ARE/ECN 240B Reference Sheet

Notation. θ_0, Θ, y_i, x_i,$s(y_i, x_i; \theta)$ and other notation pertain to the objects defined in the 240B lecture notes.

Assumption ULLN 1 $\sup_{\theta \in \Theta} |\sum_{i=1}^n f(y_i, x_i; \theta)/n - E[f(y_i, x_i; \theta)]| \xrightarrow{p} 0$, if the following conditions hold,

(i) *(i.i.d.)* $\{y_i, x_i\}_{i=1}^n$ is an i.i.d. sequence of random variables;

(ii) *(Compactness)* Θ is compact;

(iii) *(Continuity)* $f(y_i, x_i; \theta)$ is continuous in θ for all $(y_i, x_i')'$;

(iv) *(Measurability)* $f(y_i, x_i; \theta)$ is measurable in $(y_i, x_i')'$ for all $\theta \in \Theta$;

(v) *(Dominance)* There exists a dominating function $d(y_i, x_i)$ such that $|f(y_i, x_i; \theta)| \leqslant d(y_i, x_i)$ for all $\theta \in \Theta$ and $E[d(y_i, x_i)] < \infty$.

Assumption ULLN 2 $\sup_{\theta \in \Theta} |\sum_{i=1}^n f(y_i, x_i; \theta)/n - E[f(y_i, x_i; \theta)]| \xrightarrow{p} 0$, if the following conditions hold,

(i) (*Law of Large Numbers*) $\{y_i, x_i\}$ is i.i.d., and $E[f(y_i, x_i; \theta)] < \infty$ for all $\theta \in \Theta$, which implies $\sum_{i=1}^n f(y_i, x_i; \theta)/n \xrightarrow{p} E[f(y_i, x_i; \theta)]$.

(ii) (*Compactness of* Θ) Θ is in a compact subset of $\mathbb{R}^k$.

(iii) *(Measurability in $(y_i, x_i')'$)* $f(y_i, x_i; \theta)$ is measurable in $(y_i, x_i')'$ for all $\theta \in \Theta$.

(iv) (*Lipschitz Continuity*) For all $\theta, \theta' \in \Theta$, there exists $g(y_i, x_i)$, such that $|f(y_i, x_i; \theta) - f(y_i, x_i; \theta')| \leqslant g(y_i, x_i)\|\theta - \theta'\|$, for some norm $\|.\|$, and $E[g(y_i, x_i)] < \infty$.

Formula for the score statistic

$$\begin{aligned} & S \\ \equiv & \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n s_i(\hat{\theta}_R)\right)' A_{nR}^{-1} C_{nR}' \left\{ \widehat{Avar} \left(C_{nR} A_{nR}^{-1} \sum_{i=1}^n s_i(\hat{\theta}_R)/\sqrt{n} \right) \right\}^{-1} C_{nR} A_{nR}^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n s_i(\hat{\theta}_R) \right) \end{aligned}$$