“Quality standards versus nutritional taxes: health and welfare impacts with strategic firms”

Vincent Requillart, Louis-Georges Soler and Yu Zang
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September 3, 2015

Abstract

Up to now, most nutritional policies have been set up to inform consumers about the health benefits induced by more balanced diets. Reviews of the impacts of these policies show that the effects are often modest. This has led governments to implement, in more recent times, policies focused on the market environment, especially on the characteristics of the food supply. The goal of this paper is to deepen the analysis of firms’ strategic reactions to nutritional policies targeting food quality improvements and to derive a set of optimal policies. To reach this goal, we propose a theoretical model of product differentiation taking into account both the taste and health characteristics of products, and use it to assess the health and welfare impacts of taxation and MQS-based policies. The model studies how a duopoly of mono-product firms reacts to three alternative policies: an MQS policy, linear taxation of the two goods on the market, and finally taxation of the low-quality good. We find that only the MQS policy and the linear excise tax on the low-quality product are welfare increasing. The choice, however, between the two depends on the priorities of the regulator. On the one hand, for a given moderate level of improvement in health, we show that social welfare increases more with the tax policy than the MQS policy. On the other hand, for a larger increase in the health status of the population, a MQS-based policy may be preferred. Moreover, the policies have distributional effects that must be taken into account, in particular for reasons related to their social acceptability. Finally we show that policies aiming at changing the food market environment allow getting greater health benefits and welfare than policies only based on information campaigns.

JEL codes: I18, L13, Q18

Key words: taxation, MQS, product differentiation, strategic pricing, nutritional policies

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1 Introduction

Cardiovascular disease and cancers cause almost two thirds of the overall burden of disease in developed countries. A large part of these chronic diseases are due to lifestyle-related risk factors, most of them being preventable (WHO, 2009). In particular, poor dietary habits contribute to these diseases through excessive intakes of salt, carbohydrates, and fats, and insufficient intakes of fruits and vegetables (F&V). According to the WHO (2009), combined with actions reducing physical inactivity and tobacco use, preventing diet-related risk factors could lead to an increase in the average life expectancy by three to five years in high-income countries.

To deal with these public health issues, most nutritional policies aim at better informing consumers about the health benefits induced by more balanced diets. Thus, governments and public health agencies have been implementing policies that aim at promoting preventive behaviors thanks to information campaigns, and food product labeling. Reviews of these policies show that they have some positive impacts but these impacts remain quite small, at least in the medium term (Brambila-Macias et al., 2011). In addition, they are suspected to increase the health inequalities, with less-educated individuals responding less to the information (Etité, 2013).

Given the modest impacts of information-based policies, public health agencies are now considering other policies to modify the market environment in order to facilitate healthier food choices, even by non-health-sensitive consumers. A broad range of instruments have been considered, from taxation and subsidizing policies for modifying the relative prices of foods to nutrition-related standards (Brambila-Macias et al., 2011). In this context, public health agencies and policy makers urge the food industry to favor better food environment through changes in quality and variety of foods and through changes in advertising and marketing (WHO, 2012).

Which public health benefits can be reasonably expected from changes in firms’ behaviors? How can these changes contribute to reducing diet-related chronic diseases? To answer these questions, it is important to analyze how food firms react to nutritional policies as depending on the design of the policy instruments they may amplify or weaken the impacts of these policies.

Few recent works have begun to consider the strategic behaviors in prices of food firms in relation to health and nutrition policies. For instance, Bonnet and Réquillart (2013) studied the effects of the soft drink tax which has been in place since 2012 in France, taking into account the price response of soft drink producers. They showed that the price reactions of firms strongly affect the sugar intake at the consumers’ level, the reduction in sugar intake being greater with an excise tax than with an ad valorem tax. Allais et al. (2015) studied the fresh dairy product market in France. They conducted simulations to determine what could occur if fat labeling were to become mandatory in this market. They showed that firms would react to this policy by making a significant reduction in the price of the fattiest products. This price adjustment would limit consumers switching
to healthier dairy products. Another example is related to price responses to advertising bans. Dubois et al. (2013) considered the potato chips sector in the UK and conducted an ex-ante analysis of the impact of an advertising ban. They showed that, on average, prices would be lowered to limit the direct effect of the advertising ban leading to a lower reduction in potato chips consumption than expected.

Firms can also affect health outcomes induced by food consumption through their decisions related to the quality and the variety of foods. In general, the response of firms is based on market segmentation and product differentiation. Nutrition and health claims (such as ‘rich in fibers’, ‘light in sugar’, ‘enriched in vitamins’...) play this role by targeting health-sensitive consumers who have higher willingness to pay for additional health functions in foods. Market shares for those products, however, remain relatively small, about 20%. Regarding the remaining part of the market, the nutritional quality of food is more contrasted (Réquillart and Soler, 2014). For this reason, public health agencies urge the food industry to commit, in individual or collective agreements, to reduce the level of ‘bad’ nutrients in food products. Empirical studies have shown some moves, sometimes quite significant for some brands or some nutrients (Webster et al., 2011; Rahkovsky et al., 2012). Voluntary commitments of firms to improve the nutritional quality of foods, however, remain modest. For instance, in the U.K., in 2006, the food industry and the Food Standards Agency set target levels of salt for each food category. The changes in food quality implemented by food firms contributed to a reduction of the average salt intake at the consumer’s level (Griffith et al., 2014). This reduction, however, has been smaller than expected and the average consumer’s salt intake is still above the target of 6g/day (Shankar et al., 2013). In France, 35 voluntary commitments were signed by the food industry between 2008 and 2013. This policy has led to significant changes in some sectors, but the overall effect on consumers’ intake is weak (less than 10 Kcal/day/consumer) because the number of committed firms remain small (Oqali, 2012).

On the whole, the economic and public health literature tends to consider that firms have very weak incentives to enter into a product reformulation strategy aimed at improving the nutritional quality of products. Several explanations have been proposed. A first blocking point relies on the cost issue. Indeed, reformulation is likely to affect production costs. For example, changes in ingredients might affect variable costs, and development of new recipes might require R&D expenditures (Traill et al., 2012). A second reason is based on asymmetry of information between producers and consumers. Smith (2004) suggested that consumers do not really know a product’s quality, even when nutrient fact panels are available on the food packages, as these panels are difficult to understand. This asymmetric information problem results in a ‘lemons-style’ breakdown in the market for processed foods, leading to the ‘McDonald’s equilibrium’ in which low-quality covers all the market. A third reason is related to the ‘addiction assumption’. If consumption of added sugars or fat leads to addictive behaviors, then firms have strong incentives to continue to market foods with high contents in sugar or fat (Smith and Tasnadi, 2007). This assumption is plausible, but remains very controversial, even among neurophysiologists and nutritionists.
A fourth explanation relies on consumers’ expectations. Indeed, many studies show that, despite the fact that moderate changes in salt or fat content are not always perceived by consumers, once they know, many consumers reject the reformulated product because they consider that ‘healthier’ means ‘less tasty’ (Raghunathan and Naylor, 2006). In other words, for some consumers, taste is more important than health issues when they purchase foods. The assessment of nutritional policies must take into account this statement, as it influences not only consumers’ behaviors and their responses to public interventions aiming at modifying their food choices, but also firms’ decisions about price and quality, which may amplify or weaken the impacts of these policies.

An example of strategic change of the quality of products is found in Moorman et al. (2012). These authors investigated how firms responded to standardized nutrition labels on food products required by the US Nutrition Labeling and Education Act (NLEA). They found that the NLEA had a negative effect on the nutritional quality of labelled brands relative to control brands not required to have a nutrition label. The rationale behind this result is found in the correlation between taste and nutrition attributes and consumer arbitrage between taste and nutrition. If consumers believe that nutrition is negatively correlated with taste and the taste characteristic is more important than the nutrition characteristic in consumers’ choices, then the strategic response of firms to the NLEA is to decrease nutritional quality to avoid discouraging consumption. This response is reinforced by the fact that price is also a key variable for consumers, whereas more nutritious products are likely to be more costly.

To force food quality improvements, policy makers may use different instruments. A first policy relies on the implementation of minimum quality standards (MQS). The theoretical literature dealing with MQS provides mixed insights. In a simple setting, namely when differentiation between products relies on a single characteristic, quality standards seem to be quite efficient (Ronnen, 1991; Crampes and Hollander, 1995). For example, the ban upon trans-fatty acid (TFA) in New York state and in Denmark, where a mandatory maximum content of TFA was implemented in 2004, seem to have had positive impacts (Unnevehr and Jagmanaite, 2008). In a more complex setting, however, where products are differentiated along multiple characteristics, setting MQS might be counterproductive, even if the market underprovides quality (Deltas et al., 2013). Other tools such as food taxes can be designed to influence the quality chosen by the firms. For instance, the regulator might define a quality threshold. Products that have a quality higher than the threshold are not taxed whereas products that have a quality lower than the threshold are taxed. Producers of low-quality products might have an interest in reformulating their products in order to escape the tax. Such a policy seems to be efficient provided that quality thresholds are not too stringent (Duvaleix-Tréguer et al., 2012). Indeed, when the threshold is not too high, firms prefer to reformulate and then avoid the tax, leading to positive results for health and welfare.

The goal of this paper is to deepen the analysis of firms’ strategic reactions to nutritional policies targeting food quality improvements and to derive a set of optimal policies.
To reach this goal, we propose a theoretical model of product differentiation taking into account both the taste and health characteristics of products, and use it to assess the health and welfare impacts of taxation and MQS-based policies. An important challenge comes from the need to integrate these two characteristics, both of which affect consumers’ utility. Dealing with two dimensions in product differentiation models is tricky. In particular, in a duopoly setting in which firms first choose the characteristics of products and then compete in prices, it remains very difficult to determine the optimal choice of characteristics by firms in a general setting. To solve this difficulty, researchers generally impose some restrictions in the choice of characteristics and/or in the heterogeneity of consumers. Thus, to analyze nutritional policies, Duvaleix-Tréguer et al. (2012) designed a duopoly model in which products are differentiated according to two characteristics. They studied the impact of the entry of a firm on this market and considered that the incumbent firm cannot change the characteristics of its product, thus restricting the complexity of the problem. In addition, they also put some restrictions on the heterogeneity of consumers. Deltas et al. (2013) explored a duopoly operating in a market with consumers who care about both an environmental attribute and another brand-specific attribute. They developed a two-dimensional differentiation model and assumed that firms cannot choose the brand-specific attribute. They also restricted the heterogeneity of consumers, assuming they all value identically the environmental attribute. Greker (2006), who analyzed the choice between an environmental standard and a voluntary eco-label scheme by a government in a trade model, also restricted the choice of product characteristics by firms.

In the model of product differentiation we develop, we consider two mono-product firms competing in price and product characteristics. The products are differentiated along a one-dimensional axis of product characteristic (e.g., more or less salty) but the position of a product on this axis may affect consumers’ utility in two ways: through its health impact (the lower the content in salt, the greater the health benefits), on the one hand, and its taste (due to the content in salt), on the other hand.\(^1\) Thus, we take into account the linkages between the nutritional quality of food products and their taste characteristics. This complex relationship between taste and health characteristic of a product is a key point in the analysis of firms’ strategy.

Using this framework, we compare the impact of three policies - setting an MQS; setting an excise tax based on the nutrient content of the two products; and setting an excise tax based on the nutrient content of the ‘bad’ product - on consumer demand, prices, product characteristics, a health indicator, and welfare. We show that with the MQS both firms improve the quality of the products, by choosing products with a lower content of the nutrient. This policy improves health and is welfare increasing. Taxation of the two products has a positive impact on health because it provides incentives to firms to improve the quality of their products. This policy, however, might be welfare decreasing, which

\(^1\)Conrad (2005) used a similar idea that a characteristic of a product might affect consumers according to different dimensions.
makes this option unappealing for the social planner. Taxation of the low-quality product is more appealing, as the policy improves health and is welfare improving. This policy induces a change in the quality of products, which is consistent with the social planner’s point of view, provided that the tax rate is not too high. Among the three policies we analyzed, we find that only the MQS policy and the linear excise tax on the low-quality product are effective in a general sense. The choice, however, between the two depends on the priorities of the regulator. On the one hand, for a given moderate level of improvement in health, we show that social welfare increases more with the tax policy than the MQS policy. On the other hand, for a larger increase in the health status of the population, a MQS-based policy may be preferred. Moreover, the policies have distributional effects that must be taken into account, in particular for reasons related to their social acceptability. Finally we show that policies aiming at changing the food market environment allow getting greater health benefits and welfare than policies only based on information campaigns.

The paper is organized as follows. Section 2 presents the model and section 3 the benchmark equilibrium. Section 4 contains the analysis of firms’ strategic reactions to different policy interventions. In Section 5, we compare the effectiveness of policy initiatives, and Section 6 concludes.

2 The Model

2.1 Main assumptions

We develop a duopoly model of product differentiation. There are two firms $i$ and $j$ producing a product of characteristics $x_i$ and $x_j$ respectively. The characteristic is the content of the product in respect of some nutrient (e.g., salt content, sugar content, fat content, ...). Each firm chooses the content of its product on a $[0,1]$ interval. This characteristic has two effects on the product. First, it affects the taste of the product. From a consumer perspective, this is a horizontally differentiated characteristic, as some consumers might have different preferences for the taste of the product. We model this as in the standard Hotelling model, and a consumer faces a transportation cost that is a function of the distance between her location (denoted $x$) and the location of the chosen product. We assume a uniform distribution of $x$ over $[0,1]$; when $x$ approaches 1, it means that this consumer prefers a high content of this ingredient. In addition, the characteristic of the good also affects the health of the consumer. From the consumer perspective, this is modeled as a vertically differentiated characteristic. We assume that we are in a situation in which consumers have to reduce their intake of some nutrients (salt, fat, sugar) and we focus on a product category in which policies are designed to make the products healthier. As the $x_i$ characteristic of product $i$ is its nutrient content, the lower $x_i$ the better is the product from a health perspective. This health effect is a long-term effect and consumers might differ in their awareness about health impacts of their consumption. We denote $\lambda$ as the awareness of a consumer and assume that it is uniformly distributed over $[0,1]$; consumers with larger
λ give more importance to health. Moreover, we assume that there is independency in $x$ and $\lambda$, which are the two characteristics of a consumer. Thus, we capture consumers’ heterogeneity and consider different types of consumers, depending on their sensitivity to health and taste dimensions. For instance, consumers with a low $\lambda$ prioritize taste over health. To sum up, from the firm viewpoint, this model is a one-dimensional model of product differentiation. From the consumer perspective, however, it is a two-dimensional model as for a given characteristic a consumer infers two characteristics that have an effect on her utility. A consumer is thus represented by a two-dimensional random variable $(x, \lambda)$.

We denote $U_c(x, \lambda)$ as the utility consumer $(x, \lambda)$ gets when she buys one unit of product $c$, $c \in i, j$, and write:

$$U_c(x, \lambda) = v - t(x_c - x)^2 + \lambda(1 - x_c) - p_c$$

with $t$ the per-unit transportation cost. $t(x_c - x)^2$ is the transportation cost just as in the Hotelling model. This part gives the disutility of not consuming the ideal food that consumer $x$ has to suffer if she buys a product of characteristic $x_c$. $\lambda(1 - x_c)$ is the health component of the utility. $\lambda$ is the consumer’s awareness of health, and $1 - x_c$ represents the healthfulness of product $c$. The closer the food is to 1, the lower the health-related utility a consumer gets. This part of the model is similar to a Mussa-Rosen model with $1 - x_c$ being the quality of product $c$. $p_c$ is the price charged by firm $c$. Finally, $v$ is the intrinsic utility that consumers get from buying this food. As the two products differ only in their content of the specific ingredient, $v$ is the same for the two products. We assume that $v$ is large enough so that the market is covered. Thus, with this utility function, if $t = 0$, then we are in the case of a Mussa-Rosen model. If alternatively $\lambda = 0$ for all consumers, we are in the case of a Hotelling model. As compared to a pure Hotelling model, the main interest to consider the heterogeneity of consumers with respect to health, is to discuss the impact of the different policy instruments on health inequalities (see section 5.1). In the following we denote by $U_i$ and $U_j$ the utility a consumer gets when buying one unit of a product from firm $i$ and $j$, respectively. Without loss of generality, we assume $x_j > x_i$. Firm $i$ produces the ‘healthy’ product and firm $j$ produces the ‘unhealthy’ product. We will also refer to these products as high-quality and low-quality products, respectively.

By equating $U_i$ and $U_j$, we get the indifference line, along which the consumers are indifferent between consuming product $i$ and consuming product $j$. It is given by:

$$\lambda = 2tx - t(x_j + x_i) - \frac{p_j - p_i}{x_j - x_i}$$

Consumers located to the left of the indifference line buy from firm $i$, and consumers located to the right of this line buy from firm $j$. It is easy to deduce the demand $D_i$ and $D_j$ faced by firms $i$ and $j$ respectively. When consumer (0,0) buys product $i$ and consumer
(1,1) buys product j, we have:

\[ D_i = \frac{x_j + x_i}{2} + \frac{p_j - p_i}{2t(x_j - x_i)} + \frac{1}{4t} \]  

(3)

\[ D_j = 1 - D_i = \frac{4t - 1}{4t} - \frac{x_j + x_i}{2} - \frac{p_j - p_i}{2t(x_j - x_i)} \]  

(4)

From the supply side, modifying the nutrient content of food is costly. In particular, producing healthier products might increase production costs. For example, in order to decrease the salt content in ham without worsening its quality in other respects, firms will need additional processes, and this will make production more costly (He and MacGregor, 2009). In the case of TFA, which was progressively removed from several products, it has been shown that lowering the content of this ingredient was costly, as firms had to substitute it with more expensive fats (Unnevehr and Jagmanaite, 2008). To take this effect into account, and following Duvalleix-Tréguer et al. (2012), we assume that the marginal cost of production is a quadratic function of the product’s healthfulness, that is, \( 1 - x_c \) for a firm producing a product of characteristic \( x_c \). We have:

\[ c(x_c) = a(1 - x_c)^2 \]  

(5)

where \( a \) is a cost parameter.

The profit functions of a firm is written as:

\[ \Pi_f = (p_f - a(1 - x_f)^2)D_f, \ f = (i,j) \]  

(6)

To analyze competition between the two firms, we assume that they play a two-stage game. In the first stage, firm \( i \) and firm \( j \) simultaneously choose the characteristic of their products \( (x_i, x_j) \). After their choices are observed by each other and consumers, they compete à la Bertrand and simultaneously choose prices \( (p_i, p_j) \). Consumers then make their choice and profits are realized. The design of the game is standard and reflects the idea that the choice of the characteristic of a product is a long-term decision whereas the choice of a price is a short-term one.

### 2.2 Health index and welfare

The goal of the paper is to analyze how policies will affect the equilibrium prices and characteristics of the products as well their impact on health and welfare. To evaluate the impact on health, we define a health index.

As explained above, \( x_c \) is the content of a product \( c \) in respect of a nutrient that adversely affects health. The lower \( x_c \) the better it is from a health point of view. Then, \( 1 - x_i \) and \( 1 - x_j \) are indicators of the healthfulness of products \( i \) and \( j \) respectively. As
each consumer consumes one unit of either one of the two products, the health status of each individual is directly related to the quality (nutrient content) of the chosen product. In epidemiological models, the impact on health of a change in the consumption of a nutrient is evaluated thanks to a relative risk index (RR). This index links a change in the consumption of the nutrient by a consumer to a change in the probability of getting a disease. To integrate this issue in our analysis, we define the health index of a consumer consuming a product located in $x_c$ by $h_c = (1 - x_i)$. By doing so, we consider the simple case in which the RR does not vary with the initial intake.²

As we consider public health issues, we also define the aggregate index over the market. From a public health perspective, the overall population’s health depends on the nutritional quality of the marketed products and the market shares of the different products. Then we consider the following public health index $h_{ij}$:

$$h_{ij} = (1 - x_i)D_i + (1 - x_j)D_j$$

(7)

The definition of welfare also needs some discussion. Social welfare is the sum of producer surplus, consumer surplus, and tax revenues if there are any tax revenues. The issue is related to how the social planner evaluates the health impact of consumption. As explained above, we assume that consumers differently integrate the long-term impact of their consumption on health. The parameter $\lambda$ is the consumer awareness for health. A consumer with a low value for $\lambda$ does not take much into consideration the health impact of consumption whereas a consumer with a high value for $\lambda$ almost fully integrates the health impact. To evaluate the consumer surplus by the social planner, we distinguish two cases. Following the terminology used by Salanie and Treich (2009), a first case corresponds to a populist social planner who evaluates the consumer surplus on the basis of the utility function of consumers (we denote the associated level of social welfare $SW_1$). A second case corresponds to a paternalistic social planner who fully integrates the health impact of consumption (Cremer et al., 2012). In this case, the consumer surplus is evaluated by the social planner on the basis of a modified utility function. For the social planner, a consumer who consumes product $i$ gets the following utility:

$$U'_i = v - t(x_i - x)^2 + (1 - x_i) - p_i$$

(8)

The utility now integrates the ‘true’ impact of consumption on health (we denote the associated social welfare $SW_2$).

²A more general case would be to consider that the impact of a given change in consumption does depend on the initial level of consumption. More precisely, in our case of an ‘unhealthy’ nutrient, this would mean that a given decrease in the consumption of the nutrient has more impact for a consumer who already has a high level of consumption (that is, who consumes product $j$) than for a consumer who has a low level of consumption (that is, who consumes product $i$).
3 Market Equilibrium

To solve the two-stage game, we determine the perfect equilibrium by backward induction. We determine first the Nash price equilibrium in the sub-game given the characteristics $x_i$ and $x_j$. Then, we determine the optimal characteristics of products denoted by $x_i^B$ and $x_j^B$.

3.1 Prices and product characteristics at the equilibrium

At the second stage, prices are given by:

$$p_i^*(x_i, x_j) = \frac{2a(1 - x_i)^2 + a(1 - x_j)^2}{3} + \frac{(x_j - x_i)(1 + 2t(2 + x_i + x_j))}{6}$$  \hspace{1cm} (9)

$$p_j^*(x_i, x_j) = \frac{a(1 - x_i)^2 + 2a(1 - x_j)^2}{3} + \frac{(x_i - x_j)(1 + 2t(-4 + x_i + x_j))}{6}$$ \hspace{1cm} (10)

And the equilibrium locations in the first-stage game are given by:

$$x_i^B = \frac{4a - 1 - t}{4(a + t)}$$  \hspace{1cm} (11)

$$x_j^B = \frac{4a - 1 + 5t}{4(a + t)}$$ \hspace{1cm} (12)

See the Appendix for detailed calculations.

In the following, we constrain the optimal characteristics of products $i$ and $j$ to be in the range of possible locations (that is, $0 < x_i < x_j < 1$). From (11) and (12), we get $t < 1$ and $1 + t < 4a$. In addition, to ensure that consumer $(0,0)$ buys product $i$ and consumer $(1,1)$ buys product $j$, we get an additional condition $\frac{1}{2} < t$. To sum up, we restrict our analysis to the following cases:

$$\frac{1}{2} < t < 1; 1 + t < 4a$$ \hspace{1cm} (13)

The choice to consider only the cases with interior solutions is justified by the fact that we want to analyze how characteristics of products are affected by policies. With interior solutions, any firm can adapt freely the characteristic of its product, whether this corresponds to an increase or a decrease in the nutrient content of the product. Moreover, by doing so, we also focus on situations in which both products have significant market shares rather than a situation in which one of the two firms is in a 'niche' market. Under (13), we consider situations in which $t$ is quite large and then in which consumers are more reluctant to move from their preferred location. This is meaningful as it is well known that taste plays an important role in food choices. In addition, under (13), we also consider situations in which $a$ is large enough, meaning that changes in food quality are quite costly. With this assumption, we then focus on situations that are less favorable for public health.
Using (10) to (12), we get the equilibrium prices, which are function of $a$ and $t$ (Table 1). At the equilibrium, firms choose characteristics that are not symmetric, that is, $x_i^B \neq 1 - x_j^B$. This is related to the quadratic cost function as it is more costly for firm $i$ to improve the quality of its product than for firm $j$. Profits of both firms are, however, equal in this model. This comes from the fact that the absolute markups of producers $i$ and $j$ are equal ($p_i^B - c(x_i^B) = p_j^B - c(x_j^B) = \frac{3t^2}{2(a+t)}$) and demands are equal.

<table>
<thead>
<tr>
<th>Expression</th>
<th>$t \uparrow$</th>
<th>$a \uparrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i^B$</td>
<td>$\frac{4a-1-t}{4(a+t)}$</td>
<td>↓</td>
</tr>
<tr>
<td>$x_j^B$</td>
<td>$\frac{4a-1+5t}{4(a+t)}$</td>
<td>↑</td>
</tr>
<tr>
<td>$p_i^B$</td>
<td>$\frac{a+10at+49at^2+24t^3}{16(a+t)^2}$</td>
<td>↑</td>
</tr>
<tr>
<td>$p_j^B$</td>
<td>$\frac{a-2at+25at^2+24t^4}{16(a+t)^2}$</td>
<td>↑</td>
</tr>
<tr>
<td>$d_i^B = d_j^B$</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
</tr>
<tr>
<td>$\pi_i^B = \pi_j^B$</td>
<td>$\frac{3t^2}{4(a+t)}$</td>
<td>↑</td>
</tr>
<tr>
<td>$x_j^B - x_i^B$</td>
<td>$\frac{3t}{2(a+t)}$</td>
<td>↑</td>
</tr>
<tr>
<td>$h_{ij}^B$</td>
<td>$\frac{1+2t}{4(a+t)}$</td>
<td>↑ if $a &gt; 1/2$</td>
</tr>
<tr>
<td>$SW1^B$</td>
<td>$\frac{9+8t(3-4a)-26t^2}{96(a+t)}$</td>
<td>↓</td>
</tr>
<tr>
<td>$SW2^B$</td>
<td>$\frac{15+8t(6-4a)-26t^2}{96(a+t)}$</td>
<td>↓</td>
</tr>
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Table 1: Characterization of the benchmark

When the unit transportation cost increases, product $i$ moves to the left while product $j$ moves to the right, and the prices of both products increase. Thus, when $t$ increases, a consumer is more likely to purchase the product near her own location, rather than the further one. This allows producers to soften competition by moving away from each other, which allows for a price increase. In addition, marginal costs of producers are affected by the change in the characteristic of their products. Producer $i$ increases the quality of its product, generating an increase in the marginal cost and thus in price. Producer $j$ faces a different situation. Its marginal cost of production decreases as a result of the decrease in the quality of its product (that is $x_j$ increases). The effect, however, of softened competition on price is larger than that of the marginal cost change and, as a consequence, the price of product $j$ increases. This also explains that the price increase of product $j$ is lower than that of product $i$. As in a standard location model, firm profit increase with $t$.  

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An increase in the cost parameter $a$ leads to a decrease in the level of quality chosen by both firms (that is, $x_i$ and $x_j$ increase). As the cost function is quadratic, when $a$ increases, the high-quality firm $i$ suffers more than the low-quality firm $j$. Hence, the quality adjustment by firm $i$ is larger than that of firm $j$. As a result, the differentiation decreases and competition is more intense. At the equilibrium, prices decrease, meaning that the effect on price of the tougher competition dominates the cost increase. As production costs increase, profits decrease.

3.2 Health and welfare

An increase in the unit transportation cost has a positive impact on the health index when $a > 1/2$ and a negative impact if $a < 1/2$. The health index depends on the quality of products as well as on the market shares of each product. An increase in $t$ does not modify the market shares. In equilibrium, when $t$ increases, the slope of the indifference curve increases but in such a way that the market shares remain constant. The indifference curve rotates around the point $(1/2, 1/2)$. When $t$ increases, the role of transportation costs on consumers’ choice increases, and, as a consequence, some consumers with a low preference for quality now switch to the high-quality product in order to have lower transportation costs. Conversely, some consumers with a high preference for quality switch to the low-quality product. Because market shares are not affected by a change in $t$, the change in the health index is only due to the change in the quality of products $i$ and $j$. When $a > 1/2$, the increase in the quality of product $i$ is larger than the decrease in the quality of product $j$ and hence the health index increases. Welfare decreases when $t$ increases as this corresponds to an increase in the cost supported by consumers. This is true whether the welfare is evaluated using the populist or the paternalistic definition.

Finally, the health index and welfare decrease when the cost parameter $a$ increases, for intuitive reasons. With respect to the health index, this is the consequence of a decrease in the quality of both products, with market shares remaining constant. With respect to welfare, this is because there is an increase in production costs.

3.3 Optimal qualities for the social planner

As a reference for the analysis of the impact of policies, we now define the choice of qualities by a social planner. The social planner maximizes social welfare. To do so, prices are equal to marginal costs and qualities are chosen such that social welfare is maximized. The populist social planner maximizes the consumer surplus as it is defined by consumers. The optimal qualities are given by:

$$x_i^{SW1} = \frac{-1 + 12t(-1 + 4a) + 12t^2}{48t(a + t)}$$

Thus, we have: $\frac{\partial h_{ij}}{\partial t} = \frac{2a - 1}{4(a + t)^2}$. \[\text{[14]}\]
\[ x_j^{SW1} = \frac{1 + 12t(-1 + 4a) + 36t^2}{48t(a + t)} \] (15)

See the Appendix for detailed calculations.

It is easy to show that \( x_i^B < x_i^{SW1} \) and \( x_j^B > x_j^{SW1} \). In this model, price distortions do not directly play a role as, given the assumption with respect to the covered market, this essentially corresponds to a transfer between producers and consumers.\(^4\) Firms have an incentive to differentiate the products in order to soften price competition. For the social planner, there is no reason to do so and the distortions come from transportation costs and choices of qualities. The social planner’s choice tends to limit the wasted transportation costs and thus choose locations corresponding to less differentiated qualities.\(^5\)

A paternalistic social planner maximizes consumer surplus using a modified utility function (Eq. 8). The optimal qualities are given by:

\[ x_i^{SW2} = \frac{1 + 6t(-1 + 2a) + 3t^2}{12t(a + t)} \] (16)

\[ x_j^{SW2} = \frac{-1 + 6t(-1 + 2a) + 9t^2}{12t(a + t)} \] (17)

See the Appendix for detailed calculations.

It is easy to show that \( x_i^B < x_i^{SW2} < x_i^{SW1} \) and \( x_j^B > x_j^{SW1} > x_j^{SW2} \). As before, the social planner chooses less differentiated products.\(^6\) Compared with the populist social planner, the paternalistic social planner chooses higher qualities for both products. This is intuitive as this social planner considers that consumers should fully internalize the health impact of consumption in their utility whereas a populist social planner uses the consumers’ utility function, which only partially internalizes the health impact. The adjustment towards a higher quality is larger for product \( j \) than product \( i \) as a given increase in quality is less costly the lower the initial quality.\(^7\)

4 Strategic Responses to Policy Interventions

We now analyze the impact on prices, product characteristics, health, and welfare of alternative policy interventions. We analyze first the impact of an MQS policy. The MQS requires that firms operating in the market comply with a certain quality standard. In

\(^4\)Prices play an indirect role in the sense that they have an impact upon the choice of consumers between product \( i \) and product \( j \). They also have an incidence on the optimal qualities.

\(^5\)In the case of a pure Hotelling model, with identical firms, optimal locations are 1/4 and 3/4. In our model, if quality is not costly (\( a=0 \)) we find \( x_i^{SW1} = \frac{1}{4} - \frac{1+12t}{48t^2} \) and \( x_j^{SW1} = \frac{3}{4} + \frac{1-12t}{48t^2} \). The differences with the pure Hotelling model come from the fact that in our model consumers also value the quality, measured by \( 1 - x_i \) and \( 1 - x_j \).

\(^6\)If quality is not costly (\( a=0 \)) we find \( x_i^{SW2} = \frac{1}{4} - \frac{6t-1}{48t^2} \) and \( x_j^{SW1} = \frac{3}{4} - \frac{6t+1}{48t^2} \).

\(^7\)Thus, \( x_i^{SW1} - x_i^{SW2} = \frac{12t-5}{48t(a+t)} \) whereas \( x_j^{SW1} - x_j^{SW2} = \frac{12t-1}{48t(a+t)} \).
our context of an ‘unhealthy’ nutrient, the MQS is defined as the maximum content in the
‘unhealthy’ nutrient content of a product. Such type of policy was defined in the case of
TFA the use of which was progressively prohibited in different countries (Unnevehr and
Jagmanaite, 2008). That is to say that the maximum content in TFA of food products was
progressively decreased thanks to an MQS policy. Then, we analyze the impact of excise
taxes. We design two cases. We consider first a case in which both products are taxed as
a function of their nutrient content. This type of tax was introduced in 2011 in Denmark.
The tax targeted saturated fats, and was specified in DK/kg of saturated fat in the product
(Jensen and Smed, 2013). The second version of the excise tax is to consider that the tax
only applies to the low-quality product. In practice, it means that some products are taxed
and some others are not. We find examples of such taxes in the soft drink market. For
example, Hungary introduced a tax on soft drinks that contain more than 80g/l of added
sugar (ECORYS, 2014). In practice, it means that diet products are not taxed whereas
sugary products are. For taxed products, however, the level of the taxes does not vary
with the sugar content.

4.1 Minimum Quality Standard (MQS)

We assume that the regulator imposes a requirement that the nutrient content of the
products on the market should be lower than some threshold $\bar{x}$. The MQS has an impact
on the choice of characteristics only if $\bar{x} < x_j^B$. Stage 2 of the competition game is not
modified and (9) and (10) apply as they define the price equilibrium at given characteristics
of the products. Stage 1 of the game is modified as the choice of firm $j$ is constrained: we
have $x_j^S = \bar{x}$. Given that firm $j$ chooses the location $x_j^S$, the optimal choice for firm $i$ is:

$$x_i^S = \frac{4a - 1 - 4t}{6(a + t)} + \frac{x_j^S}{3} = x_i^B + \frac{\bar{x} - x_j^B}{3} \quad (18)$$

When firm $j$ faces a constraining MQS, then firm $i$ responds by lowering the nutrient
content of its product, that is, by increasing the quality of its product. We formulate in
proposition 1 the main implications of the MQS policy.

**Proposition 1.** Under an MQS policy that restricts the choice of the characteristic of
the low-quality product, and relatively to the benchmark case:

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8 The tax applied to a broad range of products, such as meat, dairy products, animal fats, and vegetable
oils. The tax only applied to the products that had a content level of saturated fat that was higher than
2.3%. For many markets, however (e.g., the market for oil), all products were taxed, as it was not possible
to completely remove saturated fats from those products.

9 France also introduced a tax on soft drinks in 2012. In the French case, however, the tax applies to all
products whether they contain sugar or not. Moreover, the level of the tax does not vary with the sugar
content. Thus, this cannot be considered as a nutritional tax (ECORYS, 2014).

10 We denote with a subscript $S$ the values of variables at the equilibrium under the MQS scenario.
• The quality of the high-quality good increases and the differentiation in the market decreases;

• The price of the high-quality good decreases whereas the price of the low-quality good increases;

• The high-quality producer is worse off and the low-quality producer is better off;

• The health index increases and so does welfare.

Proof. See the Appendix

When the choice of quality by firm $j$ is constrained and thus firm $j$ increases the quality of its product, the high-quality firm also raises the quality of its product to alleviate the more intense competition. Since improving the quality is more costly for firm $i$ than for firm $j$, the change in $x_i$ is smaller than that of $x_j$. The differentiation between the two products decreases, leading to tougher competition. The impact on prices depends on two opposite effects. On the one hand, the increase in quality increases marginal costs of production and then prices according to (9) and (10). On the other hand, the reduced differentiation leads to more intense price competition. The net effect of these two opposite effects is positive for firm $j$, that is, the price of product $j$ increases, whereas it is negative for firm $i$, that is, the price of product $i$ decreases. Firm $j$'s price and demand increase, along with a decrease in firm $i$'s price and demand level. As a consequence, firm $i$ is worse off and firm $j$ is better off. The health index increases due to the improvement in the quality of both products. This effect dominates the negative impact (on health) of the shift in the demand. Thus, some consumers switch from the high-quality good to the low-quality good. We also find that social welfare, whether it is defined by a populist or a paternalistic social planner, increases. In this setting in which the market is covered, price distortions due to market power do not affect welfare directly as this corresponds to a transfer between consumers and producers. The distortion that is reduced is the distortion due to the change in the characteristics of the products. The increase in quality of product $j$ (that is, a lower $x_j$) decreases the distortion on product $j$ as $x_j^B > x_j^{SW1} > x_j^{SW2}$. On the contrary, the increase in quality of product $i$ increases the distortion on product $i$ as $x_i^B < x_i^{SW1}$. This latter negative effect is, however, lower than the positive one due to the increase in quality of product $j$.

These results are in line with the results from Ronnen (1991) and Crampes and Hollander (1995) who analyzed the impact of imposing an MQS in a duopoly framework with vertical product differentiation and in which firms first choose the quality of their products and then compete in prices. Both papers found that the MQS acts as a commitment device for firm $j$, providing to this firm a first-mover advantage. As a consequence, firm $j$ is better off and firm $i$ is worse off. The papers by Ronnen (1991) and Crampes and Hollander (1995) differ with respect to the cost of quality. In Ronnen (1991), an increase
in quality has an impact on fixed costs, whereas in Crampes and Hollander (1995), it has an impact on variable costs, as is the case in our framework. Interestingly, in Crampes and Hollander (1995), the qualitative results depend on the quality adjustment of product $i$. When the quality of product $i$ increases less than that of the low-quality, which is the case in our model, then all qualitative results we have are similar to their results.

On the other hand, our results about the impact of MQS on product quality differ from those in Deltas et al. (2013). This paper considers a duopoly operating in a market with consumers who care about both an environmental attribute, which could be considered as the quality attribute, and another brand-specific attribute. With respect to the brand-specific attribute, the firms are assumed to locate at the two ends of a unit interval. With respect to the environmental attribute, consumers have the same willingness to pay for products’ greenness. In their setting, the consumer heterogeneity is only in terms of brand-specific horizontal attributes. Additionally, the horizontal attribute and the vertical one are independent. Using this framework, the authors found that firms’ environmental qualities are strategic substitutes. Then, the implementation of an MQS leads the high-quality firm to decrease the quality of its product, which is the opposite of our results. In our setting, on the contrary, product qualities are strategic complements and as a result, an increase in the standard leads to the augmentation of both products’ qualities.\textsuperscript{11}

4.2 Excise tax on both products

To penalize the use of the ‘unhealthy’ nutrient, the regulator sets a tax on both products. In order to penalize the low-quality product more than the high-quality product, we consider a linear excise tax with a rate $f$. That is, a product of characteristic $x_c$ faces a tax $f \ast x_c$. The profits of firms are given by:

\begin{align*}
\Pi_i &= (p_i - a(1 - x_i)^2 - f x_i)D_i \\
\Pi_j &= (p_j - a(1 - x_j)^2 - f x_j)D_j
\end{align*}

(19) (20)

The tax acts as an increase in production costs. We formulate in proposition 2 the main implications of the excise tax policy.

Proposition 2. Under an excise tax proportional to the nutrient content of both products, and relatively to the benchmark case:

- The content of the taxed nutrient decreases for both products by the same amount;
- The prices of both products increase;

\textsuperscript{11}In the benchmark case, at the equilibrium, we have,

$$\frac{\partial \Pi_i^2}{\partial x_i \partial x_j} = \frac{\partial \Pi_j^2}{\partial x_j \partial x_i} = \frac{(a + t)^2(x_j - x_i)}{a^2} > 0.$$
• The profits of producers remain constant;

• The health index increases, the welfare evaluated by a populist social planner decreases, and the welfare evaluated by a paternalistic social planner increases as long as the tax rate is not too high.

Proof. See the Appendix.

In order to limit the impact of the tax, firms choose higher quality for their products. As a consequence of the tax and the increased qualities, prices increase. Moreover, because the tax is linear, then the impact on the marginal cost of quality is identical for both firms (= f). This explains why the content in the taxed nutrient decreases for both products by the same amount. In addition, the markups remain constant, as do the market shares. The consequence is that profits do not change. Those adjustments are also a consequence of the assumption relative to the market coverage. Thus, the cost increase, which both firms face, can be transmitted to the consumers without losing consumers. The adjustments in price and qualities are such that with linear taxes the indifference curve (which defines consumers indifferent between consuming product i and j) is not affected. Because market shares do not change and the quality of both products increases, then the health index increases. Social welfare, defined by a populist social planner, decreases. On the one hand, the distortion on the low-quality decreases but on the other hand the distortion on the high-quality increases. The overall effect on welfare is negative. If defined by a paternalistic social planner, however, welfare increases, meaning that the negative impact of an increase in the distortion of the high-quality product now has less importance than in the first case.

A variant of this policy is to design a bonus malus taxation policy (keeping the characteristic that the tax/subsidy scheme is linear with respect to the content of the ‘unhealthy’ nutrient). That is to say that products that have a level of content of the ‘unhealthy’ nutrient that is lower than some threshold benefit from a subsidy. Conversely, products that have a level of content of the ‘unhealthy’ nutrient that is larger than the threshold are taxed. This type of policy was put in place in France in the car industry (Reynaert, 2014). A threshold in term of CO2 emission per km was defined. Vehicles with higher emissions than the threshold face a tax whereas those with lower emissions benefit from a subsidy. In term of nutrition, the idea is to acknowledge that the consumption of most nutrients is not ‘bad’ per se but rather it is their excessive consumption that has negative health impacts. Formally, rather than defining the tax by \( f \times x_c \) for a product of characteristic \( x_c \), the tax is defined as \( f \times (x_c - \bar{x}) \). This is thus a combination of the previous scheme and a fixed subsidy to the firm. In our model, the qualities at equilibrium are not affected by a fixed subsidy (applied to both products) or a fixed tax. Prices change by the amount of the per-unit subsidy, so that absolute markups do not change. Other variables are not affected. We only have a transfer between consumers and taxpayers. Thus, all the results presented in the case of a linear excise tax on the two products also apply to this kind of bonus malus policy.
4.3 Excise tax on the low-quality

An alternative policy is to discourage the consumption of the low-quality good by setting a tax on this product. We keep the same scheme as above, that is a linear excise tax, but apply it to the low-quality product only. Thus, product $j$ faces a tax $f \cdot x_j$. Profit of firm $i$ is given by (6) and profit of firm $j$ is given by (20). The tax acts as an increase in the production costs of firm $j$ only. We formulate in proposition 3 the main implications of the excise tax policy.

**Proposition 3.** Under an excise tax proportional to the nutrient content of the low-quality product, and relatively to the benchmark case:

- The quality of the high-quality product decreases; the quality of the low-quality product increases as long as the tax rate is not too high;
- The price of the low-quality product increases whereas the price of the high-quality product decreases;
- The high-quality firm is better off and the low-quality firm is worse off;
- The health index increases, and so does welfare.

**Proof.** See the Appendix

With the tax, firm $j$ faces a cost increase that is proportional to the nutrient content of its product. This provides this firm with an incentive to improve the quality (that is, to lower the nutrient content $x_j$) of its product. The response of firm $i$, facing a less competitive firm (because its product is taxed), is to reduce the quality of its product (that is to increase $x_i$). The change in the price of product $i$ results from three elements: a reduction in the cost due the change in quality, a less differentiated market meaning tougher competition, and an opposite effect, which is facing a less competitive firm. On the whole, the first two effects dominate the third one, so that the price of product $i$ decreases. The change in the price of product $j$ results from two elements: an increase in the cost due to both the taxation and the increase in quality and an opposite effect resulting from a less differentiated market. The first effect dominates the second one so that $p_j$ increases. The changes in profits are rather intuitive: the taxed firm is worse off and the other firm is better off.

The change in the health index results from three effects, the first two having a positive impact and the third having a negative impact. First, the quality of product $j$ increases. Second, due to the price changes some consumers switch from product $j$ to product $i$. Third, the quality of product $i$ decreases. The impact of the first two effects dominates the impact of the third one, explaining the increase in the health index with the tax. Welfare increases as a consequence in the change in quality. As explained above, distortions in this
model come from the location of the product. In this scenario, both the distortions with respect to product $i$ and product $j$ are reduced. Then welfare increases. It is, however, interesting to note that the quality of the low quality good reaches a maximum for $f = \frac{1}{2}(3t - \sqrt{t^2 + 8at} - 2t)$. If the tax rate is higher than that value, then, firm $j$ has no longer enough incentives to continue to increase the level of quality. On the contrary, it starts to deteriorate the level of quality of its product. This is the result of both the increased competition from firm $i$ as the quality of product $i$ is still increasing, and the increase in the tax rate. For firm $j$, when the tax rate is high, it becomes more profitable to lower the quality, thus relaxing competition and decreasing production costs. As a consequence, the health index decreases when the tax rate becomes high.

5 Policy Comparison

The three policy interventions analyzed in the previous sections clearly differ in their impacts on market segmentation:

- The MQS policy increases both qualities, increases the demand for the low-quality product and decreases the demand for the high-quality product;
- The tax policy on two products does not change the demands but both qualities increase;
- The tax policy on one product increases the quality of the low-quality product (as long as the tax rate is not too high), decreases the quality of the high-quality product, and increases the demand for the high-quality product.

A common feature of the three policies is to provide incentives to the low-quality firm to increase its product’s healthfulness, that is, to move left. We have shown above that all of the three instruments - MQS, a linear excise tax on the two products, and a linear excise tax on the low-quality product - ameliorate the health index. Nevertheless, depending on the instruments, the effects on the other parameters (quality of the high-quality product, demands, prices) are not identical. In terms of social welfare, both the MQS and the linear excise tax on one product increase social welfare whether welfare is defined by a populist or a paternalistic social planner. On the contrary, a linear excise tax on the two products deteriorates social welfare defined by a populist social planner whereas, as long as the tax rate is not too high, it increases social welfare defined by a paternalistic social planner. It means that only a paternalistic regulator could decide to implement this last policy, provided that the tax rate remains small, that is to say as long as the social planner does not target a large improvement in the health index.

The change in welfare is well explained by the changes in quality distortions. In this respect, a tax on the low-quality product is the only policy that reduces both the distortions on the low-quality product and the high-quality product. In order to go a step further in
the analysis and to determine which policy is the most powerful in achieving the regulator’s objective, we now compare the impact of the three policies on surplus and social welfare for a given increase in the health index. We discuss these effects in relation to the heterogeneity of consumers. Finally, we compare these policies with information-based policies.

5.1 Impacts on social welfare

In this section, we assume that the social planner has the objective of increasing health and we determine the best instrument to achieve this objective.

Due to the complexity of the analytical expressions, we perform numerical simulations to compare the different instruments. Given condition (13), we choose specific values for the parameters $t$ and $a$. We select three different levels of health index improvement. For each level of the health index, we then determine the MQS (in the case of the MQS-based policy) and the tax rates (in the case of a tax policy) that are requested to obtain at the equilibrium the targeted health index. Table 2 provides the numerical values of the different variables computed with the model. We do not comment upon the impact of each policy on the different variables, as this was done in the previous sections. Rather, we focus on the comparison of welfare impacts of the different policies. In addition, we determine the health impacts depending on the type of consumers. Indeed, an extensive literature reports large health inequalities between less and more informed (or health-sensitive) consumers in many countries (see for instance Drewnowski, 2009). Then, it is important to identify to what extent these health inequalities might be weaken or amplified depending on the policy instruments adopted by the policy makers. To do so, we split consumers into two groups depending on the weight they give to health issue. Consumers characterized by $1/2 \leq \lambda \leq 1$ are health-conscious consumers, and consumers characterized by $0 \leq \lambda \leq 1/2$ are non-health-conscious consumers.

Result 1. For a given and moderate targeted increase in the health index, taxation of the ‘unhealthy’ product increases more social welfare than with an MQS-based policy. The MQS-based policy allows, however, to reach an increase in the health index that is higher than the one reached under a taxation policy.

For a given and moderate level of the health index, the linear excise tax on the low-quality product has a larger positive impact on social welfare than the MQS. This is true whether social welfare is defined by a populist or a paternalistic social planner. These two instruments dominate the linear excise tax on two products, which is even not socially desirable for a populist social planner.

The different agents are, however, differently affected by the alternative policies. The excise tax on both products leaves total profits unchanged, whereas they decrease under

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12 We report the results for given values of $t$ and $a$. To test for the validity of the qualitative results, we tested the different scenarios for numerous values of $a$ and $t$. This allows us to draw conclusions.
Table 2: Comparison of the impact of alternative policies for a given health index outcome

the other two policies. From the industry point of view, the worst policy is the excise tax on the low-quality product (Table 3). Moreover, firm i and firm j have different preferred policies. Firm i prefers the tax policy on the low-quality product whereas firm j prefers the MQS. Conversely, the low-quality firm benefits from the MQS policy, which provides it with a first-mover advantage. On the demand side, the consumer surplus decreases with any tax policy whereas it increases under the MQS policy.

To sum up, the taxation of the two products does not seem to have strong support. It is the worst option for the social planner (and it is welfare decreasing for a populist social planner) and for the consumers. It is neither the preferred option of firm i nor of firm j.

Table 3: Ranking of the impact of alternative policies (leading to a given and moderate health index outcome) for different agents

<table>
<thead>
<tr>
<th>Policy</th>
<th>Tax both products</th>
<th>MQS</th>
<th>Tax ‘bad’ product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Industry</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Firm i</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Firm j</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Populist social planner</td>
<td>-</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Paternalistic social planner</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Taxation of the low-quality product and an MQS policy might find some support. From the social planner’s point of view, whether he is populist or paternalist, the best instrument is the taxation of the low-quality product (assuming here a linear taxation scheme). Both the consumers and the industry as a whole, however, would prefer the MQS to the excise tax on the low-quality product.

It is worthwhile to note, however, that it is possible to target higher health impacts with an MQS than with a tax on the low-quality product. Indeed, when the tax rate is low, firm \( j \) chooses to ameliorates the quality of its product. For higher level of taxation, however, firm \( j \) no longer chooses to ameliorate the quality of its product. On the contrary, firm \( j \) progressively decreases the level of quality of its product. As explained above, this is a consequence of the competition of the other firm which faces a less and less competitive firm. Thus, for a moderate gain in health, the tax policy can be preferred, but an MQS-based policy must be prioritized as soon as the social planner targets a large improvement of the health status of the population.

**Result 2.** All policies reduce health inequalities. Taxation of the low-quality product, however, reduces health inequalities in a larger extent than an MQS-based policy. The disparity in consumers’ surpluses between the most and least informed groups increases more under a tax of the ‘unhealthy’ product than under an MQS-based policy.

Regarding the health impact, it turns out that all policies are progressive in that they reduce the gap between more and less informed consumers. Indeed, in all cases the quality of the low-quality product is improved. As, on average, less informed consumers consume initially more this product, their health status is always improved. Regarding the quality of the high-quality product, it increases less than the low-quality product (MQS policy) or decreases (tax policy). As a consequence, the health status of more informed consumers is either less improved than the health status of less informed consumers, or is downgraded. Regarding the gap between consumers’ surpluses, it appears that, in comparison with the benchmark situation, all policies increase it. An MQS-based policy is, however, the least regressive policy. Indeed, the consumers’ surpluses of the most and least informed groups increases under an MQS-based policy, whereas the consumers’ surplus of the least informed group is decreased by the tax policy.

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13 The last column of Table 2 reports the results corresponding to the highest level of health index that is achievable by a policy. Taxation of the low quality product does not allow reaching such a level for the health index.

14 As explained above, under taxation of the low-quality product, the quality of the low-quality product increases only if the tax rate is not too high; this is the case we consider here.
Table 4: Comparison of the impact of market environment policies versus information policy

### 5.2 Information policy versus policies changing the market environment

Up to now, we have considered policies targeting changes in the market environment of consumers. Their goal is to favor health benefits even for non-informed and non-health-conscious consumers. In practice, however, public planners mainly develop information policies. In the following, we analyze if market environment policies perform better, in terms of health and welfare impacts, than policies exclusively based on education and information campaigns?

To determine to what extent information-based policies may be more or less effective than policies changing the market environment, we build the following scenarios. First, as in the previous sections, we consider that the consumers are heterogeneous and their information level (the parameter $\lambda$) is uniformly distributed between 0 and 1. Using numerical simulations, we determine the values of the MQS and the tax rate (assuming that the tax applies on the low-quality product) that maximize the social welfare (scenario 1 in Table 4). Second, in scenario 2, we assume that, thanks to successful information campaigns, all the consumers are informed (technically, we assume $\lambda = 1$ for every consumer) and we calculate the health index and welfare at the equilibrium.

**Result 3.** An MQS-based policy increases more the social welfare and the health status of the population than a policy only focused on consumers’ information. A tax policy is always more welfare-increasing than the information policy alone, but it does not allow reaching a higher health index.

Our model suggests that public intervention on the consumers’ environment through an MQS may be required for at least two reasons. First, the quality distortions at the equilibrium, discussed in section 3.3, still hold with perfectly informed consumers: the quality of the high-quality product is too high, and the quality of the low-quality product is too low. In other words, under an information policy distortions due to market power still exist and policies targeting changes in product quality must always complement the information policy. Second, as shown in Table 4, if we consider the two opposite policies - a policy targeting changes in the market environment through an MQS (scenario 1) versus...
a successful information policy without any change in the market environment (scenario 2) - it turns out that the former allows reaching higher levels for the health index and welfare. The tax policy is welfare-increasing too, but the health index is smaller than the one obtained with a fully successful information policy. In conclusion, it appears that an MQS must be selected against an information policy to get higher welfare and health index.

The superiority of the MQS-based policy on the information policy might be due to the fact that, in scenario 1, we assumed that at least some consumers were already informed and health-conscious (as $\lambda$ is distributed over $[0, 1]$). To test this issue, we compare a successful information policy (scenario 2) with a MQS-based policy assuming that no consumer is informed ($\lambda = 0$, scenario 3). Numerical simulations show that, even in this case, an MQS-based policy is more effective than the perfect information policy both for social welfare and the health status of the population.

6 Conclusion

Besides public health policies targeting changes in consumers’ behaviors, policy makers are implementing policies aiming at increasing the nutritional quality of foods available on the market. Indeed, many studies suggest that most of the food products available for sale are still of a low nutritional quality level, as they contain high quantities of salt, added sugars or fat. In this article, we propose an original model of product differentiation that aims at estimating to what extent tax- and standard-based policies can contribute to improve the nutritional quality of foods, and then the health status of the population.

On the supply side, a single characteristic, that is, the content of the product in some ‘unhealthy’ nutrient, the consumption of which should be limited, is used to define the product. On the demand side, consumers value a product thanks to its taste, which is a function of the nutrient content, as well as its health impact. In this case of an ‘unhealthy’ nutrient, the lower the content of the nutrient, the better the product is for health. For consumers, the characteristic of a product has thus a horizontal dimension (taste) as well as a vertical one (health). We consider that consumers are heterogeneous with respect to taste and with respect to the weight they give to the health impact. The main contribution of our analysis is to evaluate the market, health, and welfare impacts of alternative policies, integrating strategic reactions of firms both in terms of prices and product characteristics.

More precisely, we consider that firms compete in a two-stage game in which they first choose the characteristic of the products, here represented by their nutrient content or equivalently by their healthfulness, and then set prices. Thanks to the design of the model, which considers the case of a single characteristic that affects consumer’s utility in two dimensions, we do not restrict the choice of product characteristic, as is frequently the case.

In the absence of any policy intervention, we find that in equilibrium, as in the standard horizontal differentiation model, firms differentiate their products. The differentiation
between the two products increases with the transportation cost parameter whereas an increase in the production cost parameter leads firms to choose less ‘healthy’ products. Two important results are, however, that (i) the products are more differentiated than in the standard horizontal differentiation model - this is due to the consumers’ heterogeneity in both health and taste dimensions; and (ii) for a welfare-maximizing planner, the two products are too differentiated - the quality of the high-quality product is too high, and the quality of the low-quality product is too low. Thus, the model gives a theoretical background to the empirical statement of the insufficient nutritional quality of foods. In comparison to real-world observations, we could then consider that the market differentiation between ‘niche’ products based on health and nutritional claims and standard food products is not satisfying, ‘niche’ products’ being positioned at too high a level of quality and price levels, and standard products at too low a level of quality and price levels.

Alternative public interventions can be envisaged to favor welfare-increasing market segmentation. To compare the effectiveness of alternative policy interventions, we consider three indicators: a health index, which is actually a index of the healthfulness of the products, and two welfare functions depending how the social planner evaluates the consumer surplus. A populist social planner evaluates the consumer surplus on the basis of the utility function of consumers as he does not fully integrate in the consumer surplus the ‘true’ health impact of consumption. Alternatively, a paternalistic social planner fully integrates the health impact of consumption. We show that a social planner chooses locations corresponding to less differentiated products. This is because firms have an interest in differentiating the products in order to soften price competition, which is not the case for a social planner. A social planner thus seeks to limit the full costs in the market, which depends on the production costs and the transportation costs. We also show that a paternalistic social planner chooses higher qualities for both products than a populist social planner. This is because the paternalistic social planner fully internalizes the health impact of consumption, whereas the populist social planner does not.

We study three nutritional policies. A policy based on MQS that imposes a threshold upon the low-quality product and two scenarios of an excise tax proportional to the nutrient content of the product. A first case is when both products are taxed whereas the second version assumes that only the low-quality product is taxed. With the MQS, both firms improve the quality of the products, that is, they choose products with a lower amount of the nutrient. Product differentiation decreases, however, as it is more costly for the high-quality good to improve the quality of its product. Due to the first-mover advantage, the low-quality firm gains a higher profit than the benchmark model, while the high-quality firm loses under this regulation. This policy improves the health index and is welfare increasing whether welfare is evaluated by a populist or a paternalistic social planner. Those qualitative results are in line with those from the literature on MQS (e.g., Crampes and Hollander, 1995).

Taxation of the two products has a positive impact on health because it provides incentives to firms to improve the quality of their products. For a populist social planner,
however, this is not a policy option to favor because it decreases welfare. A paternalistic social planner might consider this option. For a given improvement in the health index, however, the increase in welfare is lower than the one under the other two options. Taxation of the low-quality product is more appealing, as the policy improves health and is welfare improving. This policy induces a change in the quality of products, which is consistent with the social planner’s point of view. Thus, product differentiation decreases because, on the one hand, the quality of the low-quality good increases, and on the other hand, the quality of the high-quality good decreases. This leads to lower levels of distortions in the market.

Among the three policies we analyze, we find that only the MQS policy and the linear excise tax on the low-quality product are effective in a general sense. The choice, however, between the two depends on the priorities of the regulator. On the one hand, for a given moderate level of improvement in health, we show that social welfare increases more with the tax policy than the MQS policy. But the MQS allows reaching higher targets in the health index as well as higher welfare. Therefore, from a social planner’s point of view, the MQS-based policy must be privileged. On the other hand, the policies have distributional effects that must be taken into account, in particular for reasons related to their social acceptability. On the supply side, the profit of the high-quality firm decreases and the profit of the low-quality firm increases under the MQS. The opposite occurs under the excise tax. There is thus no consensus in the industry, but the industry as a whole loses less under MQS. On the demand side, consumers’ surplus increases under the MQS whereas it decreases if the tax is applied. Moreover, the MQS policy is less regressive than the tax policy. Finally, the health inequalities are reduced by all policies, but they are more decreased by the tax policy than the MQS-based policy. We also find that an information policy, even if it is fully successful, performs less than policies targeting the market environment. The latter must always complement the information policy. A MQS-based policy always allows reaching higher levels of the health index and welfare than an information policy alone. It is not the case of the tax policy which leads to a higher welfare than the information policy, but to a lower level of health index.

An important assumption of the model is the market coverage assumption. Thus, we assume that the global demand is fixed. The demand for each firm is elastic as consumers switch from one product to another one, but the whole demand is inelastic. In practice, this means that we implicitly consider markets of products that are consumed by all consumers, which are difficult to substitute in the diet. The limit is mainly related to the impact of an increase in the quality of the ‘bad’ product, which is the cheaper one. The different policies analyzed above lead to an increase in the quality of the bad product and to an increase in its price. We assume here that it does not discourage consumption as every consumer continues to buy one of the products on the market. In practice, it is possible that some consumers stop consuming this product. This would affect their surplus but this will also affect the profit of the firm selling that product. A second limitation of the model is related to the assumption that a consumer buys one unit of a product. By doing so,
we do not capture an individual quantity effect that might occur when the characteristic of the product changes. It is possible that a decrease in the content of a ‘bad’ nutrient generates an increase in the quantity consumed as consumers might think that they ‘can’ eat more of the product as it is now safer. To deal with this second issue requires a radical change in the model to endogenize quantity decisions by consumers. Finally, a possible extension of the model would be to consider multi-products firms.

Acknowledgements
This paper benefited from comments by seminar participants at the 16th annual European Health Economics Workshop (Toulouse, France), GREMAQ seminar, 11th World Congress in Health Economics (Milan, Italy). Financial support from the ANR-French National Research Agency, project ANR-011-ALID-002-02 OCAD, and INRA-Metaprogram DIDIT is gratefully acknowledged.

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7 Appendix

All results are obtained using Mathematica.

7.1 Benchmark Model

Given the indifference line (2), we define demands as functions of $p_i$, $p_j$, $x_i$, $x_j$:

$$D_i = \frac{1}{4t} + \frac{x_i + x_j}{2} + \frac{p_j - p_i}{2t(x_j - x_i)}$$

(21)

$$D_j = 1 - \left( \frac{1}{4t} + \frac{x_i + x_j}{2} + \frac{p_j - p_i}{2t(x_j - x_i)} \right)$$

(22)

**Optimal qualities**

Using (21) and (22), we rewrite the profit function (6) and get the price equilibrium $p_i^*(x_i, x_j)$ and $p_j^*(x_i, x_j)$ in stage 2 (Eq. (9) and (10)) from the first order conditions of profit maximization. We deduce $\pi_i(p_i^*(x_i, x_j), p_j^*(x_i, x_j))$ and $\pi_j(p_i^*(x_i, x_j), p_j^*(x_i, x_j))$ which are maximized wrt $x_i$ and $x_j$ respectively. Among the set of possible solutions, we select the optimal locations (Eq. (11) and (11)) using the second order conditions:

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} |_{x_i = x_i^B} = -\frac{a + t}{2} < 0$$

$$\frac{\partial^2 \Pi_j}{\partial x_j^2} |_{x_j = x_j^B} = -\frac{a + t}{2} < 0$$

Then, $x_i^B$, $x_j^B$ are the optimal locations for firms $i$ and $j$, respectively.

**Conditions of existence**

We first impose that $x_i^B$, $x_j^B$ are within the range $[0,1]$ with $x_i^B < x_j^B$. It implies $1 + t < 4a$ and $t \leq 1$. Moreover, our definition of the demand has some implicit assumptions. Considering ($x = x00, \lambda = 0$) the indifferent consumer located in $x00$ and ($x = x11, \lambda = 1$) the indifferent consumer located in $x11$, then we assume implicitly that $x00$ and $x11$ are in the range $[0,1]$. In other words, the indifference line crosses the horizontal lines defining the consumer square. From (2), we deduce:

$$x00 = \frac{x_i + x_j}{2} + \frac{p_j - p_i}{2t(x_j - x_i)}$$

(23)

$$x11 = \frac{1}{2t} + \frac{x_i + x_j}{2} + \frac{p_j - p_i}{2t(x_j - x_i)}$$

(24)

Replace (25), (26) and $x_i^B$, $x_j^B$ into (23), (24). We get: $x00 = \frac{1}{2} - \frac{1}{4t}$, $x11 = \frac{1}{2} + \frac{1}{4t}$.  

30
It is easy to check that, that conditions on \( x_{00} \) and \( x_{11} \) imply \( t \geq 1/2 \). Conditions are summed up in (13).

**Equilibrium prices**
Substituting \( x^B_i \), \( x^B_j \) into the second stage prices (Eq. (11) and (11), we get the equilibrium prices:

\[
p^B_i = \frac{a + 10at + 49at^2 + 24t^3}{16(a + t)^2} \tag{25}
\]

\[
p^B_j = \frac{a - 2at + 25at^2 + 24t^3}{16(a + t)^2} \tag{26}
\]

To study how the equilibrium prices vary with the parameter \( t \), we write:

\[
\frac{\partial p^B_i}{\partial t} = \frac{12t^3 + a^2(5 + 49t) + a(-1 - 5t + 36t^2)}{8(a + t)^3} \tag{27}
\]

\[
\frac{\partial p^B_j}{\partial t} = \frac{12t^3 + a^2(-1 + 25t) + a(-1 + t + 36t^2)}{8(a + t)^3} \tag{28}
\]

Under condition (13), it is easy to show that (27) and (28) are positive. We also have \( \frac{\partial p^B_i}{\partial t} > \frac{\partial p^B_j}{\partial t} \). This is verified iff \( a + 4at - t > 0 \), which is true given that we have \( 1 + t < 4a \).

To study how the equilibrium prices vary with the parameter \( a \), we write:

\[
\frac{\partial p^B_i}{\partial a} = \frac{t(1 + 10t + t^2) - a(1 + 10t + 49t^2)}{16(a + t)^3} \tag{29}
\]

\[
\frac{\partial p^B_j}{\partial a} = \frac{t - 2t^2 - 23t^3 + a(-1 + 2t - 25t^2)}{16(a + t)^3} \tag{30}
\]

Given condition (13), we can draw the following curves of \( a = \frac{t+1}{4} \), \( a = \frac{t(1+10t+t^2)}{1+10t+49t^2} \) and \( a = \frac{t-2t^2-23t^3}{1-2t+25t^2} \) on the range \( \frac{1}{2} < t < 1 \) (we use the software Mathematica), and it suggest that:

\[
\frac{t + 1}{4} > \frac{t(1 + 10t + t^2)}{1 + 10t + 49t^2} > \frac{t - 2t^2 - 23t^3}{1 - 2t + 25t^2} \tag{31}
\]

as \( a > \frac{t+1}{4} \), then \( a > \frac{t(1+10t+t^2)}{1+10t+49t^2} > t - \frac{2t^2 - 23t^3}{1-2t+25t^2} \), through which we can get that (29), (30) are both negative.

**Equilibrium profits**
To get the demand at equilibrium, substitute (25), (26) and \( x^B_i \), \( x^B_j \) into (21), (22):. We have:

\[
D_i = D_j = \frac{1}{2} \tag{32}
\]
To get the profits at equilibrium, substitute (25), (26) and \(x_B^i, x_B^j\) and (32) into (6). We have:

\[\Pi_i = \Pi_j = \frac{3t^2}{4(a + t)}\]  

(33)

We also have:

\[\frac{\partial \Pi}{\partial t} = \frac{3t(2a + t)}{4(a + t)^2} > 0\]
\[\frac{\partial \Pi}{\partial a} = -\frac{3t^2}{4(a + t)^2} < 0\]

**Distance between firm \(i\) and firm \(j\)**

The distance between the two forms is a measure of the product differentiation. We have:

\[x_B^j - x_B^i = \frac{3t^2}{2(a + t)}\]

and

\[\frac{\partial (x_B^j - x_B^i)}{\partial t} = \frac{3a}{2(a + t)^2} > 0\]
\[\frac{\partial (x_B^j - x_B^i)}{\partial a} = -\frac{3t}{2(a + t)^2} < 0\]

**Health index**

The health index is computed by substituting (11), (12), and (32) into (7). When \(\alpha = 1\), we have:

\[h_{ij} = \frac{1 + 2t}{4(a + t)}\]  

(34)

and

\[\frac{\partial h_{ij}}{\partial t} = \frac{2a - 1}{4(a + t)^2}\]
\[\frac{\partial h_{ij}}{\partial a} = -\frac{1 + 2t}{4(a + t)^2} < 0\]

It comes directly that \(\frac{\partial h_{ij}}{\partial t} \geq 0 \iff a \geq 1/2\).

**Social welfare**

The indifference curve is given by:

\[\lambda(x) = 2tx - t(x_j + x_i) - \frac{p_j - p_i}{x_j - x_i}\]

Welfare, for a populist social planner, is defined as:

\[SW1 = \Pi_i + \Pi_j + \int_0^{x_0} \int_0^1 U_i d\lambda dx + \int_{x_0}^{x_{11}} \int_0^1 U_i d\lambda dx\]
\[+ \int_{x_{11}}^1 \int_0^1 U_j d\lambda dx + \int_{x_0}^{x_{11}} \int_{\lambda(x)}^1 U_j d\lambda dx\]  

(35)

32
By substituting the equilibrium values of \( x_i^B, x_j^B, p_i^B \) and \( p_j^B \), we get the expression of SW1 as functions of parameters \( t, a \):

\[
SW1 = \frac{9 - 8(4a - 3)t - 26t^2}{96(a + t)} \tag{36}
\]

from which, we get:

\[
\frac{\partial SW1}{\partial t} = \frac{-9 - 32a^2 + a(24 - 52t) - 26t^2}{96(a + t)^2} < 0
\]

\[
\frac{\partial SW1}{\partial a} = -\frac{3 + 8t + 2t^2}{32(a + t)^2} < 0
\]

Welfare, for a paternalistic social planner, is defined as:

\[
SW2 = \Pi_i + \Pi_j + \int_{x_00}^{x_{11}} U_i' d\lambda dx + \int_{x_{00}}^{x_{11}} U_j' d\lambda dx
\]

\[
= \int_{x_00}^{x_{11}} U_i' d\lambda dx + \int_{x_{00}}^{x_{11}} U_j' d\lambda dx
\tag{37}
\]

By substituting the equilibrium values of \( x_i^B, x_j^B, p_i^B \) and \( p_j^B \), we get the expression of SW2 as functions of parameters \( t, a \):

\[
SW2 = \frac{15 + (48 - 32a)t - 26t^2}{96(a + t)} \tag{38}
\]

from which, we get:

\[
\frac{\partial SW2}{\partial t} = \frac{-15 - 32a^2 + a(48 - 52t) - 26t^2}{96(a + t)^2} < 0
\]

\[
\frac{\partial SW2}{\partial a} = \frac{5 + 16t + 2t^2}{32(a + t)^2} < 0
\]

To prove that \( \frac{\partial SW2}{\partial t} < 0 \), we just need to solve the equation \(-15 - 32a^2 + a(48 - 52t) - 26t^2 = 0\) by using Mathematica, and get that \( a = \frac{1}{16}(12 - 13t + \sqrt{3\sqrt{8} - 104t - 13t^2}) \) or \( a = \frac{1}{16}(12 - 13t + \sqrt{3\sqrt{8} - 104t - 13t^2}) \). Given that \( \frac{1}{2} < t < 1 \), the previous roots are complex roots. So there is no real root for that equation. In addition, the coefficient before \( a \) is \(-32 < 0\), then we conclude that \( \frac{\partial SW2}{\partial t} \) is always negative given the conditions on \( t \) and \( a \).
7.2 Instrument: MQS

Stage 2 of the game is not modified and thus (9), (10) apply. $x_i^S$ is found by maximising $\Pi_i(p_i^*(x_i, \bar{x}), p_j^*(x_i, \bar{x}))$ over $x_i$. The optimal solution is given by (18). Following the same method as for the benchmark, we deduce prices, demands, profits, health index and welfare at the equilibrium. They are obviously function of $\bar{x}$ which is exogenously set. We provide in the following the analytical expressions of all those variables.

**Equilibrium prices:**

\[
p_i = \frac{1}{108(a + t)^2} (44a^3(\bar{x} - 1)^2 + 2t(1 + 4t(1 + \bar{x}))^2 + 4a^2(\bar{x} - 1)(1 + t(30\bar{x} - 22)) + a(5 + 20t(3 + \bar{x}) + 4t^2(67 - 14\bar{x} + 27\bar{x}^2)) \]

\[
p_j = \frac{1}{54(a + t)} (-1 + t(10 - 8\bar{x}) + 38a^2(\bar{x} - 1)^2 - 8t^2(-7 - 5\bar{x} + 2\bar{x}^2) + 2a(\bar{x} - 1)(-4 + t(-7 + 11\bar{x})))
\]

and

\[
\frac{\partial p_i}{\partial \bar{x}} |_{x_j} = \frac{8t^2 + a(3t - 1)}{6(a + t)} > 0
\]

\[
\frac{\partial p_j}{\partial \bar{x}} |_{x_j} = \frac{a(t - 1)}{2(a + t)} < 0
\]

**Demands:**

\[
D_i = \frac{1 + 4a(\bar{x} - 1) + 4t(\bar{x} + 1)}{18t}
\]

\[
D_j = \frac{-1 + 4a(1 - \bar{x}) + 2t(7 - 2\bar{x})}{18t}
\]

and

\[
\frac{\partial D_i}{\partial \bar{x}} |_{x_j} = \frac{4a + 4t}{18t} > 0
\]

\[
\frac{\partial D_j}{\partial \bar{x}} |_{x_j} = \frac{-4a + 4t}{18t} < 0
\]

**Profits:**

\[
\Pi_i = \frac{(1 + 4a(\bar{x} - 1) + 4t(\bar{x} + 1))^3}{972t(a + t)}
\]

\[
\Pi_j = \frac{1}{972t(a + t)} (1 + 4a(\bar{x} - 1) + 4t(\bar{x} + 1)) * (1 + 4a(\bar{x} - 1) + 2t(-7 + 2\bar{x}))^2
\]

and

\[
\frac{\partial \Pi_i}{\partial \bar{x}} |_{x_j} = t > 0
\]
\[ \frac{\partial \Pi_j}{\partial \bar{x}} |_{x_j^B} = -\frac{t}{3} < 0 \]

**Health index** (assuming \( \alpha = 1 \)):

\[ h_{ij} = \frac{1}{108t(a + t)}(1 + 16a^2(\bar{x} - 1)^2 + 8t(\bar{x} + 1) \]
\[ + 4t^2(31 - 19\bar{x} + 4\bar{x}^2) \]
\[ + 4a(\bar{x} - 1)(2 + t(-19 + 8\bar{x})) \]
\]
\[ (45) \]

and

\[ \frac{\partial h_{ij}}{\partial \bar{x}} |_{x_j^B} = -\frac{1}{3} < 0 \]

**Social welfare:**

\[ SW_1 = \frac{1}{7776t(a + t)}(47 + 1280a^3(\bar{x} - 1)^3 + 12t(17 + 29\bar{x}) + \\
48t^2(77 - 65\bar{x} + 20\bar{x}^2) + 32t^3(-113 + 219\bar{x} - 195\bar{x}^2 + 40\bar{x}^3) + \\
480a^2(\bar{x} - 1)^2(2 + t(8\bar{x} - 13)) + 12a(29(\bar{x} - 1) + 20t(13 - 21\bar{x} + 8\bar{x}^2) + \\
8t^2(-100 + 203\bar{x} - 170\bar{x}^2 + 40\bar{x}^3)) \]
\[ (46) \]

and

\[ \frac{\partial SW_1}{\partial \bar{x}} |_{x_j^B} = \frac{1}{72t} - \frac{t}{3} < 0 \]

\[ SW_2 = \frac{1}{7776t(a + t)}(29 + 1280a^3(\bar{x} - 1)^3 + 12t(23 + 35\bar{x}) + \\
96t^2(85 - 61\bar{x} + 16\bar{x}^2) + 32t^3(-113 + 219\bar{x} - 195\bar{x}^2 + 40\bar{x}^3) + \\
96a^2(\bar{x} - 1)^2(16 + 5t(8\bar{x} - 13)) + 12a(35(\bar{x} - 1) + 8t(61 - 93\bar{x} + 32\bar{x}^2) + \\
8t^2(-100 + 203\bar{x} - 170\bar{x}^2 + 40\bar{x}^3)) \]
\[ (47) \]

and

\[ \frac{\partial SW_2}{\partial \bar{x}} |_{x_j^B} = -\frac{1}{6} - \frac{1}{72t} - \frac{t}{3} < 0 \]

### 7.3 Linear Excise Tax - Scenario 1: Taxing both products

The analysis is similar to the analysis of the benchmark case. To determine the equilibrium, we use the new profit functions defined by (19) and (20).

**Optimal qualities:**

\[ x_i^f = \frac{-1 + 4a - 2f - t}{4(a + t)} < x_i^B \]
\[ x_j^f = \frac{-1 + 4a - 2f + 5t}{4(a + t)} < x_j^B \]

We also deduce that
\[ x_j^f - x_i^f = \frac{3t}{2(a + t)} = x_j^B - x_i^B \]

Conditions ensuring interior solutions are:
\[ \frac{1}{2} < t < 2f + 1; 4a > 1 + t + 2f \] (48)

Equilibrium prices:
\[ p_i^f = p_i^B + \frac{4f(8ta - af - t - 2tf - t^2 + 4a^2)}{16(a + t)^2} \]
\[ p_j^f = p_j^B + \frac{4f(8ta - af - t - 2tf + 5t^2 + 4a^2)}{16(a + t)^2} \]

and
\[ p_i - p_i^B = -\frac{f(-4a^2 + af - 8at + t(1 + t + 2f))}{4(a + t)^2} > 0 \]

Given the conditions over \(t, a\) and \(f\), we have \(p_j^f - p_j^B > 0, p_j^f - p_j^B > 0\) and \(p_j^f - p_j^B > p_i^f - p_i^B\).

Demands and Profits:
\[ D_i^f = D_j^f = \frac{1}{2} = D_i^B = D_j^B \]

It is easy to check that the line defining indifferent consumers is \(\lambda = \frac{1}{2} + t(2x - 1)\) which is identical to that in the benchmark case. Note that the consumer \((\frac{1}{2}, \frac{1}{2})\) belongs to this line.

\[ \Pi_i^f = \Pi_j^f = \frac{3t^2}{4(a + t)} = \Pi_i^B = \Pi_j^B \]

Health index:
\[ h_{ij}^f = \frac{1 + 2f + 2t}{4(a + t)} > h_{ij}^B \]

Welfare:
\[ SW1^f = -\frac{-9 + 24f^2 + 8(4a - 3)t + 26t^2}{96(a + t)} < SW1^B \]
\[ SW2^f = \frac{15 + 24f - 24f^2 + (48 - 32a)t - 26t^2}{96(a + t)} \]

and
\[ SW2^f - SW2^B = \frac{(1 - f)f}{4(a + t)} \begin{cases} > 0 & \text{if } f < 1 \\ < 0 & \text{if } f > 1 \\ = 0 & \text{if } f = 1 \end{cases} \]
7.4 Linear Excise Tax - Scenario 2: Taxing only product $j$

The analysis of this case leads to much more complex expressions. In order to characterize the properties of the equilibrium as compared to the benchmark, we are going to study the impact of setting a marginal tax. Technically we study $\frac{\partial X}{\partial f}|_{f=0}$ with $X$ representing any variable at the equilibrium.

In this case we can get analytically the expressions of prices at the second stage and optimal qualities. To determine prices at the second stage and optimal qualities, we use the profit functions defined by (6) for firm $i$ and (20) for firm $j$.

**Optimal qualities**:

$$x_{ij}^f = \frac{(f - 3t)(-1 + 4a + f - t)}{4(2f - 3t)(a + t)}$$  \(49\)

$$x_{ij}^f = \frac{(f - 3t)(-1 + 4a - 3f + 5t)}{4(2f - 3t)(a + t)}$$  \(50\)

From those expressions, and given the restrictions on parameters (13), that is $\frac{1}{2} < t < 1; 1 + t < 4a$) it is straightforward to show that $\frac{\partial x_{ij}^f}{\partial f}|_{f=0} > 0$ and $\frac{\partial x_{ij}^f}{\partial f}|_{f=0} < 0$. Moreover, it is easy to show that $\frac{\partial x_{ij}^f}{\partial f} = 0$ for $f = \frac{1}{8}(3t - \sqrt{t^2 + 8at - 2t})$.

**Equilibrium prices** Substituting $x_{ij}^f$ and $x_{ij}^f$ into the second stage optimal prices, we get the equilibrium prices. From which, we compute the partial derivatives. We get

$$\frac{\partial p_{ij}^f}{\partial f}|_{f=0} = \frac{-4(a^2 + t^2)(1 + t) + a(1 - t + 2t^2)}{24t(a + t)^2}$$

$$\frac{\partial p_{ij}^f}{\partial f}|_{f=0} = \frac{4a^2(3t - 1) + 2t^2(5t - 1) + a(1 + t + 14t^2)}{24t(a + t)^2}$$

It is easy to prove that under (13) $\frac{\partial p_{ij}^f}{\partial f}|_{f=0} < 0$ and $\frac{\partial p_{ij}^f}{\partial f}|_{f=0} > 0$

**Demands and Profits** From above, we deduce the demands and profits and calculate the partial derivative wrt $f$

$$\frac{\partial D_{ij}^f}{\partial f}|_{f=0} = \frac{4a + 2t - 1}{18t^2} > 0$$

$$\frac{\partial \Pi_{ij}^f}{\partial f}|_{f=0} = \frac{8a + t - 2}{12(a + t)^2} > 0$$

$$\frac{\partial \Pi_{ij}^f}{\partial f}|_{f=0} = \frac{2 - 8a - 7t}{12(a + t)}$$

\(^{15}\)The Mathematica program is available to the authors upon request.
It is easy to prove that under (13) $\frac{\partial D_i^{f_j}}{\partial f} |_{f=0} > 0$ and thus that $\frac{\partial D_j^{f_i}}{\partial f} |_{f=0} < 0$ as the total demand is fixed. We also have $\frac{\partial A_i^{f_j}}{\partial f} |_{f=0} > 0$ and $\frac{\partial A_j^{f_i}}{\partial f} |_{f=0} < 0$.

**Health index:** From the demands and optimal qualities we compute the health index and calculate the partial derivative wrt $f$

$$\frac{\partial h_{ij}^{f_j}}{\partial f} |_{f=0} = \frac{4a(1-t) + 3t + 7t^2 - 1}{36t^2(a + t)} > 0$$

**Welfare:** Using the same method than for the other policy we compute the impact of taxation on SW1 and SW2 and then calculate the partial derivative wrt $f$

$$\frac{\partial SW_1^{f_j}}{\partial f} |_{f=0} = \frac{24t^2 - 1}{96t(a + t)} > 0$$

$$\frac{\partial SW_2^{f_j}}{\partial f} |_{f=0} = \frac{1 + 12t + 24t^2}{96t(a + t)} > 0$$