University of California, Davis

Department of Agricultural and Resource Economics

M.S. Comprehensive Exam, August 2015

You have four hours for this exam after a 20 minute reading period. You do not need to use the whole time period. This exam consists of three questions. You must answer all three questions.

- Question I is worth 37.5% of the total exam score.
- Question II is worth 25% of the total exam score.
- Question III is worth 37.5% of the total exam score.

Watch the time carefully. The logic used to answer each question is important, so be sure to clearly specify your reasoning, with full sentences. Please support your answers as rigorously as possible – e.g., using diagrams or equations. If you use graphs, make sure they are clearly labeled and large enough to read easily. This is not the time to economize on paper, but keep your responses clear and concise. Make sure your writing is legible; if we can't read it, it will be assumed wrong.

I. Employment contracts

A firm offers employment contracts to a collection of potential workers. The contracts have the form $C = (w, e)$, where w is the salary paid to the worker and e is the effort that the worker must provide. The firm perfectly observes the effort provided (for instance, effort is hours worked and workers' hours are recorded). A worker of type $\theta > 0$ has the following utility function over salary and effort:

$$U_\theta(w, e) = w - \frac{e^2}{2\theta}$$

and gets a utility of zero if she is not working ($w = e = 0$). When presented with a menu of contracts, a worker chooses the contract that gives her the highest utility, and chooses to remain unemployed if all available contracts give her a negative utility. The firm's profit from employing a worker under a (w, e) contract is $\pi = e - w$.

1. Is a worker's utility increasing in w? in e?

2. What is the minimum wage that a worker of type θ will accept to supply a level of effort equal to e? (Hint: There is no optimization problem to solve here.)

Suppose that there are two types of workers, those with $\theta = 1$ (in proportion λ of the population) and those with $\theta = 2$ (in proportion $1 - \lambda$ of the population). Under perfect information, the firm can distinguish workers' types and thus offers a different contract to each type.

3. Under perfect information, what contract should the firm offer to a worker of type θ in order to maximize its profit from employing this worker? Write down the resulting contracts $C_1 = (w_1, e_1)$ and $C_2 = (w_2, e_2)$ offered to each type.

From now on, suppose that the firm cannot distinguish the type of a given worker, although it knows the proportion of each type in the population. There are N workers in the population.

4. Explain what will happen if the firm simply offers the menu of contracts (C_1, C_2) to each worker and lets them choose their preferred contract. What will the firm's total profit be?

5. Now suppose that the firm offers the same single contract to each worker, but sets the terms of the contract so as to maximize its total profit. Find the terms of this contract. Your answer should depend on λ. Interpret.

Now suppose that the firm hires a UCD graduate to figure out how to maximize its total profit. The general solution is to offer a menu of contracts to each worker, say $(\hat{C}_1, \hat{C}_2)$, such that type 1 workers choose $\hat{C}_1$ while type 2 workers choose $\hat{C}_2$. We use the notation $\hat{C}_1 = (\hat{w}_1, \hat{e}_1)$ and $\hat{C}_2 = (\hat{w}_2, \hat{e}_2)$.

6. Carefully write down the individual rationality and incentive compatibility constraints for each type.

7. (a) Using your answer to part 6, argue that type 2 workers will enjoy a strictly positive utility level. Explain intuitively why that is.

(b) Explain why type 1 workers should have zero utility from accepting contract $\hat{C}_1$.

(c) Explain why type 2 workers should be (almost) indifferent between choosing $\hat{C}_1$ and $\hat{C}_2$.

8. Using the results from part 7, write down the maximization problem of the firm with only $\hat{e}_1$ and $\hat{e}_2$ as choice variables. You should end up with an unconstrained maximization problem.

9. Check that the objective function of the maximization problem in part 8 is concave and solve this problem. Deduce the terms of the optimal menu of contracts and the resulting firm profit.

II Econometrics Question

1. Impressed by your answers to Part (1) the City of Davis has offered you a position as Human Resources Analyst (Junior Grade). Your first task is to review a consulting report paid for by your predecessor. The consulting firm, **Shaydee LLC**, surveyed 2000 residents of Davis and asked them how much they earned, their educational attainment, and other demographic variables. As best you can tell from the report, the consultants ran a regression of the form:

$$\ln\left(\text{Wage}_{i,j}\right) = \beta_0 + \beta_1 \ln\left(\text{Education}_{i,j}\right) + u_{i,j},$$

where i indexes households and j indexes wines.

   (a) What assumptions must be true for the OLS estimator b_1 to produce an unbiased estimator β_1 and how do you interpret this coefficient?

   (b) You are somewhat suspicious of the consultants' report. In particular, you know that when offering wages, employers often use Education as a signal of Ability. How does this affect your interpretation of the results above? Do you think the consultants are likely to have over or under estimated the effect of Education? Please express the bias algebraically. Clearly state any assumptions you need to make to sign any potential bias.

   (c) In discussing your concerns with your boss, they suggest using an *Instrumental Variables* approach. Specifically, they suggest using parents' educational attainment – which was collected as part of they survey as an Instrument.

      1. In your opinion, does the parents' education meet the criteria of a valid instrument? Please clearly state what makes an instrument valid as part of your answer.
      2. How do you feel about using instrumental variables under these circumstances? Is there an alternative approach you would suggest?

2. You become even more suspicious when looking at the regression results and notice that the number of observations reported by the consultants in their regression output is lower than the number of households surveyed. You obtain the raw survey responses and plot a histogram of the outcome variable **Wage**, this is in Figure 1.

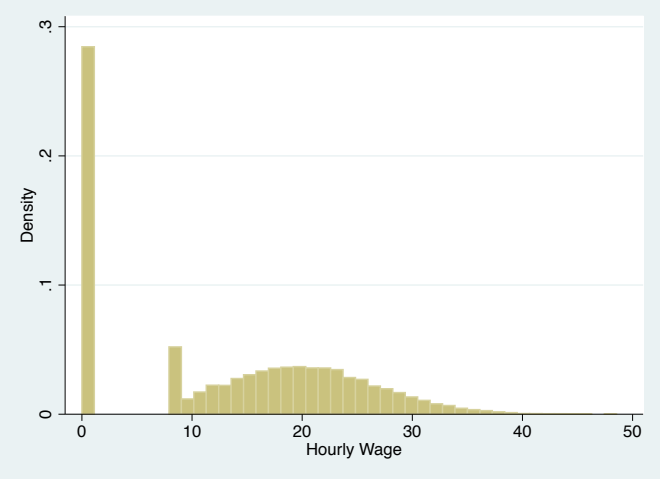
   (a) Can you explain why the number of observations in the regression and the number of households in the survey differ?

   (b) Thinking only about the issues raised by Figure 1, do you think the elasticity provided by the consultants is an over estimate or an under estimate of the true elasticity of demand?

   (c) What other econometric issues does Figure 1 raise?

   (d) What might be an appropriate strategy to analyze data of this form? Describe the method and the any assumptions or additional data you would require to implement it.

Figure 1: Distribution of Wages



III. Production functions and fertilizer in Africa

Agricultural productivity in Sub-Saharan Africa (SSA) lags behind all other regions of the world. Low fertilizer application rates are one factor that contributes to this low productivity. For example, farmers in SSA apply on average 8kg/ha of fertilizer each year compared to 200kg/ha on average in East Asia. The World Bank calls this "an escalating soil fertility crisis" in Africa.

In this problem, you will explore possible explanations for low fertilizer usage and evaluate different empirical approaches to test these explanations.

1. Suppose that farmers are risk-neutral and maximize profits. To simplify the problem initially, suppose that they each have the same fixed amount of land with uniform soil fertility and face a one input production function $y = f(N) + \varepsilon$ where $N \geq 0$ indicates the amount of fertilizer (nitrogen) they apply, $f' > 0, f'' < 0$, and ε is a stochastic production shock with $\varepsilon \sim N(0, \sigma)$. To further simplify the problem, assume that farmers apply N only at planting and that the timing of fertilizer application is uniformly optimal for all farmers.

    (a) Write out a farmer's maximization problem for a single growing season using standard notation. Introduce and define any additional notation you need. Be sure to capture all of the relevant elements of the farmer's decision.

    (b) Solve for the first order condition of this problem and use it to characterize and discuss the solution to this simple model.

    (c) How does your problem change if these farmers are risk averse? Revise your depiction of the maximization problem, solve for the new first order condition and discuss how optimal fertilizer application changes compared to the solution in 1(b).

    (d) Often production inputs can affect the variability of production outcomes as well as their expected value. To allow for this possibility, consider a modified production function of this form: $y = f(N) + h(N)\varepsilon$.

        i. Assume that fertilizer is a risk-reducing input. What then must be true about the h function?
        ii. How does this modification change the maximization problem of a risk averse farmer? How does it change his optimal fertilizer application?

2. Now assume that farmers' plots are heterogeneous in soil fertility, denoted as soil organic matter (*SOM*), and that the production function is characterized by two inputs: $y = f(N, SOM) + \varepsilon$.

   (a) What sign would you expect $\frac{\partial^2 y}{\partial N \partial SOM}$ to have? Explain.

   (b) Use a Translog production function to formulate an econometric specification you could use to estimate this production function. How would you test 2(a) using your specification?

3. Von Liebig, a 19th Century German chemist, discovered what is known as the "law of the minimum." Applied to crop production, this law states that there is some threshold level of soil fertility above which *N* limits output and below which it is other inputs that limit output. To be more precise, denote the threshold level of *SOM* as *SOM**. Then, the law of the minimum states that

$$\frac{\partial y}{\partial N}\begin{cases} > 0 \ if \ \ SOM \geq SOM^* \\ \approx 0 \ if \ \ SOM < SOM^* \end{cases}$$

   (a) Discuss carefully how this von Liebig minimum compares to the relationship you posited in 2(a).

   (b) Suppose that you and the farmer can both observe *SOM* and know *SOM**. Modify your Translog production function specification to allow you to test whether *SOM** really is a von Liebig minimum. Be sure to define any additional notation you introduce. Discuss the interpretation of the coefficients in your specification and statistical tests you would use for the von Liebig minimum.

4. Production frontiers can be useful both conceptually and empirically.
   (a) Discuss carefully how they differ from production *functions* (like those in earlier parts of this problem). Provide an example of one research question that a production frontier approach can address but a production function cannot.

   (b) Describe the econometric implications of the distinction between the two.

5. So far we have assumed that both you and the farmer can observe *SOM*. Now consider an asymmetric information case, which happens to be the most likely case of all: The farmer knows both *SOM* and the von Liebig minimum, but you can observe neither.

   (a) True/False: "Your estimated coefficient on *N* will be biased." Justify your answer carefully using concepts from econometrics.

(b) Based on what you know from this problem, will your estimated coefficient be biased upward (overstate the true effect) or downward (understate the true effect)? Explain.

6. This problem is based on a 2009 AJAE paper by Marenya and Barrett that uses data from farmers in Kenya. In that paper, they estimate the marginal value product of *N* and non-parametrically regress this on a measure of *SOM.* This regression (shown below) clearly shows the presence of a von Liebig minimum.

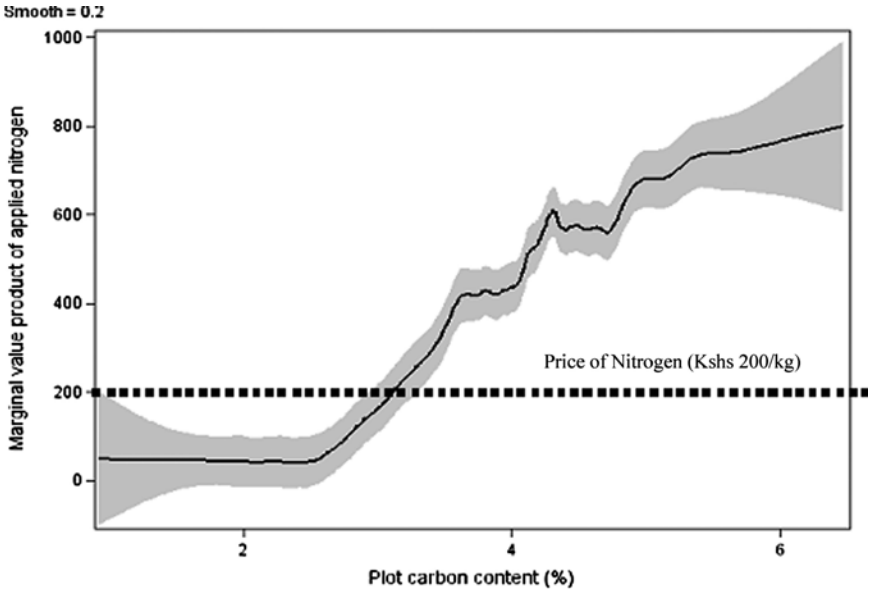


Figure 3 Estimated marginal value product of nitrogen fertilizer (Kshs/kg N) conditional on plot soil carbon content (SOM) with 95% confidence bands in gray

(a) How does this non-parametric regression compare to your proposed specifications in questions 2 and 3 above? Based on this evidence, choose the specification that is most defensible and update the specification so it would provide a parametric angle on the research question.

(b) The "escalating soil fertility crisis" in Africa is partly due to the fact that farmers have been extracting soil fertility without replenishing it with organic or chemical fertilizers. Using what you have learned in this problem, describe briefly the dynamic interactions between agricultural productivity, soil fertility and poverty in SSA.

(c) Suppose the Minister of Agriculture in Kenya asked you to provide a single policy recommendation based on this research. In one paragraph, describe carefully your policy recommendation. Be sure to use the figure above as part of your justification for this recommendation.