\mathcal{E}$, note the following: $u^1(0,0) = 0 = u^1(w^1)$ and $u^2(1,1) = 1 > u^2(w^2)$, so no singleton coalition blocks the allocation; it is impossible to increase u^2; and in order to increase u^1, one would need $x^1 \gg 0$, in which case u^2 must decrease.

   To see that $((0,0),(1,1),(0,0),(1,1))$ is *not* in the core of $\mathcal{E}^2$, consider coalition $\{1,2,3\}$ and sub-allocation

$$((\varepsilon,\varepsilon),(2-2\varepsilon, 1-2\varepsilon),(\varepsilon,\varepsilon))$$

   for $\varepsilon > 0$, small. Then,

$$u^1(\varepsilon,\varepsilon) = u^3(\varepsilon,\varepsilon) = \varepsilon^2 > 0 = u^1(0,0) = u^3(0,0),$$

   since $\varepsilon > 0$, yet

$$u^2(2-2\varepsilon, 1-2\varepsilon) = (2-2\varepsilon)(1-2\varepsilon) = 2 - 6\varepsilon + 4\varepsilon^2 > 1 = u^2(1,1),$$

   so long as ε is small enough. On the other hand,

$$(\varepsilon,\varepsilon) + (2-2\varepsilon, 1-2\varepsilon) + (\varepsilon,\varepsilon) = (2,1) = w^1 + w^2 + w^3.$$

6. Use these results to argue, informally, that the replication of agents does not affect the set of equilibrium allocations of the economy but shrinks its core.

   Answer: Exercise 2 says that in studying the set of competitive equilibria, we can dismiss the increase in dimensions due to the replication of agents. Together with Exercise 1, Exercise 2 further implies that the set of competitive equilibria is invariant to replication. Exercise 3 shows that, again, when dealing with the core we can dismiss the increase in dimensions due to replication, while Exercise 5 extends the FFTWE to replica economies, in a trivial way. But Exercise 5 implies that the core is *not* invariant: it shrinks upon replication.

   Now, from the FFTWE we know that every competitive equilibrium allocation lies in the core of the economy, but it's easy to see that many core allocations cannot be sustained as competitive equilibrium. The generalization of the arguments above, however, will show that replication of the economy *ad infinitum* will shrink the core all the way to the set of competitive equilibrium allocations of the economy.

Micro Prelim August 2018 – ANSWER KEYS

QUESTION 3

(a) First we determine the optimal response of Player 2 to any possible d_1. Choose an arbitrary d_1. If Player 2 chooses $d_2 > d_1$ he gets 0; if he chooses $d_2 = d_1$ he gets $\min\{d_2, 100M\} > 0$ and if he chooses $d_2 = d_1 - 1$ he gets $\min\{2(d_1-1), 200M\}$ (choosing $d_2 < d_1 - 1$ cannot be better than choosing $d_2 = d_1 - 1$). Note that $2(d_1 - 1) > d_1$ if and only if $d_1 > 2$, that is, if and only if $d_1 \geq 3$. Thus the best reply correspondence of Player 2, denoted by $R_2(d_1)$, is as follows:

$$R_2(d_1) = \begin{cases} d_2 = 1 & \text{if } d_1 = 1 \\ d_2 = 1 \text{ or } d_2 = 2 & \text{if } d_1 = 2 \\ d_2 = d_1 - 1 & \text{if } 3 \leq d_1 < 100{,}000{,}000 \\ \text{any } d_2 \text{ with } 100M \leq d_2 \leq d_1 - 1 & \text{if } d_1 \geq 100{,}000{,}000 \end{cases}$$

Hence there are two sets of backward induction solutions: (1) Player 1's strategy is $d_1 = 1$ and Player 2's strategy is $\begin{cases} d_2 = 1 & \text{if } d_1 = 1 \text{ or } d_1 = 2 \\ d_2 = d_1 - 1 & \text{if } 3 \leq d_1 < 100{,}000{,}000 \\ d_2 = \hat{d}_2 & \text{if } d_1 \geq 100{,}000{,}000 \end{cases}$, for an arbitrary $\hat{d}_2$ with $100M \leq \hat{d}_2 \leq d_1 - 1$; all these solutions give rise to the outcome where each charity gets \$1!
(2) Player 1's strategy is $d_1 = 2$ and Player 2's strategy is

$\begin{cases} d_2 = 1 & \text{if } d_1 = 1 \\ d_2 = 2 & \text{if } d_1 = 2 \\ d_2 = d_1 - 1 & \text{if } 3 \leq d_1 < 100{,}000{,}000 \\ d_2 = \hat{d}_2 & \text{if } d_1 \geq 100{,}000{,}000 \end{cases}$, for an arbitrary $\hat{d}_2$ with $100M \leq \hat{d}_2 \leq d_1 - 1$; all these solutions give rise to the outcome where each charity gets \$2!

(b) Let us compute the best reply correspondence of Player 2. Consider an arbitrary d_1. If Player 2 responds with any $d_2 \geq d_1$ then the total contribution to both charities is $\min\{2d_1, 200M\}$; if Player 2 responds with $d_2 = d_1 - 1$ then the total contribution to both charities is $\min\{2(d_1-1), 200M\}$. Since $\min\{2(d_1-1), 200M\} < \min\{2d_1, 200M\}$ if and only if $d_1 \leq 100M$, the best reply correspondence of Player 2 is as follows:

$$R_2(d_1) = \begin{cases} \text{any } d_2 \geq d_1 & \text{if } d_1 \leq 100M \\ \text{any } d_2 \text{ such that either } d_2 \geq d_1 \text{ or } 100\text{M} \leq d_2 \leq d_1 - 1 & \text{if } d_1 > 100M \end{cases}.$$

Thus the backward induction solutions are as follows. Player 1's strategy any $d_1 \geq 100M$ and Player 2's strategy is any function obtained from $R_2(d_1)$.

(c) **(c.1)** The payoff functions are $\pi_1(d_1,d_2)=\begin{cases}\min\{2d_1,200M\} & \text{if } d_1<d_2\\ \min\{d_1,100M\} & \text{if } d_1=d_2\\ 0 & \text{if } d_1>d_2\end{cases}$ and

$$\pi_2(d_1,d_2)=\begin{cases}0 & \text{if } d_1<d_2\\ \min\{d_1,100M\} & \text{if } d_1=d_2\\ \min\{2d_2,200M\} & \text{if } d_1>d_2\end{cases}.$$

(c.2) Any (d_1,d_2) with $d_1<d_2$ is not a Nash equilibrium because Player 2 can get more by choosing $d_2=d_1$. Similarly for any (d_1,d_2) with $d_1>d_2$. So we are left with pairs of the form $d_1=d_2=d$. For Player 1 deviating to $d_1<d-1$ is worse than deviating to $d_1=d-1$ (and similarly for Player 2). Since $2(d-1)>d$ if and only if $d_1>2$, the Nash equilibria are (\$2,\$2) (where each charity gets \$2) and (\$1,\$1) (where each charity gets \$1).

(d) In this case

- every pair (d_1,d_2) with $d_1=d_2=d$ is a Nash equilibrium: the total amount given to charity is $\min\{2d,200M\}$ and it remains the same if Player 1 increases d_1 and it remains the same or decreases if Player 1 reduces d_1 (similarly for Player 2); furthermore
- every pair (d_1,d_2) with $d_1<d_2$ and $2d_1\geq 200M$, that is, $d_1\geq 100M$ is a Nash equilibrium, and
- and every pair (d_1,d_2) with $d_1>d_2$ and $d_2\geq 100M$ is a Nash equilibrium.

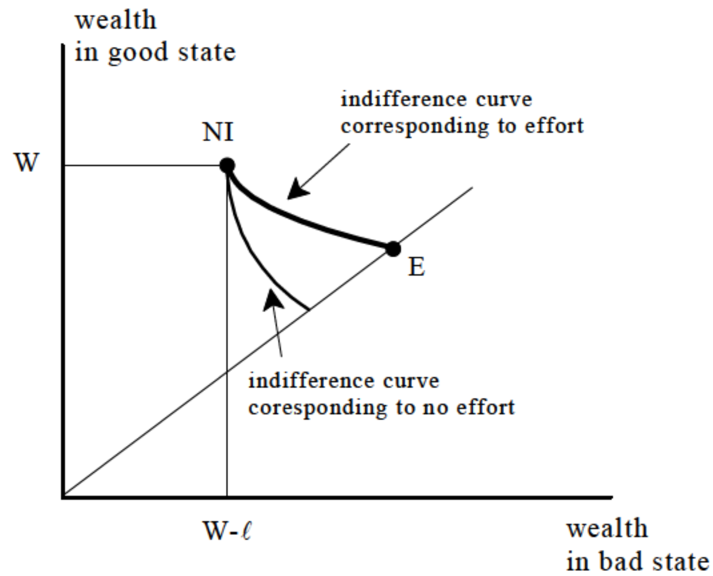
There are no other Nash equilibria.

(e) - Consider a pair (d_1,d_2) with $d_1=d_2=d$ and $2d\geq 200M$, that is, $d\geq 100M$ (so that the total is $\min\{2d,200M\}=200M$ and each player gets $\min\{d,100M\}=100M$). If Player 1 increases d_1 then the total does not change (it remains $\min\{2d,200M\}=200M$), but Player 1 gets 0 and thus is worse off. If Player 1 switches to $d_1=d-1$ then the total is $\min\{2(d-1),200M\}$; thus (1) if $2(d-1)<200M$, that is, $d\leq 100M$ then the total goes down and Player 1 is worse off and (2) if $2(d-1)\geq 200M$, that is, $d>100M$ then the total does not change and Player 1 gets $\min\{2(d-1),200M\}=200M$ and thus he is better off. The same reasoning applies to Player 2. Hence of all the such pairs only $(100M,100M)$ is a Nash equilibrium.
- Now consider a pair (d_1,d_2) with $d_1=d_2=d$ and $2d<200M$, that is, $d<100M$ (so that the total is $\min\{2d,200M\}=2d$ and each player gets $\min\{d,100M\}=d$). If Player 1 increases d_1 then the total does not change (it remains $\min\{2d,200M\}=2d$), but Player 1 gets 0 and thus is worse off. If Player 1 switches to $d_1=d-1$ (assuming that this is possible, that is, that $d\geq 2$) then the total goes down to $\min\{2(d-1),200M\}=2(d-1)<2d$ and thus Player 1 is worse off. Hence every such pair is a Nash equilibrium.
- Now consider any pair (d_1,d_2) with $d_1<d_2$. Then the total is $\min\{2d_1,200M\}$ and Player 2 gets 0. If Player 2 switched to $d_2=d_1$ then the total would remain $\min\{2d_1,200M\}$, but Player 2 would get a positive amount, namely $\min\{d_1,100M\}$. Hence no such pair is a Nash equilibrium. Similar reasoning for pairs of the form (d_1,d_2) with $d_1>d_2$.

In conclusion, **the Nash equilibria are all the pairs** (d_1,d_2) **with** $d_1=d_2=d\leq 100M$.

QUESTION 4

(a)



(b) The slope of the *e*-indifference curve ('*e*' for effort) at *NI* is

$$-\frac{p_e}{1-p_e}\left(\frac{U'(W-\ell)}{U'(W)}\right) = -\frac{1}{19}\left(\frac{\frac{1}{2\sqrt{900}}}{\frac{1}{2\sqrt{2,500}}}\right) = -\frac{5}{57} = -0.0877$$

and the slope of the *n*-indifference curve ('*n*' for effort) at *NI* is

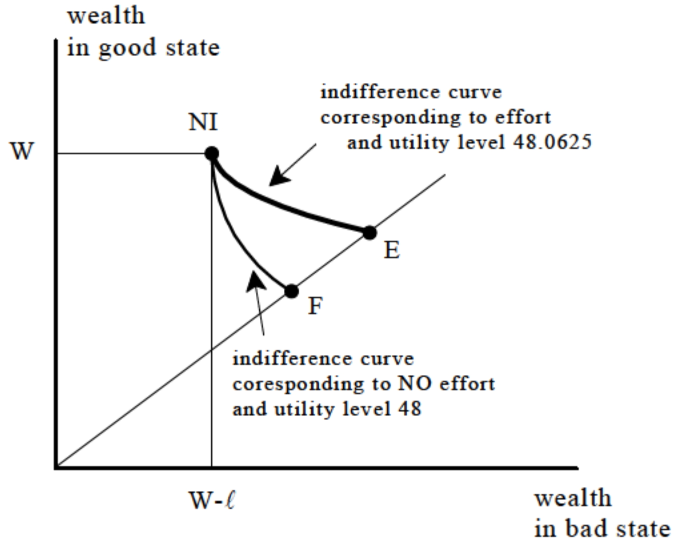
$$-\frac{p_n}{1-p_n}\left(\frac{U'(W-\ell)}{U'(W)}\right) = -\frac{1}{9}\left(\frac{\frac{1}{2\sqrt{900}}}{\frac{1}{2\sqrt{2,500}}}\right) = -\frac{5}{27} = -0.1852\,.$$

(c) $EU_n(NI) = \frac{1}{10}\sqrt{900} + \frac{9}{10}\sqrt{2,500} = 48$ and

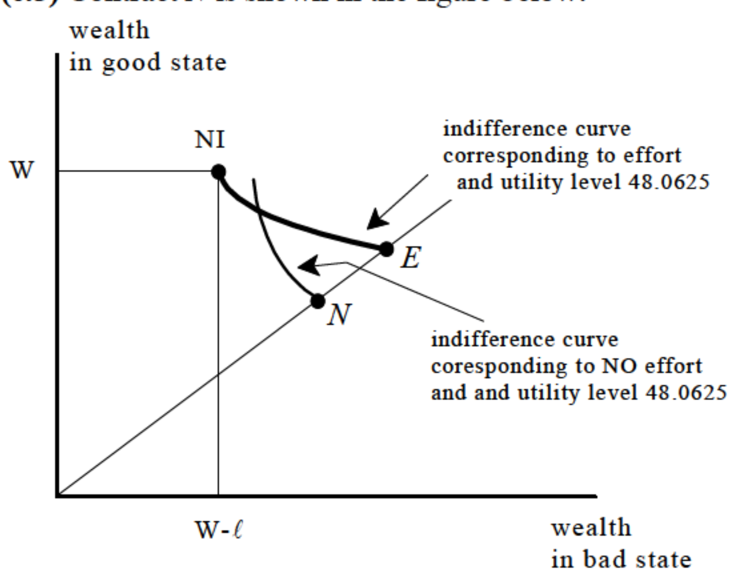
$EU_e(NI) = \frac{1}{20}\sqrt{900} + \frac{19}{20}\sqrt{2,500} - \frac{15}{16} = 48.0625$. Thus she would exert effort.

(d) $EU_n(NI) = 48$ and $EU_e(NI) = \frac{1}{20}\sqrt{900} + \frac{19}{20}\sqrt{2,500} - \frac{3}{2} = 47.5$. Thus she would *not* exert effort.

(e) Using the calculations of part (c) contracts *E* and *F* are shown in the following figure:



(e.1) The premium of contract *E* is given by the solution to $\sqrt{2{,}500-h}-\frac{15}{16}=48.0625$ which is $h=99$ (and zero deductible).
(e.2) The premium of contract *F* is given by the solution to $\sqrt{2{,}500-h}=48$ which is $h=196$ (and zero deductible).
(e.3) The customer will be indifferent between choosing *NI* with effort and contract *E* with effort and thus, given the assumption made, she will choose *E* and the monopolist's profits will be $99-\frac{1}{20}(1{,}600)=19$.
(e.4) The customer has the following options: (1) choose *NI* and no effort, with a corresponding utility of 48, (2) choose *NI* and effort, with a corresponding utility of 48.0625, (3) choose contract *F* and no effort, with a corresponding utility of 48 and (4) choose contract *F* and effort, with a corresponding utility of $48-\frac{15}{16}$. Thus she will choose option (2), that is, no insurance and the monopolist's profits will be zero.
(e.5) Contract *N* is shown in the figure below:



The deductible is zero and the premium is given by the solution to $\sqrt{2{,}500-h}=48.0625$ which is h = 189.99.

(e.6) The "reservation indifference manifold" is shown as a thick red kinked curve in the figure below: