Gains from investment timing over the business cycle:
Machine replacement in the US rental industry†

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Abstract

We develop and empirically implement a model of optimal replacement of rental equipment and apply the model to derive detailed optimal replacement strategies for five different types of rental machinery: 1) excavators, 2) high reach forklifts, 3) skid steers, 4) scissor lifts, and 5) telescopic booms. Using data on actual replacements, revenues, maintenance costs, new machine prices and resale values, we find significant potential to improve rental company profitability by strategically timing the replacement of equipment, resulting in profit gains ranging from 1 percent to over 1700 per cent. We generally find optimal replacement strategies replacing machines sooner than what we observe in our sample. However in the case of excavators the optimal policy involves replacing machines later than the mean replacement ages for the companies in our sample. We also find gains to adjusting replacement strategy to account for seasonal variation in rental demand and the timing of the business cycle due to their effects on rental revenues and the cost of replacement. For some machines, such as skid steers, we find the optimal replacement strategy is pro-cyclical. That is, a machine of a given age has a much higher probability of being replaced in a boom month than in a bust month. However for telescopic booms, we find that a *countercyclical replacement strategy* — one where replacements are concentrated in slow periods of the business cycle — can significantly increase firm profits.

**Keywords:** rental equipment, rental markets, optimal replacement

†Direct questions to John McClelland at John.McClelland@ararental.org. The data used in this study are confidential and cannot be made available to third parties as per the conditions of a nondisclosure agreement between the data provider, the authors, and the American Rental Association.
1 Introduction

There has been extensive work in economics and operations research on the optimal (profit maximizing) replacement cycles for machinery. Optimal replacement policy is analyzed as a dynamic optimization problem because the optimization occurs over time, and decision making is sequential. Decisions on whether to replace a machine are continually updated to take account of new information that changes over time such as a machine’s age, physical condition, and resale price, and business conditions including the state of the business cycle and other indicators of the demand for rental equipment.

However the type of techniques that have been developed in academic settings have not been widely applied in the rental equipment industry, for reasons that are unclear. Similar to many other industries, equipment replacement and investment decisions are more typically made informally based on experience and intuition of managers of rental equipment firms without the assistance any formal mathematical models. The goal of this paper is determine whether there can be value added from the application of the optimization techniques that are studied from a more abstract, theoretical level in academia, and whether sufficiently applied versions of these methods might be able to improve decision making and profitability in the rental equipment industry.

To be useful, any formalization of firms’ replacement investment problems as dynamic optimization problems need to account for uncertainty such as in the amount of rental revenue the machine will earn, the cost of maintaining the machine, the cost of replacing the machine (both over the cost of a replacement machine, and over the resale or scrap value of an existing machine to be replaced), and over the magnitude and duration of business cycle fluctuations which have high amplitude in the construction sector of the economy.

Replacement demand, and thus production of machinery to satisfy replacement demand, tends to be highly pro-cyclical. That is, replacement demand tends to be highest during “boom” periods when GDP growth is relatively high whereas replacement demand is lowest during “bust” periods when GDP growth is low or negative. Of course this is driven by the pro-cyclicality of construction spending, which affects both the demand for new construction machinery and demand for rental equipment. This is illustrated in figure[1] below which plots an index of the production of construction machinery compiled by the Federal Reserve Bank of St. Louis. We see that production of construction machinery decreases during recession periods, which are indicated by the vertical gray bars in the figure. Machinery production also reflects seasonal cycles and tends to be higher in the summer months and lower in the winter.
Similar cycles (both business cycles and seasonal cycles) affect rental revenues and investment in new machines by the rental industry. Rental firms face considerable uncertainty about the demand for their machines, and since this demand is highly pro-cyclical, replacement investment in this industry tends to be pro-cyclical as well. This cyclicality can lead to high prices of machines and delivery delays for new machines during boom periods when many firms in the industry (as well as other construction firms that own rather than rent their equipment) try to replace their aging equipment at roughly the same time, often just at the start of boom periods.

This cyclicality is evident in figure 2 below, which plots annual US construction spending (from the Census Bureau) and rental industry revenues and investment (from the American Rental Association). We see that all three series are cyclical, but rental industry revenues and investment are even more pro-cyclical than construction spending. For example, rental industry revenues and investment both fell during the 2001 recession and during the Great Recession (indicated by the grey shaded regions in the figure), whereas construction spending only leveled off in the 2001 recession. Rental industry revenues and investment increased faster than construction spending during the building boom from 2002 to 2006, and fell faster than construction spending did during the Great recession. Thus, the amplitude of the cycles in rental revenue and investment is greater than the already large procyclical variation in construction spending.\footnote{However we shall show in section 2 that the rental industry does not exhibit the degree of \textit{seasonal} cyclical in rental revenues or investment that we see in construction spending and in the production of construction machinery shown in figure 1}
In this paper we show that there may be profit opportunities from adopting a counter-cyclical replacement policy where more replacement investment is done during recessions or periods of weak demand for construction machinery. The weak demand for new equipment and relatively lower levels of replacement investment can reduce the price of new equipment while raising (at least in relative terms) the resale price of used equipment. This means it can be significantly cheaper for firms to replace machines in a recessionary period than in a boom period. In this paper we quantify the gains to following such a countercyclical replacement policy and derive conditions where it can be shown to be an optimal replacement policy.

Of course, the term “optimal” is relative to a number of important simplifying assumptions that make our analysis more tractable. We abstract from a number of the larger, more difficult problems that rental companies face, including setting their rental rates, and the allocation of different machines in their (location-specific) rental equipment portfolios. In addition, we abstract from capital constraints that may be important drivers of pro-cyclical investment policies at many rental firms. Thus, our analysis needs to be qualified and interpreted relative to these simplifying assumptions. The policies we derive may not be optimal in the context of a more complex, realistic and encompassing model of a rental firm’s operations that is able to account for all of these other features and constraints. However some firms in the rental industry may not be liquidity constrained and may be able to follow a counter-cyclical equipment invest-
ment/replacement strategy. Thus, our analysis might be considered as a sort of best-case analysis of the potential profit gains that might be achievable by firms that do not face strictly binding liquidity constraints and where the other simplifying assumptions in our analysis are not greatly at odds with reality.

Section 2 reviews some of the previous literature on dynamic optimization problems as they apply to optimal replacement of capital assets. Section 3 describes the data we were provided, a sample of equipment rental histories of specific machines from six (unidentified to us) rental companies with locations in six broadly classified geographical regions in the US. Section 4 provides an econometric analysis of these data, to enable us to predict rental revenues, maintenance costs, OEC and resale prices of machines, and also to characterize and predict the replacement policies used by the companies in our sample at different rental locations. Section 5 discusses the specific dynamic optimization problem we have formulated and solved, and shows how the model can be used to analyze the profitability of different replacement strategies for equipment in the U.S. equipment rental industry. Section 6 presents our results and findings, showing the optimal replacement strategies implied by our econometric predictions of rental revenues, maintenance, and replacement costs, and compares these to the actual replacement strategies used by firms in our sample. We find there may be significant gains to company profitability by altering replacement strategy, particularly with respect to strategically timing the replacement of machines over the business cycle, and in some cases the optimal policy involves a counter-cyclical replacement strategy that is rather different from the strategy currently followed by many firms in the industry.

2 Existing Literature on Optimal Machine Replacement

[Samuelson, 1937] was among the first to study asset replacement problems, which he formulated mathematically as the problem of determining the optimal time to replace a piece of equipment that is subject to deterioration by maximizing the present value of net returns from an infinite chain of assets. That is, he assumed that whenever the current machine is sold, it is immediately replaced by a new one.

Samuelson’s basic approach has been extended in various directions including allowing uncertainty, such as allowing for random failures in equipment or uncertain costs of repairing or maintaining equipment. The presence of uncertainty required more advanced optimization tools, including the use of stochastic dynamic programming which can solve a wide range of dynamic optimization problems involving sequential decision making in the presence of various types of uncertain “shocks” including machine failure,
and macroeconomic shocks that affect overall demand for rental equipment.

Rust [1987] showed how dynamic programming approach could be applied to real-world problems, under the assumption that firms were behaving optimally. Rust used this method to study the problem of bus engine replacement at the Madison Metropolitan Bus company, which periodically replaces bus engines with new or rebuilt engines to improve the reliability of its fleet of buses. McClelland et al. [1989] developed an optimal control model for assessing replacement policies when the assets can be rejuvenated. Both of these analyses are extremely relevant to equipment rental companies which are ultimately subject to the decisions that are made by key individuals (e.g. fleet managers) who, in addition to outright replacement, can consider alternatives such as overhauls, rebuilds, and reconditioning of machines that involve rejuvenation of varying degrees that could be more cost-effective than outright replacement.

Though the assumption that firms maximize profits is close to sacred in economics (along with the assumption that all consumers maximize utility), a growing line of work in behavioral economics has lead to an increasing recognition among economists that profit or utility maximizing behavior cannot be taken for granted. The dynamic optimization problems that real world firms and consumers are assumed to solve are highly complex and often very difficult, if not impossible to solve when sufficiently realistic versions are formulated mathematically. There is a problem, known as the curse of dimensionality, (a term coined by one of the developers of dynamic programming, Richard A. Bellman, is his early book on the subject, Bellman [1957]) that suggests that there are some dynamic optimization problems that cannot realistically be solved even using the most clever algorithms and fastest available computers. Even though Rust [1997] has shown that it is possible to break the curse of dimensionality in some situations using randomized algorithms, for dynamic optimization problems involving continuous decisions (e.g. how much to spend on a refurbishment) Chow and Tsitsiklis [1989] have shown the curse of dimensionality is present and cannot be “broken” regardless of the type of algorithm or computer that might be used to solve it. Thus, it seems quite reasonable that firms may resort to approximations or rules of thumb to circumvent the difficulty of solving complex dynamic optimization problems. The Nobel Prize winning economist Herbert Simon used the term satisficing to describe how firms and individuals may actually operate when confronting complex problems, an idea that originated in his seminal empirical study of firm behavior, Administrative Behavior (Simon [1947]).

However to the extent that firms do use rules of thumb and learn by “trial and error” how to improve their profitability over time, there may be an opportunity to use formal dynamic programming methods for
sufficiently simple problems to help improve their profitability. Cho and Rust [2010] studied the operations of a large Korean car rental company using a similar approach that is used in this study. Specifically, they compared the firm’s actual replacement policy to the policy predicted from the solution to a dynamic programming problem. Cho and Rust [2010] found a puzzling result: *rental rates are flat*, i.e. they do not decline with age or odometer value. Car rental companies often justify such a policy by replacing all of their cars quite rapidly, often after only one year and with less than 20,000 miles. The Korean car rental company that Cho and Rust [2010] studied held their cars longer than most American car rental companies, selling them when they were on average 2.7 years old and 75,000 kilometers on their odometers. However Cho and Rust [2010] found the company could significantly increase its profits (more than doubling it for some makes/models) by keeping its rental cars roughly twice as long as the company kept them under its *status quo* replacement policy.

To address the company’s concerns that its customers would not rent cars that were “too old” Cho and Rust [2010] proposed discounting the rental rates of the older cars to induce customers to rent them. The company was initially skeptical that this alternative strategy could increase its profitability, but it decided to carry out a *controlled experiment* to test the predictions of the Cho and Rust [2010] analysis. The company chose four of its rural rental locations as “treatment locations” where the rental replacement and pricing policy suggested by Cho and Rust [2010] was put into effect. The operating profits in the treatment locations were compared with those in six “control locations” where the company’s *status quo* replacement and rental pricing policy continued to be followed. The experiment revealed that the discounts for renting older cars were highly attractive to many of the company’s customers, but did not greatly reduce rentals of new cars. This increased utilization rates and overall rental revenue from the older cars in the treatment locations by significantly more than it had expected. The unexpected increase in rental revenues combined with the substantial savings in replacement costs by keeping its rental cars longer, demonstrated that the company could significantly increase its profits by keeping its rental cars longer combined with age-based discounts to incentivize its customers to rent the older vehicles in its fleet.

One primary question to be addressed in the initial stages of research is whether the replacement policies that companies use are approximately optimal. That is, are firms’ decisions approximately the same as the decisions that a computer model that formally attempts to maximize profits would recommend? If the decisions made by firms are based on rules of thumb and industry experience, there is a possibility that formal modeling and optimization of replacement decisions could identify profit opportunities for firms.
However it is important to qualify that any computer model depends on a number of assumptions and if the assumptions are wrong, the predicted “optimal” replacement policy implied by these assumptions may not be optimal in practice. Thus, it is important to test the validity of these assumptions by comparing the profitability of firms under their status quo replacement policy to the counterfactual “optimal” replacement policy predicted by the computer model.

The first and least costly way to do this is via stochastic simulations. That is, we develop a computer environment that can simulate the evolution of rentals, replacements, and other variables for a hypothetical firm operating under its status quo replacement policy. These simulations can also allow for different simulated sequences of macroeconomic shocks that affect rental revenues, equipment resale prices, and replacement decisions by the firm. Then, using the same sequence of macroeconomic shocks we can simulate a counterfactual replacement policy such as the “optimal” replacement policy predicted by our dynamic programming model. For various scenarios and horizons, we can then determine whether the simulations of this optimal replacement policy really do result in higher simulated profits than the firm’s status quo replacement policy.

Of course computerized simulations still depend on a number of assumptions and if these assumptions are wrong, even the simulated outcomes under the firm’s status quo replacement policy might differ from the replacement decisions that the firm would actually undertake under similar conditions. Further, the environment in which the firm actually operates may be significantly more complex and reflect features that are not modeled or anticipated in the computerized simulations. Thus, the ideal way to test whether any profit opportunities that are suggested from our analysis from adopting a counterfactual “optimal” replacement policy is to undertake a controlled experiment similar to the one described in Cho and Rust [2010]. If the controlled experiments reveal clear profit gains to adopting an alternative replacement policy (as they did in the Cho and Rust [2010] study), we can be much more confident that the profit gains would also be realized if the alternative replacement policy were implemented company-wide.

However some of the profit gains we identify come from exploiting patterns in resale prices of machines over the business cycle, particularly for the tendency of replacement costs to be lower during recessionary or “slow” periods for the rental business. We have shown above that most of the industry follows a highly pro-cyclical replacement strategy that tends to generate and reinforce these pricing patterns. However if enough firms were to try the alternative replacement policies we suggest and eventually adopt them as their new replacement policy, a sufficiently large change in firm behavior could have an impact on re-
sale prices of machines and in effect, “arbitrage away” some of the profit opportunities we identify in this analysis. Our analysis is therefore only fully valid in an environment where the number of firms that alter their replacement policies to take advantage of these “arbitrage opportunities” is not large enough to affect market prices. A more sophisticated analysis would be required to account for changes in market pricing patterns if we expect an industry-wide shift in replacement policies to occur.

In the conclusion we will discuss other limitations and important caveats, as well as extensions to the model to account for other questions and problems (such as the optimal size and composition of the rental fleet) that have not been addressed in this study.

3 Data

We were provided anonymized data on OEC (original equipment cost) and monthly rental revenues, and a separate sample of resale values (mostly from auction prices) and cumulative maintenance costs (from initial purchase until sale) for five types of rental equipment: 1) excavators, 2) high-reach forklifts, 3) scissor lifts, 4) skid steers, and 5) telescopic booms. The data are anonymous in the sense that though there are company and geographical identifiers for each of the machines in our sample, we were not informed of the identities of any of the companies that own these machines: the companies are only identified by letters ‘A’ through ‘F’.

For reasons we discuss below, we do not believe that the companies in our sample are representative of the overall populations of equipment rental companies operating in various parts of the US over the sample period January 1, 2011 to March 1, 2013. In particular, the data do not come from a random sample of rental companies and the machines we study do not represent the universe of all types of rental equipment but instead constitute some of the most commonly rented machines that companies have relatively large numbers of.

Our data also contain region identifiers for six different regions of the US (identified as regions 1 to 6) where the companies in our sample operate rental locations. In the rental revenue data set we have an equipment number identifier that enables us to track rental revenues earned by each individual machine in our sample, as well as its OEC and acquisition date and cumulative maintenance costs. We do have finer geographic identifiers: a variable that also indicates the specific urban area, but we do not know if a specific machines are “bound” to specific rental locations in these markets, or whether they are allowed
Table 1: Summary of the Equipment Sample

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Number of observations</th>
<th>Average age (years)</th>
<th>Average OEC</th>
<th>Average monthly rental revenue</th>
<th>Average monthly maintenance cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excavators</td>
<td>192</td>
<td>2.4</td>
<td>$141,042</td>
<td>$4,091</td>
<td>$144</td>
</tr>
<tr>
<td>High-Reach forklifts</td>
<td>581</td>
<td>4.1</td>
<td>94,325</td>
<td>2,645</td>
<td>150</td>
</tr>
<tr>
<td>Telescopic Booms</td>
<td>655</td>
<td>6.3</td>
<td>50,984</td>
<td>1,197</td>
<td>84</td>
</tr>
<tr>
<td>Skid Steers</td>
<td>307</td>
<td>2.8</td>
<td>22,806</td>
<td>992</td>
<td>65</td>
</tr>
<tr>
<td>Scissor Lifts</td>
<td>971</td>
<td>4.9</td>
<td>16,179</td>
<td>735</td>
<td>56</td>
</tr>
</tbody>
</table>

to “float” between different locations in the same market area. Due to confidentiality concerns, we are unable to provide more specific information on the companies, the makes of machines in our sample, or the locations within the US where are data come from.

Table 1 summarizes the number of machines in each of the five categories in our equipment sample. We see that excavators are the most expensive of the five types of machines we analyze, and they are the youngest, with an average age of only 2.4 years. Not surprisingly, they earn the highest monthly revenue and have the second highest average monthly maintenance costs among the five types of machines in our sample. We have the most data on scissor lifts, and these machines are the cheapest in terms of OEC, and also earn the lowest average monthly revenue and have the lowest average monthly maintenance costs.

Table 2 summarizes the data in our sample of equipment sales. Since we have no information on whether any of the machines in our equipment sample had been sold during the period covered in our equipment data set (or after it), we were provided a separate data set on resale prices (typically at auction) of a sample of 5565 other machines of the same five types as in our equipment sample that were sold between January 1, 2011 and November 27, 2013. We do not have any information on the rental histories or total rental revenue for these machines, however we do have the cumulative maintenance costs for each of these machines at the time they were sold.

Clearly the average age of machines that were sold is greater than an average age of machines in the current operating inventory of a rental company. For example, the excavators in our sales sample were sold when they were on average 6.2 years old, whereas the average age of excavators in our equipment sample is only 2.4 years old. This difference reflects a wave of purchases of new excavators in 2012: only 47 of the 192 machines in our equipment sample were purchased prior to 2010, and 72 (or nearly 40% of
Table 2: Summary of the Sales Sample

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Number of observations</th>
<th>Average age at sale (years)</th>
<th>Average OEC</th>
<th>Average monthly maintenance cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excavators</td>
<td>534</td>
<td>6.2</td>
<td>$116,423</td>
<td>$253</td>
</tr>
<tr>
<td>High-Reach forklifts</td>
<td>934</td>
<td>7.3</td>
<td>84,625</td>
<td>222</td>
</tr>
<tr>
<td>Telescopic Booms</td>
<td>996</td>
<td>8.0</td>
<td>47,992</td>
<td>109</td>
</tr>
<tr>
<td>Skid Steers</td>
<td>1607</td>
<td>6.2</td>
<td>18,378</td>
<td>108</td>
</tr>
<tr>
<td>Scissor Lifts</td>
<td>1494</td>
<td>7.0</td>
<td>15,539</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3: Acquisition dates of machines in the equipment sample

<table>
<thead>
<tr>
<th>Machine type</th>
<th>Number of machines acquired in calendar year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excavators</td>
<td>0</td>
</tr>
<tr>
<td>High-Reach forklifts</td>
<td>0</td>
</tr>
<tr>
<td>Scissor Lifts</td>
<td>0</td>
</tr>
<tr>
<td>Skid Steers</td>
<td>0</td>
</tr>
<tr>
<td>Telescopic Booms</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>13</td>
</tr>
</tbody>
</table>

The excavators) were purchased after January 1, 2012. This is in part a reflection of the phenomenon we noted in the introduction that rental companies tend to replace more of their rental equipment at the start of boom periods than during recessions.

In fact only 37 machines in our equipment sample were acquired during the depths of the recession in 2009, whereas 516 machines were purchased in 2012, which nearly equaled the pre-recession peak in machine purchases in 2007 when 648 new machines were acquired. In general, there are relatively small numbers of acquisitions of all types of machines in 2002 and 2003 because machines acquired in those years would have been between 8 and 11 years old over the period of our equipment sample, and thus older than the mean age at which companies sell these machines. However in a steady state replacement cycle (i.e. in the absence of macro shocks and their effects on replacement decisions) we would expect to see between 12 and 17 percent of the machines in a company’s inventory replaced each year. These are the oldest 12 to 17 percent of the machines the companies hold, corresponding to mean times to replacement varying between 6 and 8 years.
For example for excavators, if they are replaced every 6 years and if we assume there are no cyclical effects and a time-invariant demand for rentals, then in a steady state environment with no macro shocks affecting acquisitions or sales of machines, we would expect that the average age of excavators would be about 3 years old and about 16% of these machines would be replaced each year (i.e the 1/6th of the inventory of excavators that would turn 7 years old each year). This implies that for the 192 excavators in our sample, we should expect to see about 24 machines replaced each year in steady state. If the rental companies in our sample followed a strict policy of replacing excavators when they reach 6 years old we would see no acquisitions prior to 2005 but observe acquisitions of 24 machines every year starting in 2005 and going forward, so that by 2011 (the first year of our equipment sample) the 24 machines would be 6 years old and due for replacement during that year.

While we do see very few replacements in 2002, 2003 and 2004, instead of a constant steady state flow of replacements we see a highly non-stationary pattern of replacement investment. The total number of replacements is well below the steady replacement rate level during the recession years prior to 2010, and is significantly higher than the steady state replacement level after 2010 after the Great Recession ended and the US economy was on a path to recovery. The same cyclical pattern of replacements is evident for all five the machine types in our equipment sample: relatively high levels of replacements (i.e. more than the expected number of replacements in a steady state situation) occurred in 2006 and 2007 (which were relatively strong years for the US economy), but there was pronounced collapse in the number of replacements in 2008 as the financial crisis struck the US economy. The low rate of new machine acquisitions continued into 2009 and 2010 during the depths of the ensuing recession. However as the US economy began to emerge from recession in 2011 and 2012 we see a rebound in the number of new machine acquisitions. The number of machines acquired in 2012 were nearly as large as the peak number of acquisitions prior to the recession. The main exception was telescopic booms: even by 2012 only 41 new telescopic booms were acquired, less than the steady state replacement level of 110 units, and far below the pre-recession peak replacement investment in telescopic booms of 179 and 174, respectively, in 2006 and 2007.

Thus, it is evident from our simple analysis of the acquisition dates of the machines in our equipment sample in table 3 that there is pronounced cyclical variations in decisions of rental companies of when to acquire rental machinery. We presume there is corresponding cyclicality in sales decisions, but unfortunately our sales sample covers sales of machines between January 2011 to March 2013, so we are unable
to study whether there is a rough balance between sales of machines and purchases of new machines over the 11 year time span covered in table 3.

However the table provides clear evidence that new machine purchases are highly pro-cyclical, and this is consistent with the pronounced pro-cyclical variations in construction machinery production that we illustrated in figure 1 in the introduction. This emphasizes the need to account for macroeconomic uncertainty into our model of replacement investment. In principle, cyclical variations in construction spending and new construction activity should generate corresponding variations in construction equipment rental demand, which rental companies experience as pronounced pro-cyclical variations in rental revenues. We will show that this is indeed the case. However it is useful to have a variable that captures the cyclical demand in construction activity and in turn, cyclical variation in the demand for rental equipment.

We used the monthly times on US total construction spending (not seasonally adjusted, but adjusted for inflation using the Consumer Price Index) compiled by the US Census Bureau as our measure of overall construction activity that drives demand for rental equipment. We show below that rental revenues are correlated with a construction spending index that we derived from this time series, and the correlation is most strong for excavators. Figure 2 plots total (not seasonally adjusted, but inflation-adjusted) construction spending (in billions of 2015 dollars) and a 12 month moving average of this spending. The grey vertical bars indicate the periods of recession, as dated by the National Bureau of Economic Research. There are two main recession periods in the timespan of this series (which runs from January 1993 to present): 1) the 2001 recession between March 2001 and November 2001, and 2) the “Great Recession” between December 2007 and June 2009.

We see that construction spending is highly seasonal, varying by as much as 25% from the peaks in the summer months to the troughs in January and February. In addition to the seasonal variability there are longer term cycles that correspond to booms and recessions in the overall economy. However the “construction cycle” appears to lag the usual business cycle. For example we see that although the Great Recession is dated as having occurred between December 2007 and June 2009, construction spending did not bottom out until the end of 2011. Many construction projects take substantial time, and funds for these projects are often allocated well ahead of the time that the projects are actually undertaken. This could explain why construction spending remained relatively high even in the midst of the Great Recession: much of it could have represented “overhang” of projects already in progress that began in the
construction boom preceding the Great Recession.

Figure 4 compares the Census construction spending time series and rental industry revenues, based on data compiled by the American Rental Association. We see that both series follow roughly similar trends and are pro-cyclical but rental revenues are more pro-cyclical (i.e. have greater amplitude over the business cycle) but are less subject to the substantial seasonal fluctuations that occur in construction spending. Since the data in figure 4 are at the quarterly time frequency, there is some averaging or smoothing of higher frequency fluctuations that may occur at monthly and shorter time intervals.

Figure 5 plots average rental revenues for the five types of machines in our sample at the highest frequency we have available — monthly. The left panel of figure 5 plots the average revenue earned by all machines of each type in our sample. That is, it plots the ratio of total rental revenues in each month by the number of machines in our sample in that month. The right hand panel plots average rental rates for full month rentals. We calculated this by multiplying the average daily rental rate (the average of the ratio of the monthly rental revenue for each machine divided by the number of days in the month the machine was rented) times the number of days in the month. Of course the latter numbers will be higher than the average rental revenues earned because a typical machine will not be rented for every day of every month. That is, the “time utilization” of the machines in a rental company’s inventory will typically be less than 100%, so the right hand panel of figure 5 represents the best case estimate of monthly revenues if a company were
Figure 4: US construction spending and rental industry revenues, quarterly time series

Figure 5: US construction spending versus average monthly revenues for machines in our sample
able to achieve a 100% utilization (rental) rate for all of its machines.

The dashed black line in figure 5 is the Census construction spending series, which has been rescaled for purposes of comparison by dividing total construction spending (in billions of dollars per month) by 20. We see that this index tracks the variation in average monthly revenues for excavators fairly well, but average monthly revenues of other machine types tend to be less variable (especially over months of the year) than for excavators. The right hand panel of figure 5 also shows that rental rates are also far less variable than the construction spending index.

Overall we see that construction spending is more variable over the year than average rental revenues for most of the machine types in our sample, except for excavators, which seems sensible since excavators are primarily used in construction. Average monthly revenue for rented excavators is even less variable than construction spending however. This suggests that the rates of rental equipment (e.g. monthly, weekly or daily rental rates) were fairly stable over this period, and much of the variability in average revenue earned by all machines is due to variability in time utilization — i.e. the fraction of machines in a company’s inventory that are actually being rented.

This conjecture is confirmed in figure 6 which plots indices showing the growth in US construction spending, rental industry revenues, and total revenues for the six companies in our sample. We normalized the indices to 1 at the start of our sample by dividing each of the data series above by their values for the month of January, 2011. We see that the growth of revenues in our sample of six firms grew at a much faster pace over this period than total revenues in the rental industry as a whole, and much faster than overall construction spending in this post-recession recovery period. Total revenues for the 6 firms in our sample nearly tripled, whereas rental industry revenues as a whole increased by only 15%, and total construction spending increased by 20%. We also see that some of the seasonal cycles in overall construction are reflected in oscillations in the growth of revenues for the companies in our sample, though with reduced amplitude. Overall rental industry revenues seem relatively unaffected by the cyclical variation in construction spending, however.

Figure 7 illustrates the growth in the number of machines in our sample. The left hand panel plots the total number of machines of each type in our sample, and the right hand panel plots these series as ratios of number of each type of machine in the first month of the sample, January, 2011, making it easier to see the growth in each type of machine. We see that though we only have a relatively small number of excavators in our sample, the number of excavators nearly tripled by the end of our sample in March,
Figure 6: US construction spending and rental revenue indices

![Construction and Rental Revenue Indices](chart)

Figure 7: Growth in the number of machines over the sample period

![Number of Machines and Growth](chart)
2013. The number of skid steers increased by 250%, whereas the number of scissor lifts only increased by 25% and the number of telescopic booms barely increased over this period.

In conclusion, though we are unsure of the precise method by which our sample was drawn, it does not appear representative of the growth in rental revenues for the rental industry as a whole over the period of our sample, and rental revenues and investment in new machines in our sample appear to co-move more strongly with overall construction spending than do rental revenues and investment for the industry as a whole. In particular we see significant seasonal and cyclical fluctuations in rental revenues and investment, and well as significant idiosyncratic variations in revenues that vary over firms, regions, and machines.

Even though our data may not be representative of the rental industry as a whole, we believe it has considerable value for analyzing the equipment replacement and investment decisions of the specific companies in our sample and assessing the extent to which there are potential profit gains from optimizing replacement decisions. Before we can do this optimization, however, we need to develop econometric models that can be used to predict and simulate rental revenues, maintenance costs, and the prices of new and used rental equipment — the topic of the next section.

4 Econometric analysis of equipment rental and sales data

This section summarizes the results of our econometric analysis of rental revenues, maintenance, and replacement costs. Our goal is to develop econometric models that can capture the variability in the data that we noted in the previous section, and which can predict how these quantities vary by age and over the business cycle and during different months of the year, as well as across companies and regions of the country. In addition, we used a separate sample on sales of machines to predict the factors that lead these companies to sell their machines. These econometric models are essential to our ability to mathematically model replacement policies and derive optimal policies by dynamic programming, as well as to simulate firm behavior under firms’ status quo replacement policies.

Our econometric analysis is adapted to the periodicity of our data set — monthly. With more detailed higher frequency data, e.g. daily data, it is possible to estimate correspondingly more detailed econometric models such as models of time on-rent and off-rent (we also refer to these as rental spells and lot spells below) where machines cycle between random length periods when they are out with a customer being rented and lot spells where the machine is back at the company, either being maintained, or waiting to
be rented by some other customer. We are not able to conduct an analysis of the duration of rental and lot spells in this study since our data do not provide information on the duration of individual contracts. We only observe the total number of days each month a particular machine is rented, and the total rental revenues it earned that month. We also do not observe individual maintenance expenses on particular machines in the month they were incurred. We only observe the cumulative maintenance expenses on each machine from the date it was acquired until it was sold (in the sales data set) or until March 30, 2013 (in the equipment data set). Despite this, our econometric model will be able to predict expected maintenance costs for individual months, and show how these increase as a function of the age of the machine.

4.1 Rental Revenues

We start by analyzing rental revenues. Our analysis in the previous section noted that variation in rental revenues for different machines is a product of variation in rental rates, but mostly due to more substantial variation in utilization rates — both between-month and within-month variation. Since our objective in this first stage analysis is not to question how firms set rental rates or determine the overall quantity of their inventories of particular types of machines, we treat the variation in rental rates and utilization rates of machines as \textit{exogenous} and restrict our attention to trying to best predict overall rental revenues without trying to separately forecast rental rates and utilization rates. Given this, we estimated regressions of rental revenues of the form

$$\log(R_{i,c,t}) = X_{i,c,t}\beta + \varepsilon_{i,c,t}$$

where $R_{i,c,t}$ is the rental revenue earned by machine $i$ of company $c$ at time $t$. If we assume that the unobserved error terms in this regression equation, $\varepsilon_{i,c,t}$, are normally distributed, this results in a “lognormal” model of rental revenues where predicted revenues are equal to

$$E\{R_{i,c,t}|X_{i,c,t}\} = \exp\{X_{i,c,t}\beta + \sigma^2/2\},$$

where $\sigma^2$ is the variance of the residual unpredictable component $\varepsilon_{i,c,t}$ of monthly rental revenues.

The regression equation (1) was estimated for each of the five types of rental machinery (excavators, high reach forklifts, scissor lifts, skid steers and telescopic booms) separately, so we will have five separate sets of $(\beta, \sigma^2)$ coefficient estimates for each of the five machine types. We will be more specific about the $X_{i,c,t}$ variables entering the regression equation (1) shortly, but it includes company dummy variables,
regional dummy variables, an index of construction spending, dummies for different makes of machines, monthly dummy variables, and the age of the machine, measured in months.

However as we noticed in section 3 above, there are some months where a particular machine will not be rented at all, so its revenues will be zero, $R_{i,c,t} = 0$. Since we cannot take the logarithm of 0, we estimated the regression equation (1) only for the subset of machines that earned some positive amount of rental revenue in a given month. As is well known, the predicted revenues from a regression that uses only positive values of the dependent variable will be subject to an upward “selection bias”. That is, the regression will result in a prediction or conditional expectation that not only depends on $X_{i,c,t}$, but also conditions on the event that revenues are positive, which we can express as $E\{R_{i,c,t}|X_{i,c,t}, R_{i,c,t} > 0\}$. This prediction will generally be higher than conditional expectation $E\{R_{i,c,t}|X_{i,c,t}\}$ that does not condition on the knowledge that revenues during the month in question will be positive. This latter conditional expectation is the relevant one for predicting revenues, since at the start of any month the firm will generally not know whether a particular machine it owns will be rented during that month.\(^2\)

We estimated a separate logistic (also known as a binary logit) model to predict the probability that a particular machine would have zero rental revenue in a given month. Let $P(X_{i,c,t})$ be the probability that a machine $i$ owned by company $c$ with observable characteristics $X_{i,c,t}$ will have zero revenues in month $t$. We have

$$P(X_{i,c,t}) = \frac{\exp(X_{i,c,t}\gamma)}{1 + \exp(X_{i,c,t}\gamma)},$$

(3)

where $\gamma$ is a vector of coefficients to be estimated (via the method of maximum likelihood). Then using the estimated $\beta$, $\sigma^2$ and $\gamma$ parameters, we can write the conditional expectation (or best prediction) of rental revenues during the month as

$$E\{R_{i,c,t}|X_{i,c,t}\} = [1 - P(X_{i,c,t})] \exp\{X_{i,c,t}\beta + \sigma^2/2\}.$$  

(4)

Intuitively, equation (4) adjusts for the upward bias in predicted rental revenues from the lognormal regression (1) that uses only data from machine-months with positive rental revenues by multiplying by what we called the between-month time utilization rate, $1 - P(X_{i,c,t})$, which varies between 0.8 to 0.95, and is about 95 percent on average. Note that the other source of variation in time utilization, which we referred

\(^2\)Of course, in a spell-based model if a machine is in the middle of a rental spell at the start of the month, the firm will have a belief about how much longer the current rental spell will last, and thus have more information for forecasting utilization and rental revenue for that particular machine for the rest of the month. In this case, the firm does know that the machine will earn positive rental revenue during the month, and thus the conditional expectation of rental revenues that conditions on the event that rental revenues are positive is the relevant conditional expectation for the firm to use in this case.
to as within-month time utilization, is captured indirectly via its effect on rental revenues in the regression (1).

In the interest of space, we omit the presentation of regression estimates of $\beta$ and $\sigma$ from the rental revenue regression equation (1), and instead we summarize the main conclusions we draw from these regressions and then present our predictions of revenues graphically for each machine type. One of the main findings is that expected revenues (conditional on the machine being rented during the month) decline with age of the machine: the estimated machine age coefficients are negative and statistically significantly different from zero for all five machine types. However in the case of telescopic booms, the age coefficient is sufficiently small that we can consider rental revenues to be virtually “flat” as a function of machine age, something that will be more apparent in our graphs of expected revenues to be shown shortly.

The decline in revenues with machine age could reflect two different effects: 1) a reduction in the rental price for older machines, or 2) a reduction in the “within-month” time utilization of the machine with age. The latter effect refers to a decline in the days in the month an older machine is rented compared to a newer machine. Most of the expected revenue decline comes from the latter effect, since rental rates (whether measured at a daily level or on a per month basis) do not appear to decline with age. The decline in utilization with age may reflect a larger amount of time that older machines are “offline” for maintenance each month, or a preference among customers (or rental company employees) for renting out the newer machines instead of older ones when newer ones are available.

We also included the logarithm of the machine OEC cost and dummies for the make of the machine to see whether rental companies are able to earn more revenues for certain makes of equipment, or for machines that cost more. With the exception of excavators, the regression coefficient for the OEC cost variable is positive, indicating that indeed, rental companies earn a return in the form of higher rental revenue for renting machines that cost more. The higher OEC may reflect additional features or capabilities of particular makes and models of rental equipment that the rental companies can charge extra for, such as telescopic booms that have extra long reach, or can handle higher loads, etc.

We also find significant, positive coefficients on all the make dummies we included in the regression. The coefficient estimates can be interpreted (roughly) as the percentage increase in monthly revenue a company can earn by renting a machine of a particular make. For example, the “Make 3” dummy for excavators indicates that these excavators earn about 5.8% rental revenue premium, other things equal.

We also see regional and company variation in rental revenues. Excavators earn about 16% less rental
revenue in the west, high reach forklifts earn about 16% higher rental revenue in the southwest, and skid steers earn about 20% higher rental revenue in the southwest. Of the six companies in our sample, company F earned about 7% less rental revenue on excavators relative to the excluded company (D, we excluded a dummy for this since the sum of dummy variables for all 6 companies is perfectly collinear with the constant term in the regression), whereas company F earns a 25% rental revenue premium relative to the excluded company (A) for rentals of scissor lifts. In some cases (high reach forklifts, skid steers and telescopic booms) we excluded more than one company dummy (i.e. normalized their coefficients to zero) when there were too few observations for particular combination of company and machine to accurately identify a “company effect”.

We also included monthly dummies to capture seasonal variation in demand for rental equipment, as well as “macro dummies” to capture business cycle effects on the demand for rental equipment. Recall that we used the Census construction spending index and we created a trichotomous indicator for values of construction spending that are low, medium and high. We classify the low values of construction spending as “bust” periods for rental demand, whereas when construction spending is high, we classify this as a “boom” period for rental demand, and the remaining intermediate states we classify as “normal”. We excluded the dummy for low construction spending to avoid perfect collinearity with the constant term, so we estimated coefficients on the “normal” and “boom” dummy variables. We see that for all five machines, as expected, rental revenues are significantly higher in “boom months”. For example excavators earn 10% more revenue, high reach forklifts earn about 19% more revenue, and scissor lifts earn about 24% more revenue compared to a month where the construction spending is in a “bust” state. Even in normal times, rental revenues are 8%, 9% and 10% higher than in the bust state for excavators and high reach forklifts, and scissor lifts, respectively.

We also see the effects of seasonality on rental demand via the regression coefficients of the monthly dummy variables (we excluded a dummy for January to avoid perfect multicollinearity). For excavators, rental revenues are lowest in November through February, but highest from May to October, which generally matches the pattern of seasonality in overall construction spending. The seasonal patterns in rental revenue is different for the other machines. For example telescopic booms are predicted to have peak revenues in January.

Overall, the level of predictive power of our rental regressions is low, with estimated $R^2$ values ranging from 4.4% for telescopic booms to 6.8% for scissor lifts. The estimated values of standard deviations of the
regression residuals are correspondingly high, ranging from $\sigma = 0.47$ for high reach forklifts to $\sigma = 0.616$ for skid steers. These high residual standard deviations reflect the considerable level of idiosyncratic variability in rental revenues across machines and across time. Figure 8 illustrates the variability in rental revenues by providing a scatterplot of all rental revenues and the regression prediction from equation (2) for different values of the macro shock variable and particular choices of company, region, month, and model of machine. We see that expected rental revenues decrease with the age of machine, and that expected revenues are shifted by the macro variable in the expected way, i.e. revenues are highest in booms than in normal times, which are higher than expected revenues in the bust state.

The decline in expected monthly revenues as a function of the age of a machine raises another question: is this decline driven mostly by a fall of “within month” utilization of machines, or by lower rental rates for older machines? To get more insight into this we did parametric and non-parametric regressions of the imputed monthly rental rate for the five machine types in our sample. The imputed monthly rental rate is calculated by dividing the revenue earned by the machine during a given month by the number of days it was rented during the month to get an imputed daily rental rate. We then constructed an estimated or imputed monthly rental rate by multiplying the imputed daily rental rate by the number of days in that month. This simple imputation procedure is subject to some obvious shortcomings. For example rental companies may charge a lower per day rental rate for machines that are rented for longer durations such as for weeks or months at a time, and charge a higher daily rental rate for “day to day” rental contracts. If this is the case, our imputation procedure could overestimate the actual rental rate a company would charge for a machine that was rented for an entire month.

We can, of course, separate our data by the number of days in the month a machine was rented to see if the imputed daily rental rate is lower for machines that are rented for a larger fraction of the month. However our data do not allow us to tell, for example, whether a machine that was rented for 30 days in a given month was rented to only one customer for one 30 day rental spell, or to three different customers for three 10 day rental spells. Given these limitations, we only use the imputed monthly rental rates to give us a qualitative impression of how rental rates change with the age of the machine, recognizing that our imputation procedure may overstate the degree of variability in actual rates for full month rentals. The main conclusion we draw is that rental rates are flat as a function of age. Thus, it follows that there must be a decline in the utilization of older machines that drives the decline in monthly revenues with age that our regression estimates predict.
Figure 8: Predicted versus actual rental revenues for the five machine types in the sample

Predicted versus actual monthly rental revenues

**Excavators**

- Actual, boom, mean: 4369
- Actual, normal, mean: 4148
- Actual, bust, mean: 3880

**Hi-Reach Forklifts**

- Actual, boom, mean: 2810
- Actual, normal, mean: 2650
- Actual, bust, mean: 2512

**Scissor Lifts**

- Actual, boom, mean: 1049
- Actual, normal, mean: 1009
- Actual, bust, mean: 917

**Skid Steers**

- Actual, boom, mean: 791
- Actual, normal, mean: 728
- Actual, bust, mean: 693

**Telescopic Booms**

- Actual, boom, mean: 1490
- Actual, normal, mean: 1401
- Actual, bust, mean: 1293

Predicted versus actual monthly rental revenues
Overall, we conclude that while there are a number of company, seasonal, macro and model effects that help to predict the large variability in rental revenues we find for the machines in our sample, most of the age-related decline in expected revenues comes from a decline in utilization of older machines. That is, older machines are rented for fewer days in any given month than newer machines. We do not have enough data or knowledge of the detailed mechanics of how equipment rentals work to to explain the decline in expected revenues with age, though the predicted effect seems reasonable. One possible explanation is that older machines are more likely to be in the shop for maintenance and this might be a reason why older machines are rented for fewer days — because they are not available to be rented. However another possibility is that customers prefer to rent newer machines when a newer machine of a desired type is available, so the decline in utilization with age might be driven by customer preferences or the desire of rental company employees to cater to their customers by renting the newest machines in their fleet whenever they are available.

There is another aspect to utilization of machines that we previously referred to as the “between month utilization rate”, which we defined as 1 minus the probability that a machine will not be rented at all during a given month. Recall that in our data set, it is not uncommon to find entire months where machines earn no rental revenue at all. We regressed the between month utilization rates implied by the estimated value of the $\gamma$ parameters in equation (3) and with the notable exception of skid steers, utilization rates are predicted to decline with the age of the machine. However we did not find significant effects of the macro state on utilization rates. We did find significant variations in utilization depending on the make of the machine, the company, the region, and the month of the year.

The top left panel of figure 9 shows that utilization rates of excavators and high reach forklifts decline the most with age, and as we noted, for skid steers, utilization rates are predicted to slightly increase with age. Beyond this, the figure illustrates the substantial variation in utilization rates depending on the make of the machine, the company, region, and make.

The product of the estimated utilization rates and the revenue a machine can be expected to earn conditional on it being rented during the month is the quantity we referred to as the *unconditional expected revenue function* given in equation (4) above. Figure 10 plots the expected revenue functions for the same companies, regions, and machine models that we plotted the utilization rates for in figure 9 above. We see that in every case expected unconditional revenues predicted by our econometric model decline with the age of the machine, and revenues decline the most with age for excavators and high reach forklifts, and
Figure 9: Predicted between month utilization rates for machines in the sample

- **Predicted utilization for Excavator**
  - Make 1 in region 3 for company A
  - Make 2 in region 4 for company C
  - Make 3 in region 6 for company E

- **Predicted utilization for Hi-Reach Forklift**
  - Make 3 in region 2 for company B
  - Make 5 in region 4 for company D
  - Make 6 in region 6 for company F

- **Predicted utilization for Scissor Lift**
  - Make 1 in region 2 for company A
  - Make 2 in region 4 for company C
  - Make 3 in region 6 for company E

- **Predicted utilization for Skid Steer**
  - Make 1 in region 2 for company C
  - Make 2 in region 4 for company E
  - Make 2 in region 6 for company F

- **Predicted utilization for Telescopic Boom**
  - Make 1 in region 2 for company A
  - Make 2 in region 4 for company C
  - Make 3 in region 6 for company E
Figure 10: Unconditional expected monthly revenues, from equation (4), for machines in the sample.
this is driven mostly by the rapid decline in utilization rates since figure 8 showed that conditional on being rented during the month, rental revenues of excavators and high reach forklifts do not decline as rapidly with age.

Similarly, the bottom panel of figure 8 shows that conditional on being rented during the month, rental revenues of telescopic booms do not decline rapidly with age. Thus, the main factor behind the decline in unconditional expected rental revenues is the decrease in between month utilization rates. For example, the bottom panel of figure 10 shows that a new make 1 telescopic boom owned by company A in region 2 can expect to earn about $1200 per month in rental revenue, but expected revenues drop relatively quickly with age. This is driven by a drop in within-month utilization rates, since we from the corresponding bottom panel of figure 9 that the between month utilization rates for these machines are nearly 1 regardless of age. That is, even older make 1 telescopic booms will be rented at some time in any given month, but older machines tend to be rented for a smaller fraction of time during the month, and this drives the fall in unconditional expected revenues with age. On the other hand for excavators and high reach forklifts, we find both between-month and within-month utilization rates decline with the age of the machine.

4.2 Maintenance Costs

Our data also provided information on maintenance costs, but not on on month by month basis that would allow us to track variability over time in when maintenance costs are incurred, but only as a cumulative level of maintenance expenditures over the life of the machine. For the machines in our equipment sample we have total maintenance costs incurred from the date the machine was acquired until the end of our sample in March, 2013. In our separate data set on sales of machines, we also have the total maintenance costs from acquisition date until the date the machine was sold. That is, our data provide machine level data on the quantities $m_{i,t,c}$ given by

$$m_{i,t,c} = \frac{1}{t} \sum_{s=1}^{t} m_{i,s,c}$$

where $m_{i,s,c}$ is the actual maintenance costs incurred by machine $i$ that is $s$ months old and is owned by company $c$. Unfortunately we only observe the lifetime average maintenance costs $\bar{m}_{i,t,c}$ and not the individual monthly maintenance costs incurred in each month of a machine’s life, $\{m_{i,s,c}|s = 1, \ldots, t\}$.

We regressed the logarithm of $\bar{m}_{i,t,c}$ on a collection of explanatory variables $X_{i,t,c}$, and as expected, the age of the machine is the most important explanatory variable explaining average monthly maintenance costs, and in the expected direction. Average monthly maintenance costs are the highest and rise most
quickly with age for excavators and high reach forklifts. We also find that maintenance costs differ significantly across different models. For example the average monthly maintenance cost of a make 2 scissor lift is $62.50, which is over 20% more than the corresponding value for a make 1, $50.40. For skid steers, make 1 machines are predicted to cost $116 per month to maintain, compared to only $78 per month for a make 2. Due to the averaging of maintenance costs, the $R^2$ from our regressions are higher than they were for predicting rental revenues, though there is still significant variation in idiosyncratic or machine-specific maintenance costs, even when averaged over the life of the machine.

Our simulation and dynamic programming model will need to predict maintenance costs for each month in a machine’s life, not just the average monthly maintenance cost over its entire life. However if we can predict how average monthly maintenance costs increase with age, we can also predict how expected maintenance costs increase with age in every specific month in a machine’s lifetime. To see this, let the function $f_{i,c}(t)$ given by

$$f_{i,c}(t) = E\{\bar{m}_{i,t,c}\}$$

be the prediction from our regression model of average monthly maintenance costs of a machine that is $t$ months old. Then $tf_{i,c}(t)$ is the predicted total or cumulative maintenance costs for this machine from acquisition to month $t$. Similarly, $(t-1)f_{i,c}(t-1)$ is the predicted value of cumulative maintenance costs from acquisition to month $t-1$, and thus the difference of these quantities,

$$E\{m_{i,t,c}\} = tf_{i,c}(t) - (t-1)f_{i,c}(t-1)$$

is the implied prediction of the expected maintenance costs $m_{i,t,c}$ company $c$ will incur on a machine $i$ that is $t$ months old during month $t$. It is easy to see that average monthly maintenance costs are an average of the presumably low maintenance costs when a machine is new and the higher maintenance costs when the machine is older, we will have $E\{m_{i,t,c}\} > f_{i,c}(t) = E\{\bar{m}_{i,t,c}\}$, and expected maintenance costs incurred in each month rise more quickly with the age of the machine, causing average monthly maintenance costs to increase with the machine age as well.

Figure 11 plots our estimates of current monthly expected maintenance costs, $E\{m_{i,t,c}\}$ and average monthly maintenance costs over the life to date of the machine, $E\{\bar{m}_{i,t,c}\} = f_{i,c}(t)$. We see that expected monthly maintenance costs (indicated by the blue curves in figure 11) do indeed increase significantly faster with the age of machines than the lifetime average maintenance cost (indicated by the red lines in the figure). Monthly maintenance costs are highest for excavators and high reach forklifts: a 100 month
old excavator and high reach forklift are predicted to cost over $1000 and $750 per month, respectively, to maintain. In comparison, a 100 month old skid steer, scissor lift, or telescopic boom are predicted to cost less than $250 per month to maintain.

With our estimated maintenance costs and expected revenue predictions, we can generate predicted expected profits for different machines, companies, months, and macro states. Let $\tau$ denote the type of machine (including the specific make, such as a make 1 excavator) and let $c$ index a specific company (A, B, C, D, E, or F), $r$ index the region of the country, $m$ index the current month of the year, $a$ index the age of the machine in months, and $s$ index the macro state (boom, normal or bust). Our econometric model allows us to predict revenues for machines that depend on each of these variables, and we can let $E\{R_i|a, \tau, c, r, m, s\}$ denote the expected revenues for a specific machine $i$ that is $a$ months old, of type and make $\tau$, owned by company $c$ and in service in region $r$ during month $m$ in macro state $s$. Similarly, $E\{m_i|a, \tau\}$ denotes the expected maintenance costs for this machine. Note that our regressions for maintenance costs exclude month effects and macro shocks as well as company and region dummy variables, so the variables $(c, r, m, s)$ do not enter the expected maintenance cost function. Let $\Pi_i$ denote the gross profits earned by machine $i$ in a given month, i.e. the difference between rental revenues less maintenance costs for a specific machine $i$ with characteristics $(a, \tau, c, r, m, s)$. Then the expected profits for this machine is given by

$$E\{\Pi_i|a, \tau, c, r, m, s\} = E\{R_i|a, \tau, c, r, m, s\} - E\{m_i|a, \tau\}. \quad (8)$$

Figure 12 plots the expected monthly gross profits $E\{\Pi_i|a, \tau, c, r, m, s\}$ as a function of machine age $a$ for different example machines $(\tau, c, r, m, s)$ where we fix the month as January ($m = 1$). We see that profits are generally highest in the boom months ($s = 3$, indicated by the red lines in figure 12) and lowest in the bust macro states ($s = 1$, indicated by the blue lines in the figure). We also plot a horizontal zero profit line, so the age where the profit curves intersect the zero profit line indicates the “breakeven age” beyond which the firms can expect to lose money if they continue to hold the machine.

We see that the profits from renting a new excavator are the highest of the machine types shown. We forecast that a new make 2 excavator owned by company C in region 2 will earn about $5000 in gross rental profits in its first month of life if the macro state is in a boom ($m = 3$) and about $4500 if it is a bust state ($m = 1$). The breakeven age for this machine ranges from 140 to 150 months depending on the macro state. That is, our model predicts that this firm will expect to lose money if it keeps this machine beyond this breakeven threshold. In the next subsection we econometrically model the equipment sales decisions
Figure 11: Current Monthly versus Average Monthly Maintenance Costs

Predicted versus actual monthly maintenance costs make 3 Excavators

Predicted versus actual monthly maintenance costs make 3 Hi-Reach Forklifts

Predicted versus actual monthly maintenance costs make 3 Scissor Lifts

Predicted versus actual monthly maintenance costs make 1 Skid Steers

Predicted versus actual monthly maintenance costs make 3 Telescopic Booms
Figure 12: Expected Monthly Profits by Age of Machine

Expected profits for make 3 of Excavators of company C in region 4

Expected profits for make 5 of Hi-Reach Forklifts of company D in region 4

Expected profits for make 3 of Scissor Lifts of company E in region 6

Expected profits for make 1 of Skid Steers of company A in region 2

Expected profits for make 3 of Telescopic Booms of company A in region 2
of the companies in our sample. Our econometric model predicts that company C will have replaced this machine well before it hits this breakeven threshold: in a boom state, the median age at which we predict that company C will replace this machine is 75 months (i.e. there is a 50% probability that a 75 month old make 2 excavator will be replaced in a boom state), and the median age of replacement in a bust or normal macro state is about 100 months old. By the time the machine reaches the lowest breakeven threshold of 140 months, the probability company C will replace this machine is very close to 1.

Now consider the expected profit curves for a make 3 scissor lift owned by company A in region 6, shown in the middle left panel of figure [12]. This scissor lift is predicted to be less profitable than the excavator that we considered above: the expected monthly gross profit for a new make 3 scissor lift ranges from about $600 in a boom state to $500 in a bust state, and the profit falls off faster with age, and the breakeven point where gross expected profits turn negative occurs when the machine is about 70 months old. As we show in the next section, our econometric model predicts that company A keeps this scissors lift well beyond this breakeven age range: the median age of replacement in boom states is approximately 120 months and in bust states, 140 months. Thus, if our projections of expected revenues and maintenance costs is correct, our econometric analysis already suggests that company A could increase its profits by replacing this scissor lift substantially earlier than it currently does. We would expect that companies should replace their machines before the breakeven point for expected profits, but an important additional consideration must be factored into the calculation of when it is best to sell a machine: namely, replacement costs.

4.3 Machine OEC and Resale Prices

The increase in maintenance costs with age combined with the decline in rental revenues with age are the two obvious factors that motivate rental companies to sell their older machines and replace them with new ones. But there is a key missing piece of information that helps firms determine the best time to do this: namely the expected cost of replacement. These costs depend on predictions of the prices of new machines (OEC prices) that the company purchases and the secondhand resale or auction prices of the older machines the company sells. In this section we present our econometric predictions of these prices based on data on the OEC prices of the machines in our equipment sample, and the OEC and resale prices of the machines in our sales sample.
We estimated a logarithmically transformed regression of the form

\[ \log(P_i) = X_{i,t,c} \alpha + \epsilon_i \]  

where \( P_i \) is the OEC price of machine \( i \) for the observations in our sample of purchases of new machines, and \( P_i \) is the resale (possibly at auction) price received for the machines that were sold. Separate regressions were run for the five different types of machines in our sample and the regression coefficient estimates of \( \alpha \) and the estimated residual standard error \( \sigma = \sqrt{\text{var}(\epsilon_i)} \). We use \( X_{i,t,c} \) to denote the explanatory variables in the regression, which include the age of the machine \( t \), dummies for the company \( c \), and make of machine, and dummies for the macro state \( s \), the month of the year \( m \). We initially estimated a simple specification that included a simple linear term in the age of the machine, \( a \). This implies a constant rate of price depreciation for machines over their lifetimes. However we found we could significantly improve the fit of the model by allowing depreciation rates to vary with age. We estimated a spline specification that depreciation rates to differ depending on whether a machine is aged 0 to 40 months, and another depreciation to hold for machines over 40 months old. We also interacted these depreciation with dummy variables for the macro state being either a normal month \( (s = 2) \) or a boom month \( (s = 3) \).

The predicted OEC and resale values from our estimated regression model\[9\] are illustrated in figure\[13\]. Focusing first on predicted OEC prices, we see that OEC prices are predicted to be higher in boom months in the case of telescopic booms, high reach forklifts, and scissor lifts, but lower in boom months in the case of excavators and skid steers. Turning to resale prices, the model predicts rapid early price depreciation in the case of excavators, high reach forklifts, and telescopic booms, but in the case of scissor lifts our model predicts depreciation rates that are lower for machines that are under 40 months old than for machines older than 40 months old. Also, our model predicts that the OEC prices of skid steers in boom months are lower than OEC prices in normal and bust months, and this results in a lower price depreciation rate for skid steers under 40 months old relative to machines that are over 40 months old in boom months.

Our regressions had relatively small number of observations (ranging from a low of 760 observations for excavators to a high of 2320 observations for scissor lifts) and so some of our OEC and resale price predictions may be affected by outliers or machines that sold for especially high or low values due to special configurations or other idiosyncratic reasons. Thus, we advise caution in the interpretation of some of our predictions of optimal replacement strategies that result from these estimates, since the dynamic programming and simulation models treat these as providing reliable predictions of the prices it can buy and sell machines of different makes under different macroeconomic conditions during different months.
Figure 13: Predicted OEC and resale prices for the 5 machine types, unrestricted specification
of the year. We would feel more comfortable if we had more data on OEC and resale prices to determine if our predictions are reliable.

We do note that the $R^2$ of our OEC/resale price regressions are substantially higher than the low $R^2$ we obtained in our regression estimates for monthly rental revenue. The lowest $R^2$ is 0.755 for skid steers and the highest is 0.843 for scissor lifts. Thus, our model provides relatively accurate price predictions, even though as you can see from the scatterplots of resale and OEC prices in figure 13, there is still a fair amount of apparently idiosyncratic variability in the OEC prices companies pay for new machines and the resale prices they obtain for their used machines. We will discuss ways in which our model can account for such idiosyncratic variability in both machine prices and rental revenues in the next section.

4.4 Status Quo Machine Replacement Policy

We conclude this section by econometrically estimating a model of companies’ decision of when to sell their machines under their status quo replacement policies. There may be many idiosyncratic, unobserved factors that affect precisely when a company decides to sell a particular machine, but it seems clear that the age of and make of machine as well as the macro state $s$ will be among the most important observable factors that affect companies’ decisions. To this end we estimated a binary logit model of companies’ decision of when to sell particular machines. Let $P_s(X_{i,a,c})$ be the probability that a machine $i$ that is $a$ months old and owned by company $c$ with observable characteristics $X_{i,a,c}$ will be sold. Under the logit specification we have

$$P_s(X_{i,a,c}) = \frac{\exp(X_{i,a,c}\delta)}{1 + \exp(X_{i,a,c}\delta)},$$

where $\delta$ is a vector of coefficients to be estimated (via the method of maximum likelihood).

It is important to understand that the data set used to estimate the $\delta$ coefficients is a pooled dataset that follows the rental histories of 2706 distinct machines provided in the “equipment sample” plus 5565 additional machines that is part of separate “sales sample.” It would be impossible to estimate the $\delta$ coefficients using the equipment sample alone since the machine lifetimes (i.e. the duration from initial acquisition until sale) are censored — that is, all of the machines in the equipment sample were operating and none had been sold at the end of the 27 month time interval on March 1, 2013 in the equipment sample. However we were given a separate data set of 5565 machines that were sold between January 1, 2011 and November 30, 2013. This latter data set is uncensored in the sense we see the sales date for every one of the machines in this sample, but we have a potential problem of choice based sampling — i.e. the sales
sample was specifically drawn to provide observations on machines that had been sold. The sales sample is also left censored — i.e. we do not observe the full rental histories for these machines as we do for the machines in our equipment sample.

However the sales sample does contain the acquisition date for each of the 5565 machines that were sold, and this enables us to backcast and reconstruct some information about these machines over their entire lifetime, except for rental revenues. For example knowing the acquisition date, we can determine information such as the month of the year and the value of Census construction spending index (which we have used to construct our trichotomous macro state variable s) in every month of the machine’s lifetime between its acquisition date and sale date. Obviously in every month before the machine was sold, we know that it was not sold. Thus, our backcasted sales sample is more analogous to a separate panel data set on histories of machines, albeit one that has been endogenously sampled. However we believe the bias in our results by simply pooling the data sets and treating them as an exogenously stratified sample is small — we get reliable (i.e. unlikely to be badly biased) estimates of $\delta$.

In any event it would be completely impossible to estimate $\delta$ using only the censored data in the equipment sample. Adding the backcasted sales panel adds hugely to the number of observations at our disposal. We have full histories (i.e. except for month by month maintenance costs, but with month by month rental revenues) in the equipment sample, so this constitutes 55308 machine-machine observations where a sale hasn’t yet occurred. When we add the sales sample, we add another 5565 observations on machine-months where a sale has occurred. Then with the backcasted data, we add another 443,642 machine-months before the month these machines are sold. Thus, the pooled sample has 499,515 machine month observations for all five machine types in total, and only 5565 of these are machine-month observations where machines had been sold. As long as the sales sample is a representative sample of the machines that companies have sold and the equipment sample is a representative sample of machines that the companies currently have in operation (conditional on the restriction that we are focusing on just 5 types of machines that tend to be the most numerous and frequently rented machines in these companies’ rental portfolios), we believe that our estimated econometric model of company sales decisions will give us reliable predictions of how age, the month of the year, and the current macro/demand conditions affect the probability they will sell individual machines.

---

3We will investigate the possibility of estimating $\delta$ using a modified likelihood function that accounts for the choice-based nature of the sales sample in future extensions of this work.
Once again due to space constraints, we omit the presentation of the estimated $\delta$ parameters but illustrate our estimates graphically. Our specification allowed the age variable $a$ to be interacted with company dummies and macro state dummies $I\{s = 2\}$ (for normal state) and $I\{s = 3\}$ (for boom state) as well as make, month and regional dummy variables. Since every month we observe a machine in our sample of rental revenue contributes an observation where the firm doesn’t sell the machine, we augmented the sample with the information on machines that were sold from our sales data set which contribute observations where the companies did sell machines. These combined samples on sales decisions are therefore much larger than the samples we had to estimate rental revenues, maintenance costs and OEC/resale prices. Our sample size for estimating the $\delta$ parameters range from a low of 43,361 observations for excavators, to a high of 145,216 observations for scissor lifts. The large number of observations enabled us to estimate a correspondingly richer model with more variables and interactions, such as interactions between machine age and company dummies (to capture company-specific age-replacement profiles) and interactions with the macro state dummies $I\{s = 2\}$ and $I\{s = 3\}$.

Given that there are over 30 $\delta$ coefficients, it is easier to illustrate the variation in implied sale probabilities $P_s(X_{i,a,c})$ that are predicted by our logit model in equation (10) above graphically in figures 14 and 15 below. In each figure we plot examples of sales probabilities for different makes and companies and regions, but in each case all for the month of January. Generally we do not find significant seasonal effects on sales probabilities and the month dummy variables are generally insignificantly different from zero. The main variation in sales probabilities is over machines, makes, companies, and macro states.

In all cases, the probability of selling a machine is increasing in the age of the machine, and in all cases the probability of selling a machine in a boom month is always higher than the probability companies sell the machine in a normal or bust month. For excavators, high reach forklifts and scissor lifts we also see that the probability of selling a machine is the lowest in a bust month. In normal months the probability of selling (shown by the black curves in the figures) is in-between the upper red curve for sales probabilities in boom months and the lower blue curve for sales probabilities in bust months. This finding is consistent with the investment behavior we observed and commented on in the introduction and in section 2, namely, that investment in rental equipment is pro-cyclical.

However we do note that our results technically only allow us to conclude that sales of machines are procyclical since we cannot observe whether every sale of a machine is matched by a replacement purchase of a new machine. However we believe that sales of existing machines are closely connected
Figure 14: Predicted sales probabilities for excavators

- Replacement probabilities for make 1 of Excavator by company A in region 3
- Replacement probabilities for make 2 of Excavator by company C in region 4
- Replacement probabilities for make 3 of Excavator by company E in region 6
with purchases of new machines, and in most cases a sale of an old machine will be followed by a purchase of a new machine to replace it. If so, this does imply a pro-cyclical pattern to new equipment purchases as well, something we also found in section 2, particularly in our analysis of the procyclical pattern in the number of machines acquired in different years in our sample.

Figure 14 displays a wide variation in the ages at which three different companies (A, C and E) sell three different makes of excavator in three different regions. The make 1 excavators are replaced the earliest, with the first sales occurring after only 25 months. The sales probability increases to 1 by the time these excavators reach 100 months, and the median age of replacement (i.e. the age at which half of the these excavators have been replaced) is about 50 months. However the make 2 excavators do not start to be sold until they are 50 months old, and the median age of replacement ranges from about 75 months (in boom times) to 100 months (in normal and bust times). Firm E replaces make 3 excavators even later, and there is a much greater variation in replacement probabilities in different macro states. The median age of replacement in boom times is 100 months, in a normal month it is about 175 months, and in a bust month, 200 months. Another way to say this is that a 175 month old make 3 excavator is virtually certain to be replaced in a boom month, but has a 50/50 chance of being replaced in a normal month, and has about a 20% chance of being replaced in a bust month.

Figure 15 displays the probabilities that three different makes of scissor lifts are sold by three different companies (C, A and E) in three different regions. Here we see dramatic differences in the sales policies in the three cases. Firm C sells its make 2 scissor lifts the earliest, with a median age of sale ranging from 80 months in boom times to about 105 months in a bust. Firm A does not sell its make 1 scissor lifts until they are significantly older, with a median age of sale ranging from 120 months in a boom to about 150 months in a bust. Firm E holds make 3 scissor lifts even longer, with a median age of sale ranging from 180 months in boom times to 225 months in a bust.

In summary, our econometric analysis has revealed a number of important insights into the operations of the rental companies in our sample: 1) rental revenues decline fairly rapidly as machines age, 2) maintenance costs increase rapidly with age, 3) the prices of old machines decline with age, but not as rapidly as we are used to for other durable goods such as automobiles, and 4) we find significant effects of our indicator of the macro state on rental revenues and OEC and resale prices of machines. As expected, expected rental profits are highest in boom months and lowest in bust months, but our analysis of machine sales decisions confirm that sales are strongly procyclical, i.e. for any given age, a machine is significantly
Figure 15: Predicted sales probabilities for scissor lifts

Replacement probabilities for make 2 of Scissor Lift by company C in region 4

Replacement probabilities for make 1 of Scissor Lift by company A in region 2

Replacement probabilities for make 3 of Scissor Lift by company E in region 6
more likely to be sold in a boom month than in a bust month. Our results allowed us to characterize the “breakeven ages” for different machines, i.e. the age at which expected profit from renting machines first reaches zero. Generally we find that companies are selling their machines well before they reach this breakeven age, but we did identify cases where some companies keep some makes of machines well beyond this breakeven age, and thus appear to be holding machines that generate expected losses (e.g. make 1 scissor lifts owned by company A in region 2).

However we cannot provide a full assessment of the optimality (or lack thereof) of the status quo sales/replacement policies of these companies until we show how to also take replacement costs into account. We turn to this analysis in the next section.

5 Model

In this section we formulate a mathematical model of optimal replacement of rental equipment. We will show how to derive optimal rental policies — i.e. dynamic strategies that specify when a company should replace individual rental machines in order to maximize the expected discounted profits from owning an infinite sequence of machines. That is, our mathematical model will tightly link sales of an old machine with the purchase of a new replacement machine. We assume that for each of the companies and each of the five types of machines in our sample, companies feel there is a stable, predictable “core demand” for the machines, so it is reasonable to assume that for these core machines, the companies will almost immediately replace any existing old machine it sells with a new one of similar make/model/configuration. Of course companies may have a total fleet or inventory of machines of a given type that goes up and down with business cycle conditions: they may hold more machines of a given type in boom times when demand is strong and hold fewer machines in bust times when demand is weak.

It is possible to extend the model we develop below to accommodate situations where a company may sell an old machine but not replace it (or delay when it buys a replacement), as well as situations where a rental location that is experiencing strong growth buys new machines without selling other older machines, resulting in a model with a variable total fleet size. However to accurately model such situations we would need a way to predict total demand for machines of each type at specific rental locations. If we can predict the demand for machines of each type (and possibly more detailed predictions by make/model and age of machine), it would be feasible to solve a more general version of the “replacement problem” that
could enable us to determine a dynamically optimal policy for adjusting fleet size. Intuitively, in periods where we predict demand for machines will exceed the available stock of machines in “inventory” a rental company may be motivated to buy additional machines for a temporary period of time to meet the excess demand by its rental customers. Conversely in hard economic times, if the company forecasts a sustained period of low demand (and thus excess supply of machines of a given type), it may be optimal for it to “downsize” by selling some of its older idle machines.

5.1 Caveats and Simplifying Assumptions

Unfortunately in order to build a more realistic dynamic model of overall fleet size management, we would need better measures of the overall demand for machines of specific types at specific locations, something we are unable to construct from the data we were provided. So our best alternative is to formulate model of replacement policy for an assumed set of “core machines” that the firm knows it will always need to have, in good times and in bad. Later, if we are provided more data to provide finer grained prediction of the demand for machines of specific types at specific locations, we can extend our analysis to a fuller dynamic optimization that can be used to determine the optimal fleet size over time.\[4\]

We believe that important insights can be gained by considering initially a smaller “subproblem” of the overall fleet and portfolio optimization (and the related problem of setting rental rates for machines). If the overall management of a firm is approximately optimal (i.e. profit maximizing), it should not be possible to obtain significantly more profits by varying the management strategy of various subproblems of the firm’s overall optimization problem. Conversely, suppose we find that there are ways to change policies the firm currently uses for specific well defined subproblems (such as when a firm should sell an existing machine and replace it with another one), that results in significantly greater profits. Provided this alternative strategy does not significantly impinge on or constrain or change other parts of the firm’s operations and operating policy, it makes sense for the firm to consider adopting the improved policy since it results in greater expected profits even though the change may not be enough on its own to result in a “globally optimal” solution to the company’s overall management strategy for a given rental location.

As we indicated in the previous section, a simplistic analysis of “breakeven ages” — i.e. the age at

\[4\]As we discuss in the conclusion there are even bigger questions of portfolio management — which “portfolio” of machines should a company a at specific rental location subject to demand, space, and capital constraints at that location? So even the question of optimal fleet size for a particular machine type such as excavators is only a “subproblem” of the larger problem of optimal management of individual rental locations.
which the current expected rental profit equals 0 — is unlikely to be a reasonable basis for determining an optimal replacement policy because it completely ignores a critically important additional cost of maintaining a fleet of rental machines: the cost of replacing them. Thus, our perspective is to consider the policy for when a firm should replace a currently held piece of rental equipment with a new one, with the objective of maximizing the expected discounted stream of future profits from a potentially infinite sequence of machines.

When a replacement occurs, the firm incurs a large net cash outflow to cover the cost of replacing the existing old machine. This replacement cost, of course, is the OEC of the new replacement machine less the price the firm receives from selling or auctioning the old machine, less any additional taxes or transactions costs. Our analysis will focus on policies that maximize the expected discounted value of gross cash flows. We ignore taxes and the allocation of corporate fixed costs such as the cost of buying or renting the land where the rental locations are, wages of employees, and corporate overhead. We believe that tax provisions such as like kind exchange (which enable companies to “roll over” capital gains from the sale of an old machine into the taxable basis of the new replacement machine) enable firms to defer many capital gains taxes associated with the replacement of machines into the indefinite future. For this reason we do not think that the timing of realization of capital gains for tax purposes is an important factor to consider in our first cut analysis of the strategic timing of machine sales.

Though some companies may be subject to a corporate income tax, this tax appears to us (at least to a first approximation) to simply proportionally reduce the profits earned by specific machines, but otherwise has no obvious impact on the optimal timing of equipment replacements. Thus we also ignore accounting details such as the depreciation of the initial acquisition cost of a machine on a straight line or accelerated depreciation formula for tax accounting purposes. These tax depreciation expenses do offset some of the taxes the firm will pay on its gross cash flows similar to the maintenance costs are a tax deductible expense, but we do not believe that accounting for the tax benefits of equipment depreciation would materially affect the optimal replacement strategies we calculate below.

Our infinite-horizon model of optimal replacement of rental equipment is formulated on a monthly basis, to match the periodicity of our data. It is possible to formulate and solve models on a finer time scale, including the daily level, where the rental state of each machine (i.e. time on and off rent) is tracked on a day by day basis. An example of this type of model is Cho and Rust [2010], which focuses on optimal replacement of rental cars of a large Korean rental car company, and it distinguishes between long term and
short term rental contracts. A finer time scale is desirable for more detailed tracking and management of a fleet. For example, it is important for management to know, on any given day, how many of its machines are out being rented, how many are in the shop for maintenance and repairs, and how many are sitting idle in the lot waiting to be rented.

Since the data we were provided are at monthly rather than daily frequencies, we decided to develop a simplified monthly model. Though this model will not be able to answer some questions, we believe a monthly model can be useful for gaining new insights into replacement policy and potentially identifying opportunities to improve profitability. The model we develop below is “generic” in the sense that it can apply to a variety of equipment types. The data needed to operationalize and solve the model for any specific type of machine or equipment at a monthly time scale are 1) rental revenues, 2) maintenance costs, and 3) prices of new and used versions of this machine. We assume that maintenance is done in an optimal manner and constitutes the minimal cost necessary to keep the machine in safe working condition. With additional data, it is possible to extend the model to optimize over maintenance decisions, such as the timing of major upgrades, refurbishments, rebuilds, engine replacements, or other investments in “renewing” existing machines such as considered in [Rust (1987)] and [McClelland et al. (1989)]. We abstract from accidents that result in a total loss of the machine and assume that the rental company has insurance coverage that covers such losses and enables it to purchase a replacement machine of equivalent age and condition at no additional cost (e.g. zero deductible insurance). This assumption could be easily relaxed in future versions of the model.

We also arbitrarily restrict our replacement policy to constrain the firm to replace an old machine with a *brand new machine* which is acquired at the OEC price. However it is possible to relax this constraint and allow the firm to replace an existing old machine with another used but newer machine. Or we can fully relax the choice and allow an existing old machine to be replaced by an *even older machine*. We would conjecture that the latter policy is not generally an optimal one, but it does depend on how machine resale prices vary with machine age and over seasons and the business cycle. We will show below that the dynamic programming solution is able to find and exploit “arbitrage opportunities” that may exist in the shifting pattern of OEC and resale prices of machines. Older (i.e. not brand new) replacement machines might be acquired, for example, at a wholesale auction for used machines.

There may be even greater profit opportunities from relaxing the replacement constraint in this fashion, but we judged that allowing such replacement strategies might be regarded as too big a departure from
standard operating practice in the industry to have credibility. There are also considerations of “market thickness” — whether there are sufficiently many used machines available at any given age available to be purchased at auction or in secondary markets for used equipment. If markets were thick enough, our preliminary calculations suggest that there can be significant additional profit gains from strategies that allow firms to replace a machine with another used machine. However since we are unsure of market thickness for used machines, we did not attempt to systematically evaluate the additional profit gains from relaxing our “replace used machine with a brand new machine” constraint that we imposed in the results we report in this paper.

A final caveat is that in order to keep this version of the model simple, when there are several different makes/models of the same piece of equipment, we solve the problem under the assumption that the company always buys the same make/model for the replacement unit. Thus we abstract both from the possibility of future technological change and a gradual evolution in the makes and models of rental equipment, and we ignore the potential to “arbitrage” across different makes/models to choose the make/model that provides the highest discounted profit to the rental company. While this might seem to be a restrictive assumption, we are able to solve for optimal replacement strategies for individual makes and models of equipment and calculate the discounted profits of each different type. Thus, if one type of equipment generates higher discounted profit, the implication is obvious: the rental company should buy the make/model that generates the highest profits. The only case for buying multiple different makes/models is if there is intertemporal price variability in different types of machines, so that there may be temporary “good deals” on specific machines that make them the best choice at that price, even though they may not be the the most profitable machine for the company to buy at the normal prices available for these machines.

In general, however, our model assumes that the only intertemporal variability in machine prices is in the OEC price available in different parts of the country and at different parts of the business cycle. As we have already shown, the construction machinery sector is a notoriously cyclical industry since demand for new machines follows the pronounced cycles in construction spending. Thus, in recession periods, machine manufacturers may reduce the OEC prices to try to stimulate more sales, but during boom periods manufacturers may experience order backlogs and raise OEC prices. Our model can account for these cyclical variations in OEC prices and profitability exploit predictable price patterns and comovements such as buying new machines during recession periods when OEC prices tend to be lower, and selling used equipment in boom period when the prices of used machines tend to be higher. This enables the rental
company to, in effect, “buy low and sell high” by strategically timing the replacement of its machines over the business cycle to further increase its profitability.

5.2 Mathematical formulation of the model

Now we are in a position to describe our model mathematically. The only decision we consider at this stage is the binary decision of whether to replace an existing piece of equipment (which is identified as a specific type and make of machine). There are three main state variables in our model: \((a, s, m)\) where \(a\) denotes the age of the machine in months, \(s\) denotes the “macro state” which takes the values 1, 2 or 3 where 1 denotes a recession or low demand month for the construction sector (which we have referred colloquially as a “bust month”) when the firm expects rental demand for its machines to be low too, and 3 denotes a month when construction demand and therefore rental demand is at its peak, which we have described as a boom period. If a month is not a bust or boom month we refer to it as a “normal month” during which the firm can expect its “normal” demand for rental equipment.

The variable \(m\) denotes the current month of the year (Jan, Feb, . . . , Dec). We include this to capture seasonal effects on rental demand, such as the fact that demand for construction equipment is typically higher during the summer months than in the winter months, a pattern we typically found in our econometric analysis of rental revenues and rental equipment utilization in section 3. We allow the macro state variable \(s\) to evolve over time conditional on its past value and the month of the year. That is, we assume that \(\{s_t\}\) evolves as a stochastic process, where the (discretized) value of the construction spending index \(s_t \in \{1, 2, 3\}\) at each time \(t\). We assume this process is Markovian with transition probability \(\pi(s_{t+1}|s_t, m_t)\) where \(\pi\) is a transition probability that specifies whether the probability that the macro state variable \(s_{t+1}\) will take the value 1, 2 or 3 in period \(t+1\) conditional on its value \(s_t\) in period \(t\), and also on which month time \(t\) corresponds to, \(m_t\). Obviously the month variable cycles deterministically according to the law of motion \(m_{t+1} = \text{mod}(m_t + 1, 12)\) according to modulo arithmetic, i.e. the next month after December \((m_t = 12)\) is January, which is not \(m_t + 1 = 13\) but rather \(m_t = 1 = \text{mod}(13, 12)\), the remainder when we divide 13 by 12 using integer arithmetic.\[^5\]

It is obvious why we want to keep track of the age of the machine: our econometric analysis shows that in general rental revenues for a machine decline with its age whereas maintenance costs increase with

\[^5\] Paarsch and Rust [2009] introduce a “cyclic inversion algorithm” to substantially speed up the solution of dynamic programs that involve stochastically cycling state variables such as \(s_t\) and deterministically cycling state variables such as \(m_t\). We can use this algorithm to speed up the numerical solution of the dynamic programs we describe below.
age. Because of these two effects, it is generally the case that there will be a specific age or replacement threshold beyond which the firm will want to replace an existing old piece of rental equipment by a new (or newer one). We include the macro state \( s \) and the month of the year \( m \) as additional state variables for the obvious reason that rental revenues vary over the business cycle and over months in the year, since in the construction industry the peak period of demand is in the summer months, so construction demand for many types of rental machines, and thus rental revenues, tend to be higher in those months.

The OEC price and the price of used equipment also varies with the business cycle and may also vary over different months in the year. In general, whenever we can econometrically uncover a solid relationship between our state variables and the relevant revenues, maintenance costs, and prices of new and used equipment, we want to include the key variables into our optimization model that help predict variations in revenues and costs that affect company profits. Our model can then adapt an optimal replacement policy to exploit or “arbitrage” these predictable variations and relationships in OEC prices, used machine price, rental revenues, and maintenance costs to help firms earn greater profits.

There are other “implicit state variables” in our model that are geography based: we have data on rental revenues, maintenance costs and the prices of used equipment in different regions in the country and there can be different statistical relationships governing rental revenues and resale prices that hold in widely varying parts of the country, say between the Southwest and the Northeast. These differences can persist due to the transportation costs of moving heavy equipment across the country, and thus there can be differences in rental revenues and even in the prices of used rental equipment that will not be arbitraged away by moving rental equipment from low profit regions in the country towards high profit regions.

Let \( V(a,s,m) \) be the expected present discounted value of profits of a given make/model of rental equipment at a given rental location in a specific region of the country. In terms of our notation in section 3, we have suppressed \( \tau \) which indexes the type and make of a specific machine to simplify the equations of the model. We have also suppressed the company and region indices \((c,r)\), but all of our results provided in the next section will account for different machine types, makes, companies, and regions. That is, in our analysis the value function and decision rule implied by the model also depends on \((\tau,c,r)\) in addition to \((a,s,m)\), however the variables \((\tau,c,r)\) are assumed to be time invariant whereas the variables \((a,s,m)\) change over time, something we denote by time subscripts \((a_t,s_t,m_t)\) which denotes a machine of age \( a_t \), in macro state \( s_t \) in month \( m_t \) at time \( t \).

There is a functional equation for \( V \) known as Bellman’s equation that we solve in order to determine
the optimal rental equipment replacement policy for the firm

\[
V(a,s,m) = \max \left[ R(a,s,m) - C(a) + \beta \sum_{s' = 1}^{3} V(a + 1, s', m'), \right.
\]

\[
\left. \max_{a' \geq 0} R(a', s, m) - C(a') + P(a', s, m) - P(a, s, m) + T(a) + \beta \sum_{s' = 1}^{3} V(a', s', m'), \right]
\]

where \( m \) is the index of the month of the year and evolves according to \( m' = m + 1 \mod(12) \) as discussed previously.

Equation (12) states that \( V(a, s, m) \) is the maximum of two different options: 1) keeping the existing machine of age \( a \), or 2) selling the existing machine \( a \) and purchasing a new machine \( a' \), but choosing the best such replacement age \( a' \). This is indicated by the second max operation in the right hand term of equation (12). The function \( R(a, s, m) \) denotes the expected rental revenues earned from rentals of a machine of age \( a \) in macro state \( s \) in month \( m \), and \( C(a) \) denotes the expected costs of maintaining that machine. The function \( P(a, s, m) \) is the market price function that specifies the price the rental company expects to receive by selling its existing machine of age \( a \) when the macro state is \( s \) and the month is \( m \).

When it replaces the old machine, it buys another replacement machine of age \( a' \) for a price \( P(a', s, m) \) and of course this is a cash outflow for the company and it why this enters with a negative sign in equation (12).

The symbol \( T \) denotes the expected value of any transactions costs associated with trading the current used machine of age \( a \) for a replacement unit of age \( a' \). These costs could include transportation costs of hauling out the machine to be sold and bringing in the new replacement unit, any licensing or transfer costs, and any auction or sales fee to an intermediary that assists the company in making this transaction. Finally the symbol \( \beta \in (0, 1) \) denotes the company’s discount factor which is related to the company interest rate (or risk-adjusted “opportunity cost of capital”) \( r \) via the equation

\[
\beta = \frac{1}{1 + r}
\]

The discount factor can also incorporate a risk of bankruptcy or closure of a specific rental location provided we assume there is a constant probability \( p \) that this will occur. Then a component of the discount factor \( \beta \) will include a “survival probability” \( (1 - p) \) which corresponds to the probability that the rental location avoids “dying” in any given month, which would result in a permanent termination of further cash flows.

As we noted above, we solved the dynamic programming problem by solving the Bellman equation (12) but under the constraint that when a firm replaces an existing machine, it can only purchase a new
machine. This amounts to the restriction that \( a' = 0 \) in equation (12), in which case \( P(a', s, m) = P(0, s, m) \) is the OEC price, i.e. the cost to the firm of purchasing a brand new machine to replace the existing used machine. When we relax this constraint, the resulting value function will only increase: this reflects that providing the firm with additional options can only increase and will never decrease its present discounted profits. But as we noted above, we decided to impose the constraint that a used machine must be replaced by a brand new one. We did this because we are not sure of how “thick” markets for used machines are, and because of our desire to avoid conclusions that might be regarded as too radical or unconventional to firms in this industry, which to our knowledge, do generally follow a strategy of replacing used machines with brand new ones.

The Bellman equation can be used to solve for the function \( V(a, s, m) \) and from this function we can derive the optimal replacement policy \( d(a, s, m) \) where \( d(a, s, m) = a \) if it is optimal to keep the current machine, and \( d(a, s, m) = a' \) if it is optimal to trade the current machine of age \( a \) and replace it with another machine of age \( a' \). The equation for \( d(a, s, m) \) is given by

\[
d(a, s, m) = \begin{cases} 
  a & \text{if } EV(a, s, m|0) > EV(a, s, m|1) \\
  a' & \text{otherwise}
\end{cases}
\]  

(13)

where \( a' \) is the age of the optimal replacement machine from the bottom formula in the Bellman equation (12) and \( v(a, s, m, 0) \) is the expected payoff to keeping the current machine given in the top expression of equation (12)

\[
v(a, s, m, 0) = R(a, s, m) + C(a) + \beta \sum_{s' = 1}^{3} V(a + 1, s', m')
\]  

(14)

and \( v(a, s, m, 1) \) is the payoff to replacing the current machine with the best replacement machine \( a' \) given in the lower formula of equation (12)

\[
v(a, s, m, 1) = \max_{a' \geq 0} \left[ R(a', s, m) - C(a') + P(a', s, m) - P(a, s, m) + T(a) + \beta \sum_{s' = 1}^{3} V(a', s', m') \right].
\]  

(15)

These equations assure that if the firm follows the optimal replacement rule \( d(a, s, m) \) the firm will obtain the profit maximizing payoff \( V(a, s, m) \), and this value will represent the expected present value of following an optimal replacement policy for a machine that is currently in state \((a, s, m)\). If \( v(a, s, m, 0) > v(a, s, m, 1) \), then \( d(a, s, m) = a \) and the difference in values \( EV(a, s, m|0) - EV(a, s, m|1) \) represents the gain (in discounted profits) from keeping the current machine instead of replacing it with an optimally chosen replacement unit. Conversely if \( v(a, s, m, 1) > v(a, s, m, 0) \), then it is optimal for the firm to sell the
current machine of age \( a \) and buy an optimally aged replacement unit \( d(a,s,m) = a' \) and \( v(a,s,m,1) - v(a,s,m,0) \) represents the gain in expected discounted profits from selling the existing machine of age \( a \) with a new machine of age \( a' \). If transactions costs are positive, \( T(a) > 0 \), we can prove the obvious result that it is never optimal for the firm to replace a machine of age \( a \) with another machine of age \( a' \): this simply results in a loss of expected discounted profits of \( T(a) \).

Typically it will be the case that \( a' = 0 \), i.e. the firm replaces an old machine with a brand new one. In most of our results, we place an arbitrary restriction that when a firm replaces an old machine it must buy a brand new replacement machine. But this is a restriction we can relax. If there is a sufficiently “thick market” the company might be able to do better by buying a slightly used machine rather than a brand new machine, especially if there is rapid early price depreciation for rental machinery similar to what we observe in the market for used cars. However the key is whether there is a sufficiently active market in very new but not brand new machines. If not, it may not be feasible to buy slightly used machines that are only a few months old. This is part of the reason why we opted to constrain the choice of replacement to only \( a' = 0 \), i.e. to constrain firms to buy brand new replacement machines at OEC.

5.3 Valuing existing, potentially suboptimal replacement strategies

Let \( \mu(a,s,m) \) be the probability a firm replaces machines under its existing or status quo replacement policy. We have already econometrically estimated these probabilities in section 4.4 above. We assume that the status quo replacement policies always involve replacing any machine that a company sells with a brand new model, i.e. \( a' = 0 \). We are interested not to know only the behavior corresponding to firms’ existing replacement policies (i.e. which ages, months and macro states make the firm most likely to replace an older machine), but we also want to know the value of expected discounted profits implied by these policies. Let \( V_\mu(a,s,m) \) be the present discounted value of gross profits (i.e. pre-tax and not accounting for any allocation of corporate fixed costs, etc. as discussed above). Then we have the following recursive linear equation for \( V_\mu(a,s,m) \) which is analogous to the Bellman equation (12):

\[
V_\mu(a,s,m) = \left[ 1 - \mu(a,s,m) \right] \left[ R(a,s,m) - C(a) + \beta \sum_{s' = 1}^{3} V_\mu(a+1,s',m') \right] + \mu(a,s,m) \left[ R(0,s,m) - C(0) + P(0,s,m) - P(a,s,m) + T(a) + \beta \sum_{s' = 1}^{3} V_\mu(1,s',m') \right].
\] (16)
By construction, we have $V(a,s,m) \geq V_\mu(a,s,m)$, i.e. the optimal value function $V(a,s,m)$ from the solution of the Bellman equation \[12\] is always at least as great as the value of any other replacement policy $\mu(a,s,m)$, given by the value $V_\mu(a,s,m)$. Thus, the difference $V(a,s,m) - V_\mu(a,s,m) \geq 0$ represents the gain in discounted profits to the firm from switching from its existing status quo replacement policy $\mu(a,s,m)$ to the optimal replacement policy $d(a,s,m)$. However it is important to keep in mind that $d(a,s,m)$ is only “optimal” in a restricted sense: we are taking as given the firm’s policies over how it prices its rental equipment and its overall fleet management and rental location “portfolio allocation” — i.e. how it allocates the limited space and capital in a given rental location over different types, makes and models or rental equipment.

5.4 Accounting for Idiosyncratic Shocks

Note that the only state variables in the dynamic program we are solving \([12]\) are $(a,s,m)$. This implies that all machines of a given type, model, and age will be treated the same in any given month $m$ and macro state $s$. If we further restrict old machines to be replaced with brand new ones, we can reinterpret the optimal decision rule $d(a,s,m)$ as a (degenerate) probability distribution function where $d(a,s,m) = 0$ if it is not optimal to replace the current machine (i.e. it is optimal to keep it) and $d(a,s,m) = 1$ if it is optimal to replace the machine. Thus a key difference between $d(a,s,m)$ and the firm’s status quo replacement policy $\mu(a,s,m)$ is that $d(a,s,m)$ only takes the values 0 or 1 whereas $\mu(a,s,m)$ will generally have values strictly between 0 and 1 as we saw in figures \([14] and \([15] in section 4.4.

From the standpoint of a model where $(a,s,m)$ is the only information a firm uses to make replacement decisions, it is never optimal for it to “randomize” and replace the machine with a probability $\mu(a,s,m)$ that is different from both 0 and 1. The reason is that in any given state, the expected discounted profits of keeping the current machine relative to replacing it are either strictly positive, or strictly negative (a tie almost never occurs). If the expected gain in profits from replacing the machine, a quantity we denoted as $v_\mu(a,s,m,1) - v_\mu(a,s,m,0)$ where the expected values $v_\mu(a,s,m,1)$ is the expected value of replacing the machine under the firm’s status quo replacement policy, a quantity given by equation \([15]\), and $v_\mu(a,s,m,0)$ is the value of keeping the current machine under the status quo replacement policy $\mu$ given in equation \([14]\), but with $V_\mu$ also substituted for $V$. If the difference $v_\mu(a,s,m,1) - v_\mu(a,s,m,0) > 0$ is positive, then the firm ought to replace the machine with probability 1, and if the difference is negative, then the firm ought to replace the machine with probability 0 (i.e. keep it).
How do we “rationalize” the actual replacement behavior of equipment rental firms then? We believe replacement behavior that appears “probabilistic” to us as observers is probably not actually probabilistic to firm that has additional information about any individual machine that we do not observe in our data set. Certainly the firm observes the states of its machines in continuous time and can decide to sell a machine at any point within a month depending on the information it observes that we do not observe (modulo the constraint that if a machine is out with a customer being rented, the firm may not be able to replace it until the customer returns it, unless the firm can deliver a suitable replacement unit).

As we noted in our econometric analysis in section 4, there are a number of idiosyncratic revenue and cost shocks that can be very large that could affect a firm’s replacement decision. For example, an inspection of a machine may reveal the need for a costly engine replacement or other repair that may not be economically worthwhile if the machine is sufficiently old. Or a machine may not be rented for many days in a given month, leading to lower than expected rental revenues in that month. A final possibility is that the firm may receive an offer to buy one of its machines for a price that is much higher than the price $P(a, s, m)$ that it would expect to get in an auction or sale. Any of the revenue shocks, maintenance cost shocks, or resale price shocks could be large enough to make it optimal for the firm to replace the machine when such shocks are also taken into consideration, even though in terms of expected future profits, not taking these idiosyncratic factors into account it is not optimal to sell the machine: i.e. $\nu_{\mu}(a, s, m, 1) < \nu_{\mu}(a, s, m, 0)$.

Since our econometric analysis was able to identify the variances of the idiosyncratic shocks to monthly revenues, maintenance costs, and resale prices, we can extend our model to allow for these shocks. Let $(\varepsilon_r, \varepsilon_c, \varepsilon_p)$ denote a vector of shocks to rental revenues, maintenance costs, and resale prices, respectively. We will treat these as multiplicative shocks that are lognormally distributed with mean equal to 1 and standard deviations equal to the standard deviations of the residuals in our regression results in section 4. Then the firm’s information, taking into account the idiosyncratic shocks $(\varepsilon_r, \varepsilon_c, \varepsilon_p)$ is given by the vector $(a, s, m, \varepsilon_r, \varepsilon_c, \varepsilon_p)$ and there will be a version of the Bellman equation that holds for this expanded set of state variables and the value function for the problem that includes these idiosyncratic shocks is $V(a, s, m, \varepsilon_r, \varepsilon_c, \varepsilon_p)$ and the decision rule that is implied by this value function is $d(a, s, m, \varepsilon_a, \varepsilon_r, \varepsilon_p)$ and for almost all values of the vector of state variables $(a, s, m, \varepsilon_r, \varepsilon_c, \varepsilon_p)$ this decision rule (expressed as a probability of replacement) will be either 0 or 1.

Note that for any given value of the full vector of state variables, $(a, s, m, \varepsilon_r, \varepsilon_c, \varepsilon_p)$ when we “integrate out” the idiosyncratic shocks $(\varepsilon_r, \varepsilon_c, \varepsilon_p)$ of the decision rule, we obtain conditional on the “observed”
variables \((a,s,m)\) a conditional probability of replacement that will be strictly between 0 and 1, which we denote by \(\mu^*(a,s,m)\) given by

\[
\mu^*(a,s,m) = \int_{\varepsilon_r} \int_{\varepsilon_c} \int_{\varepsilon_p} d(a,s,m,\varepsilon_r,\varepsilon_c,\varepsilon_p) \Phi_r(\varepsilon_r) \Phi_c(\varepsilon_c) \Phi_p(\varepsilon_p),
\]

(17)

where \(\Phi_r, \Phi_c, \Phi_p\) are the lognormal cumulative distribution functions of the random variables \((\varepsilon_r, \varepsilon_c, \varepsilon_p)\), which we also assume are mutually independent random variables. Thus, the fact that we generally have \(0 < \mu^*(a,s,m) < 1\) does not imply that the underlying decision rule \(d(a,s,m,\varepsilon_r,\varepsilon_c,\varepsilon_p)\) is suboptimal. If we also interpret the firm’s status quo decision rule \(\mu(a,s,m)\) as also constituting our best estimate of a decision rule that depends on the additional information \((\varepsilon_r, \varepsilon_c, \varepsilon_p)\), it follows that \(\mu(a,s,m)\) is also generally between 0 and 1 does not imply that the underlying decision rules that firms use (which depend on more information that we actually observe) are necessarily suboptimal either.

For completeness we provide the Bellman equation that incorporates the idiosyncratic shocks below

\[
V(a,s,m,\varepsilon_r,\varepsilon_c,\varepsilon_p) = \max \left[ R(a,s,m)\varepsilon_r - C(a)\varepsilon_c + \beta EV(a,s,m|0), \right. \\

R(0,s,m) - C(0) + P(0,s,m) - P(a,s,m)\varepsilon_p + T(a) + \beta EV(a,s,m|1) \right].
\]

(18)

where we have

\[
EV(a,s,m|0) = \int_{\varepsilon_r'} \int_{\varepsilon_c'} \int_{\varepsilon_p'} \sum_{s=1}^{3} V(a+1,s',m',\varepsilon_r',\varepsilon_c',\varepsilon_p') \Phi(\varepsilon_r') \Phi(\varepsilon_c') \Phi(\varepsilon_p'),
\]

(19)

\[
EV(a,s,m|1) = \int_{\varepsilon_r'} \int_{\varepsilon_c'} \int_{\varepsilon_p'} \sum_{s=1}^{3} V(1,s',m',\varepsilon_r',\varepsilon_c',\varepsilon_p') \Phi(\varepsilon_r') \Phi(\varepsilon_c') \Phi(\varepsilon_p').
\]

(20)

At the monthly time scale of our model, there is some question as to whether it is better (i.e. more “realistic”) to allow for the presence of idiosyncratic shocks, or ignore them. The timing of the monthly model can be described as follows: on the first day of each month, the manager of a rental location observes the age of a current machine and the value of the macro shock \(s\) and makes a decision based on the information he/she has at the start of the month whether or not to replace the machine. From the model perspective, it is natural to assume that the manager is making his/her decisions \textit{ex ante} that is, before the manager “sees” the realized values of rental revenues, maintenance costs, or the actual resale price he/she will obtain if they decide to sell the machine. So in this \textit{ex ante} perspective, the manager does not actually observe the idiosyncratic shocks \((\varepsilon_r,\varepsilon_c,\varepsilon_p)\) at the start of the month, and must make their decisions based on the expectations of these shocks. If this is the perspective that seems more “realistic” then the solution that ignores the idiosyncratic shocks given in equation \([12]\) is the appropriate framework to use.
However if we consider that rental companies are actually operating in continuous time and can make decisions at any time during the month, not just on the first day of the month, then it seems more realistic to allow for the idiosyncratic shocks. Information on realized rental revenues, maintenance costs will be arriving all the time during the month, and will affect the rental manager’s decisions. Further, as we noted above, rental companies may sometimes get offers to sell one of their machines at “retail” for a price that is much higher than what they could expect to get if they sold the machine in an auction. So the manager may be able to see such offers and make a more informed decision by taking this information into account. In this sense, it would be better to use the version of the model that allows for the idiosyncratic shocks as it is able to better approximate the sort of information available to managers of rental location in the true, continuous time reality rather than in the artificial monthly time intervals of our model, which as we noted are largely dictated by the periodicity of our data set.

In the next section we will report results from the version of the model without the idiosyncratic shocks, but in future work we will also solve and simulate the model when we allow for idiosyncratic shocks, in order to see if it results in any major changes in our conclusions. But for the first version of this model, we judged it would be easier to explain and communicate results using a model that abstracts from idiosyncratic shocks, and it also simplifies the numerical solutions for the value functions $V(a,s,m)$ and $V_\mu(a,s,m)$ and the optimal decision rule $d(a,s,m)$.

6 Results

In this section we solve the dynamic programming problem described in the previous section to determine optimal replacement policies, using the econometric predictions of rental revenues, maintenance costs, OEC prices and resale values that were presented in section 4. We describe the optimal replacement policies in some detail and compare them to the probabilistic replacement rules $\mu(a,s,m)$ that we econometrically estimated in section 4.4. We emphasize that there is no single “optimal replacement policy” but rather we calculated optimal replacement policies that are individually optimized for each company, machine make, and rental location. In the notation of the previous section, our calculated optimal replacement rules do not only depend on $(a,s,m)$ (i.e. the machine age $a$, macro state $s$, and month $m$), but also on $(\tau,c,r)$ where $\tau$ indexes the machine type and make, $c$ indexes a specific company, and $r$ indexes the rental location.
6.1 Excavators

The first results we present are for excavators. Figure 16 displays the optimal replacement policy and the corresponding optimal value function for a make 4 excavator owned by company A in region 6. We also compare the optimal replacement policy to the one firm A actually uses, as well as the implied discounted value of profits that firm A can expect to earn from an infinite sequence of these machines under the status quo. The top panel of figure 16 shows the optimal replacement thresholds by month, for the three different values of the macro state variable. We see that there is some variability in these thresholds across different months of the year, and this variation reflects predictable differences in rental revenues and replacement costs in different months. Generally, the DP model predicts that the replacement thresholds are the lowest in boom months ($s = 3$) and highest in normal months. Thus, for excavators we find the optimal replacement policy is pro-cyclical but with one key difference. Interestingly, the replacement threshold in a bust month lies in between the thresholds applicable in a boom month and a normal month. That is, under the optimal policy the firm is least likely to replace a machine in a normal month rather than in a bust month.

Thus, if January was a boom month, the DP model predicts that it is optimal to replace the excavator once it is 60 months old or older. However in a bust month, the model predicts that the firm should not sell the machine until it is more than 150 months old, and in a normal month, the firm should not replace the machine until it is more than 190 months old. The middle panel compares the calculated optimal replacement policy with the policy firm A actually follows. We see that in all three macro states, the firm replaces machines when they are roughly 60 months old, with a median age of replacement ranging from about 57 months old in boom times to about 70 months old in normal or bust months. Thus, company A is not varying its replacement policy over the business cycle as much as the DP model predicts is optimal in order to take advantage of predictable changes in replacement costs and rental revenues over the business cycle.

The third panel of figure 16 plots the value functions as a function of the machine age and macro state and compares them to the corresponding value functions for company A under its status quo replacement policy. We see that the value functions corresponding to the optimal policy are uniformly higher. Specifically if we compare the solid red line (the value of the machine under the optimal policy as a function of age when the economy is in a boom month) with the dashed red line (the value of the machine under the status quo), we see that the solid red line is uniformly higher at all ages. If we consider a brand new ma-
Figure 16: Optimal replacement policy for a make 4 excavator owned by company A in region 6
chine, i.e. at age 0, the value function is \( v(0, 3, 1) = 435984 \), i.e. the firm can expect to earn a discounted profit of $435,984 \textit{over an infinite horizon} by following the optimal replacement policy. This works out to a monthly gross profit of $3618. In comparison, the expected discounted value of profits under company A’s \textit{status quo} replacement policy is \( v_\mu(0, 3, 1) = 421528 \), i.e. it expects to earn $421,528 over an infinite horizon involving an infinite sequence of purchases and replacements of excavators. This works out to a monthly profit of $3498. Thus, the optimal policy results in an increase in discounted profits of 3.4\% \( (0.034 = (3618 - 3498)/3498) \).

To get some perspective on what we mean by “value”, note that the average expected discounted profits the rental company expect to earn if it owns a new make 4 excavator (averaged over the different months and macro states) is $436,519. It is important to realize that this number does not only include the discounted profits from the initial make 4 excavator from its initial purchase until replacement, but it also includes the entire stream of discounted profits from the sequence of all future make 4 excavators the company will purchase and rent to customers. We can calculate the expected discounted value of profits for the \textit{current machine only} and is roughly half as large: $232,449. The average of the predicted OEC costs to buy a new make 4 (again averaged over the different months and macro states) is nearly $140,000. The valuations we calculated above presume that company A already owned a brand new make 4 excavator. If it did not already own it, we would have to subtract the OEC cost to buy a new make 4 to calculate the net profit to the company. Thus, company A expects to earn a gross discounted profit of $232,449-140,000=$92,448 from its ability to rent \textit{this} make 4 excavator to its customers. However if we consider company A as infinitely lived with the ability to rent a sequence of make 4 excavators to customers extending into the infinite future, the expected discounted value of these profits are about 3 times the OEC of the initial make 4 excavator. This suggests that Company A’s southwest location is a relatively profitable rental location, because in effect company A can expect to get more than a 3 to 1 return on its investment of about $140,000 for its first of a sequence of make 4 excavators.

Of course, the use of an infinite horizon model involves some assumptions that one might question, such as whether the prices and rental revenues of make 4 excavators will remain the same over the infinite future. We can reasonably expect rapid technological progress and perhaps not too far in the distant future there may be new models of make 4 excavators that are fully \textit{robotic} (i.e. they do not require human operators or can be directed and run remotely), and may have a number of other technological improvements that change their OEC and resale prices as well as the rental revenues company A can
expect to earn. We have not attempted to adjust our predictions for such possible technological changes as they are too hard to predict. Our defense for failing to do this is that since future profits are discounted, what happens far ahead in the future does not have a big impact on our calculations, which are dominated by profits received in the near term future which we can more confidently predict.

In order to obtain more insight into how the optimal replacement policy enables the firm to earn greater profits and whether these increased profits can actually be realized in the near term we resort to the technique of monte carlo simulations. That is, we can simulate detailed future paths of revenues, maintenance costs, machine sales and purchases, as well as macro shocks that affect these quantities, under the optimal replacement policy and under companies’ status quo replacement policies. Further, we can do these simulations over shorter horizons to determine if the optimal replacement policy can actually deliver profit gains in a reasonably short period of time (as opposed to delivering most profits far off in the future when it is far less certain that the predictions of our model will still be valid). Further, the simulations provide more insight into actually how the optimal replacement policy can deliver improvements in profitability.

Table 4 provides a breakdown of average simulated gross profits for the make 4 excavators of Company A in one of its locations in region 6 into its key components: revenues, maintenance costs, and replacement costs. The table shows discounted and undiscounted values separately, and all are reported on a per machine basis, which is an average over all 50 simulated machines, each simulated for 60 periods, and finally averaged over 20 independent replications with different sequences of stochastic shocks such as macro shocks drawn in each replication. We see that because the optimal policy entails keeping these excavators longer than company A keeps them under its status quo replacement policy, revenues are lower and maintenance costs are higher under the optimal replacement policy than what company A would expect under the status quo. However replacement costs are the biggest expense for Company A, and by keeping these

Table 4: Detail on simulations of make 4 excavators of Company A, region 6

<table>
<thead>
<tr>
<th>60 month simulation</th>
<th>Discounted values</th>
<th>Undiscounted values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal</td>
<td>Status Quo</td>
</tr>
<tr>
<td>Revenues</td>
<td>$231,439</td>
<td>$238,260</td>
</tr>
<tr>
<td>Maintenance costs</td>
<td>14,246</td>
<td>8,951</td>
</tr>
<tr>
<td>Replacement costs</td>
<td>34,134</td>
<td>64,405</td>
</tr>
<tr>
<td>Gross profits</td>
<td>$183,059</td>
<td>$164,903</td>
</tr>
</tbody>
</table>
Table 5: Gain in discounted profits from optimal replacement policy: excavators

<table>
<thead>
<tr>
<th>Make/Company</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>9.6%</td>
<td>2.7%</td>
<td>1.9%</td>
<td>5.1%</td>
<td>2.8%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Make 2</td>
<td>4.7%</td>
<td>1.2%</td>
<td>1.2%</td>
<td>2.1%</td>
<td>1.8%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Make 3</td>
<td>3.1%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>2.5%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Make 4</td>
<td>3.5%</td>
<td>1.8%</td>
<td>1.6%</td>
<td>1.4%</td>
<td>3.1%</td>
<td>1.8%</td>
</tr>
<tr>
<td>Make 5</td>
<td>10.4%</td>
<td>2.9%</td>
<td>2.1%</td>
<td>5.5%</td>
<td>3.0%</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

excavators longer and strategically timing their replacement over the business cycle, the optimal policy is able to reduce replacement costs by nearly 50%. As a result, the optimal replacement policy leads to an 11% increase in discounted profits and a 11.8% increase in undiscounted profits on average in our 20 simulation replications.

Table 5 reports the percentage increase in discounted profits from switching from the status quo replacement policies to the optimal policies for 30 different combinations of company/location and makes of excavators. We solved separate dynamic programs that are specific to each company/region and make of excavator, and then simulated the optimal replacement policy over a 60 month (5 year) horizon for a fleet of 50 excavators. We initialized each simulation by drawing a set of initial ages of excavators that roughly match the age distribution of excavators at the beginning of our sample. Since the outcome of the simulations depend on the particular sequence of macro shocks that are simulated over the ensuing 60 month simulation horizon, the results in the tables are averaged and reported on a per machine basis and the averages are taken over 20 independent replications of sequences of future macro shocks. Note that for comparability between the optimal policy status quo replacement policies, we used the same initial conditions in each simulation and in each simulation replication the 50 machines simulated under the status quo replacement policy were subjected to the same sequence of macro shocks as the corresponding fleet of 50 machines that were simulated under the optimal policy. Thus, our simulations provide a type of controlled experiment that would be very difficult to carry out in the real world.

The ratio of the present discounted profits to the OEC of each make/company combination can be considered as rates of return on the initial investment of an excavator at a specific location. For the example above of a make 4 excavator, the value to OEC ratio is 3.07, or expressed as a percentage rate of return this amounts to 207% — i.e. the initial investment of the OEC price to buy the first excavator results in a total payoff in terms of expected future profits (on the current excavator and all future ones the
company will replace it with) that is 3.07 times the initial OEC investment.

We generally find very high rates of return for all companies, makes of excavators, and for all regions of the country. Thus, though excavators are expensive (i.e. their OEC can be $140,000 or higher) they also generate high gross rental profits, and as a result they are an attractive investment, earning rates of return on initial investment in the OEC cost of the first excavator of 200% or higher. In other words, the ratio of the value of expected discounted profits to the initial OEC for excavators is generally 3 or higher, and in some cases over 4 to 1.

The companies that we found to earn the highest returns are B and D: this may partly reflect “locational rents” — i.e. higher rate rates, rental revenues, and higher time utilization of their excavators if their locations are in an area where there is a high demand for excavators and their locations are closer or more convenient for more customers. We also found that make 3 excavators earn higher returns than the other makes: i.e. they have higher value to OEC ratios than other makes of excavators. On the other hand, make 4 excavators generally produced the lowest rates of return.

Table 6 summarizes the average undiscounted profits earned per machine in the various simulations we did for different companies and makes of excavators. Comparing to the infinite horizon simulations, we generally find larger percentage increases in profits in our simulations than from our infinite horizon calculations of the value functions in table 5. For example for the make 4 excavator owned by Company A, table 5 predicts a 3.5% gain in expected discounted profits over an infinite horizon, whereas table 6 predicts that the average per machine undiscounted profits increase by 11.7% over our 5 year simulation horizon.

Precisely how does the optimal replacement policy result in profit increases? Table 4 already provided some insights into this: the optimal policy involves keeping excavators longer than the firms keep them under their status quo replacement policy, and though the older machines earn less revenue and have higher maintenance costs, the reduction in replacement costs more than makes up for these other factors, resulting in higher profits overall. This same pattern is repeated for other makes of excavators and for other companies and regions. We generally see that the optimal replacement policy involves keeping machines longer than companies keep excavators under the status quo. For example for the make 4 excavator owned by company A, the simulated mean age of replacement under the optimal replacement policy is 76.8 months which is nearly 50% higher than the mean age at which these excavators are replaced by company A under its status quo replacement policy. The standard deviation replacement ages is much
Table 6: Gain in undiscounted profits: excavators simulated for 60 months (20 replications)

<table>
<thead>
<tr>
<th>Make/Company</th>
<th>A</th>
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<th>C</th>
<th>D</th>
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<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>20.6%</td>
<td>4.8%</td>
<td>3.3%</td>
<td>12.5%</td>
<td>6.4%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Make 2</td>
<td>15.7%</td>
<td>1.4%</td>
<td>2.9%</td>
<td>6.3%</td>
<td>0.9%</td>
<td>2.9%</td>
</tr>
<tr>
<td>Make 3</td>
<td>9.4%</td>
<td>1.5%</td>
<td>4.0%</td>
<td>4.0%</td>
<td>1.3%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Make 4</td>
<td>11.8%</td>
<td>1.5%</td>
<td>2.4%</td>
<td>3.5%</td>
<td>1.4%</td>
<td>-0.2%</td>
</tr>
<tr>
<td>Make 5</td>
<td>23.9%</td>
<td>2.8%</td>
<td>3.6%</td>
<td>12.4%</td>
<td>2.9%</td>
<td>11.5%</td>
</tr>
</tbody>
</table>

larger under the *status quo* than it is under the optimal replacement policy (34.1 versus 8.4).

We find that the main gain in profits comes from a significant reduction in *replacement costs* which is achieved in part by increasing the age when the excavators are replaced, but there are additional gains due to *strategic timing of replacement over the business cycle*. Because expected revenues decline with age and maintenance costs increase with age, keeping machines longer before replacing them decreases profits somewhat, but this decline in profits is more than compensated by a much larger decrease in replacement costs. For example, for the make 4 excavators owned by company A, our simulations reveal that average undiscounted revenue per excavator over the 20 replications over a 60 month simulation horizon decrease by 3% to $293,320 under the optimal policy from $301,712 under the status quo. Maintenance costs increase by 57%, from 11,264 to 17,749 per machine. However average replacement costs fall by nearly 50%, from 80,837 under the *status quo* to only 41,317 under the optimal replacement policy. The fall in replacement costs overwhelms the fall in current operating profits (i.e. monthly revenues less maintenance costs) due to the fact that average age of the fleet increases under the optimal replacement policy, so overall profits (net of replacement costs) increase by nearly 12%.

Figure [17] shows the simulated paths of replacements and average fleet age for a single replication of the simulations we did for the make 4 excavator owned by company A in one of its locations in region 6. The dashed black lines in each panel illustrate the simulated sequence of macro shocks in this particular simulation replication. We see that replacements tend to be more *clustered* under the optimal replacement policy than under the *status quo* and this is largely because of the idiosyncratic factors driving replacements under the *status quo*. Note that replacements occur in this simulation only in the boom states (i.e. when the dash line is at its highest point indicating boom macro states) whereas replacements occur in smaller, more regular bunches under the *status quo* replacement policy. This reflects the *strategic timing* motive, which in the case of excavators dictates replacing older machines in boom periods to take advantage of the
Figure 17: Simulation of fleet of make 4 excavators owned by company A, region 6
lower replacement costs in these states.

Why are replacement costs lower in boom states? Recall the top left panel of figure[13] which shows the predicted OEC and resale prices from our econometric model. Our econometric model predicts that OEC prices are lower in boom months, whereas resale prices of excavators that are 40 months or older are higher in boom months. Together these two effects imply that the cost of replacing an older make 4 excavator is lower in a boom month compared to a normal or bust month. So in effect, our dynamic programming algorithm has “discovered” a profitable way of strategically timing replacements of machines to take advantage of predictable movements in machine prices over the business cycle. In this case, the dynamic programming algorithm determines that the optimal replacement cycle is in fact procyclical.

Note that the concentrated nature of replacements under the optimal replacement policy is partly for strategic reasons but also due to the fact that we are simulating a model that does not allow for idiosyncratic shocks. Replacements will be less concentrated in specific months in a simulation of a model that allows for idiosyncratic shocks. However we did not make any assumptions about quantity discounts that may be available to rental companies that undertake block replacements of machines. If such discounts are available, it is possible to modify our model to allow the company to take advantage of them, and this can result in greater clustering or replacements.

### 6.2 High Reach Forklifts

High reach forklifts are not generally as expensive or profitable as excavators, and this is reflected in our machine valuations below: our model predicts that the profits companies earn on an average high reach forklift are significantly lower, and the returns to investment are generally lower for high reach forklifts than for excavators. Due to the rapid rate at which rental revenues fall off with age for forklifts, combined with the relatively slow rate of price depreciation, the DP model predicts that the optimal time to sell a high reach forklift is much sooner than what firms do in practice. We find that moderate increases in profitability can be expected in the long run from adopting the optimal replacement policy, ranging from a low of 3% to a high of 12%. How much of these expected gains are realized in the short run, such as a 5 year horizon? Our simulations show that it is possible for the DP solution to do worse than the companies would have done using their status quo replacement policy in some cases.

This is due to a number of reasons, but the major one is that the optimal strategy that we calculate does
Table 7: Gain in discounted profits from optimal replacement policy, high reach forklifts

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<thead>
<tr>
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<td>7.9%</td>
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<td>4.2%</td>
</tr>
<tr>
<td>Make 2</td>
<td>5.6%</td>
<td>4.6%</td>
<td>3.0%</td>
<td>3.7%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Make 3</td>
<td>7.6%</td>
<td>6.0%</td>
<td>3.1%</td>
<td>4.5%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Make 4</td>
<td>12.3%</td>
<td>9.9%</td>
<td>4.5%</td>
<td>7.1%</td>
<td>5.3%</td>
</tr>
<tr>
<td>Make 5</td>
<td>11.1%</td>
<td>8.5%</td>
<td>4.0%</td>
<td>6.5%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Make 6</td>
<td>9.0%</td>
<td>6.8%</td>
<td>3.6%</td>
<td>5.7%</td>
<td>3.9%</td>
</tr>
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</table>

far more replacements in a concentrated fashion that leads to “spikes” of replacements rather than a smaller number of replacements on a more or less regular basis, reflecting the importance of the idiosyncratic factors at play in the status quo replacement policy. For some sequences of macro shocks the optimal solution can result in a large number of replacements near the end of the 60 month simulation horizon and this can result in a “loss” relative to the status quo since there is not enough time in the remaining simulation horizon for the firm to recoup the cash outlay for the spike in replacements in the form of higher subsequent profits.

For example in one simulation, there were only a few simulated replacement that company A was predicted to have done under its status quo replacement policy during the 60 month simulation horizon. However for the simulated optimal replacement strategy, there were no simulated replacements until 43 there is a massive replacement of all 50 machines. This massive investment in month 43 was prompted by the fact that this was a boom month, and the optimal policy entails replacing machines older than 68 months, and by month 43 of the simulation all 50 machines were beyond this threshold, so the optimal replacement policy entailed replacing all of them in one fell swoop.

Of course there is a massive cash outflow in month 43 to pay for all of these replacements, but but there are simply not enough months left in the arbitrarily terminated simulation horizon for this mass investment to be recouped. For these reason in this simulation, the optimal policy involves a significant loss relative to what is realized under the status quo investment rule.

Table [7] displays the expected gains to adopting the optimal replacement policies calculated individually for each company/make combinations in the table (but with company F excluded because it did not have any high reach forklifts in its locations in region 6 in our data set). We see that in general, the expected percentage gain in expected discounted profits range for a low of 3.0% for make 2 high reach forklifts of
Figure 18: Optimal replacement policy: make 1 high reach forklift, company A, region 6
Table 8: Gain in undiscounted profits: high reach forklifts simulated for 60 months (20 replications)

<table>
<thead>
<tr>
<th>Make/Company</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>-1.8%</td>
<td>4.2%</td>
<td>3.2%</td>
<td>3.4%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Make 2</td>
<td>-0.8%</td>
<td>2.1%</td>
<td>-1.6%</td>
<td>-0.4%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Make 3</td>
<td>-8.1%</td>
<td>0.5%</td>
<td>-2.6%</td>
<td>4.9%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Make 4</td>
<td>3.0%</td>
<td>3.0%</td>
<td>-4.7%</td>
<td>5.1%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Make 5</td>
<td>-9.0%</td>
<td>-4.3%</td>
<td>-0.7%</td>
<td>-1.2%</td>
<td>-1.5%</td>
</tr>
<tr>
<td>Make 6</td>
<td>-0.2%</td>
<td>-1.5%</td>
<td>5.2%</td>
<td>2.4%</td>
<td>5.2%</td>
</tr>
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</table>

company C to a high of 12.3 percent gain for make 4 high reach forklifts of company A. Of the 6 makes of high reach forklifts in our data set, the makes 4 and 5 delivered the highest discounted profits over all of the 5 companies considered, and generally company C had the highest profitability across all makes, compared to the other companies. We found the highest gains to adopting the optimal replacement policy for firm A, and firm B was generally the least profitable of the five firms analyzed for each of the 6 makes of high reach forklifts we analyzed.

The mean at of replacement is often about 60% as high as mean ages at which these companies currently replace high reach forklifts. On average the optimal policy results in replacements when machines are about 57 to 60 months whereas under the status quo machines are replaced when they are significantly older, around 90 months or more for all companies except company E which also replaces its high reach forklifts when they reach 60 months or so on average. Firm C also tends to replace its high reach forklifts earlier than the other companies except E, and its mean age of replacement varies from 72 to 79 months for the 6 makes we considered.

Table 8 provides the average profits earned by different companies for different makes of high reach forklift over the 60 month simulation horizon. We see a much more mixed pattern in these simulations where in 14 out of the 30 cases considered, the optimal replacement policy resulted in lower average profits for the 50 machines in the simulations and 20 replications than were obtained under the status quo replacement policy. The losses under the optimal policy ranged from a low of −0.4% in the case of make 2s owned by company D, to a high of −9.0% in the case of make 5s owned by company A.

Why such a difference in results between table 8 which shows the possibility of losses in the short run from adopting the optimal policy versus table 7 which shows only gains when policies are evaluated on an expected value basis over an infinite horizon? We believe it is due to what we already illustrated:
replacements tend to be more concentrated and clustered under the optimal policy, and sensitive to the
timing of macro shocks (particularly the occurrence of boom periods which trigger the firm to replace all
machines over 55 to 65 months old), and a relatively short evaluation horizon (e.g. the 60 month period
in our simulations) will result in a relatively frequent occurrence of large spikes in replacements toward
the end of the 60 month simulation horizon, which results in the artificial appearance that the optimal
replacement policy does worse than the firm is able to do using its status quo replacement policy.

We also point out that despite the fact that the optimal decision rule calculated by our dynamic pro-
gramming model does not allow for idiosyncratic factors that affect replacements that the status quo re-
placement rules reflect, the variability (standard deviation) of age of replacement is actually generally
higher under the optimal replacement policy than under the status quo which reflects the clustering and
spikiness in replacements under the optimal policy.

6.3 Skid Steers

There are only two makes of skid steers in our data set and no observations of skid steers of company B
in any region of the US in our data, so we restricted our comparison to companies A, C, D, E and F in
table 9 below. Our main finding is that while skid steers have lower OEC, the rental revenues per skid steer
and also gross profits (rental revenues less maintenance costs) are also lower on a per machine basis as we
already illustrated in the middle right panel of figure 12.

But in terms of return we find skid steers to be significantly more profitable than high reach forklifts,
and even more profitable than excavators. Under the optimal replacement policy, the rates of return range
from a low of 218% for make 1 skid steers owned by company E to a high of 447% for make 1 skid steers
at company D’s locations in region 6. Thus, for whatever reason, skid steers appear to deliver significantly
higher rates of return compared to excavators and high reach forklifts.

The other main conclusion we draw is that there appear to be significantly larger expected gains for
adopting an optimal replacement strategy for skid steers than we typically found for excavators or high
reach forklifts. That is, the percentage gains in profitability from adopting the optimal policy relative to
the firms’ status quo replacement policies is generally significantly larger for skid steers. The percentage
gains in profit range from a low of 13.6% for make 1 skid steers of company C, to a high of 59.2% for
make 2 skid steers owned by company E. Compare this to high reach forklifts: the highest percentage gain
from adopting the calculated optimal replacement policy for any of the company/make combinations we
considered was 12.3% for make 4 excavators of company A. For excavators, the highest percentage profit gain we found was 23.9% for make 5s owned by company A in its locations in region 6.

The reason why skid steers are less profitable on a absolute per machine basis can be understood by viewing the static gross profit function (but not including replacement costs) shown in the middle-right panel of figure 12. A new make 2 skid steer is expected to generate less than $800 for firm A in its location in region 6 even in a boom month, whereas a new make 5 high reach forklift can bring in over $2000 in a boom month, and a new make 2 excavator can be expected to bring in over $5000 in a boom month. Thus it is not surprising that the discounted expected profit valuations of skid steers are significantly less than for excavators or high reach forklifts.

Figure 19 compares the optimal value functions and replacement policies with the status quo replacement policy of company D in one of its locations in region 6. We find that even though the static profit breakeven age of skid steers is about 120 months, and though companies replace these machines over a range of age between 55 to 95 months (depending on company and make of skid steer), the optimal policy entails a much shorter mean age of replacement, ranging from 22 to 29 months, depending on which company and make of skid steer we consider.

In the first panel of figure 19 shows the optimal replacement thresholds by month and macro state for a make 2 skid steer owned by company D. The lower red line is the replacement threshold in boom months and we see that in a boom month it is optimal to replace any skid steer older 30 months and in some cases at low as 18 months old. This is a big acceleration in the turnover of skid steers, but paradoxically, it is driven by the desire to maximize current profits by keeping a relatively young fleet of skid steers because the replacement costs of doing this is favorable in boom states.

To understand why, look at the middle-right panel of figure 13. This graph shows the expected OEC and resale prices for a make 1 skid steer, and the red line plots the forecasted prices in a boom month. We see that the model predicts that OEC is lower in a boom month relative to a normal or bust month, but the resale prices for Bobcat skid steers between age 20 and 80 are higher in a boom relative to normal or bust.
Figure 19: Optimal replacement policy for make 1 skid steer owned by company D, region 6

Optimal replacement thresholds for make 1 Skid Steer
Company A in region 6

Replacement probabilities for make 1 Skid Steer in January
Company A in region 6

Value functions for make 1 Skid Steer in January
Company A in region 6
times. This reduces the slope of the red line, but this slope just indicates the replacement cost and thus we conclude that it is much cheaper for company D to replace relatively new make 1 skid steers in a boom month relative to a normal or bust month.

The optimal replacement strategy takes direct advantage of what it believes to be a stable and predictable price pattern in the rental market. So it is not surprising then that the DP model sets a very low replacement threshold of 18 months in a boom month, but relatively high thresholds of about 90 and 120 months in a normal or bust month, respectively. The middle panel of figure 19 shows this and it compares these thresholds to the replacement policy that company D is predicted to follow.

We see that company D also follows a pro-cyclical replacement investment strategy (i.e. it tends to replace more machines in a boom state than a bust state), but it does not respond as aggressively, i.e. it does not make such a wide variation in replacement ages over different macro states and thus, if our predicted price pattern really holds, we are suggesting that company D is not taking maximal advantage of a predictable variation in prices that should enable it to earn more money by strategically timing its replacements over the business cycle. Note that the dashed red line in the middle panel of figure 19 shows the probability that firm A replaces make 1 skid steers in a boom state as a function of the age of the machine. The probability of selling the machine first becomes positive at about 80 months, and is nearly 1 by 190 months. The median age of replacement in a boom state is about 130 months and this is over 6 times as large as the optimal threshold in a boom state.

Of course the optimal replacement threshold do bracket this 130 month median age of replacement in the normal and bust states, respectively. So sometimes the optimal policy can entail replacing machines that are 120 months or older, provided that the company had experienced a long previous string of normal or bust months, but without any intervening boom month. Then the simulation could predict that machines could be allowed to age more than the 18 to 30 month threshold where replacement of machines is optimal in a boom month. But it is relatively rare not to experience at least one boom month in a sequence of 120 or more months, and thus on average, simulations of the optimal policy lead to most replacements being done when machines are relatively young.

The mean age at replacement under the optimal policy varies from a low of 22 for company D, to a high of 95 months for company E. However there is much wider variation in the status quo replacement ages across different companies and makes than we find for the optimal policy. For example, while company E is predicted to have a mean age at replacement of 95 months for make 1s, the mean age of replacement for
Table 10: Gain in undiscounted profits: skid steers simulated for 60 months (20 replications)

<table>
<thead>
<tr>
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<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>24.1%</td>
<td>21.7%</td>
<td>42.3%</td>
<td>76.9%</td>
<td>45.2%</td>
</tr>
<tr>
<td>Make 2</td>
<td>20.0%</td>
<td>13.7%</td>
<td>30.7%</td>
<td>63.0%</td>
<td>29.5%</td>
</tr>
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</table>

make 2 skid steers is only 46 months. This swamps the variation in the optimal policy, for which the mean age of replacement is 27 months for make 1s and 28 months for make 2s.

Table 10 shows the average profits earned in 20 replications of a simulation experiment where 50 machines of each make are simulated for 60 months, starting from equivalent age “initial conditions” for the initial fleet of machines at the start of the experiment. We see some very large simulated gains in this case relative to what we saw for high reach forklifts. This is because in these simulations, the profit gains are coming from a much more active and frequent replacement investment under the optimal policy relative to the status quo.

For example in the average of the 20 simulation replications we did for make 1 skid steers owned by company D, the optimal replacement policy generated higher revenues, lower maintenance costs and only slightly higher replacement costs due to the much higher volume of replacement investment. Specifically, there was an average of 196 machines replaced by the optimal policy during the 60 month simulation horizon, compared to only 39 machines under the status quo. So there was a 5-fold increase in the number of replacements, but the average per-machine replacement cost over this 60 month period was $8328 (average over all 50 machines in the the 20 replications) versus $7287 under the status quo. So replacement costs increased only 14% when the total number of replacements increased by over 500%! The optimal strategy is able to do this seemingly amazing feat by concentrating all replacements in boom months when replacement costs are predicted to be much cheaper than in normal times.

Because the increase in replacement costs was relatively minor, the move to more frequent replacement of skid steers raised average per machine rental revenue over the 60 month horizon by 31% (from $48456 under the status quo to $63453 under the optimal policy), and reduced maintenance costs by 40% (from $4168 under the status quo to $2482 under the optimal policy). Thus, the overall profits over the 60 month horizon increased by 42%, from an average of $37001 under the status quo replacement policy to $52642 under the optimal replacement policy.

Though one might question whether the underlying price pattern that the optimal policy is exploiting
(i.e. lower OEC and higher resale values of make 1 skid steers in boom months) is “real” and would be a persistent price pattern that the firm could always count on being able to take advantage of, we believe the DP solution algorithm very cleverly exploits the assumptions given to it and returns a replacement policy that exploits the perceived price patterns.

While we do not yet ourselves know if we really believe that the optimal policy that prescribes such frequent replacement of machines is really a good idea, especially if the prediction from our econometric model happened to be wrong or generate predictions that are too sensitive to a OEC or resale price outliers. However if the firm experienced that it was persistently unable to realize the lower OEC prices or the higher resale values tat our econometric model predicts, then it would be easy to adjust its predictions and the predicted optimal replacement strategy accordingly. Specifically, given new data, if the implied revised regression predictions did not imply the significant reductions in replacement costs in boom months (due to OEC price being higher or resale prices lower in reality in boom months, contrary to what our model currently predicts), then we could re-solve the DP model using the updated, more accurate price data, and this would no doubt lead the model to significantly raise the age thresholds at which replacement is optimal, and this would reduce the amount of replacement activity and probably result in behavior that is more in line with the replacement behavior we see under the status quo for these companies.

6.4 Scissor Lifts

Now consider scissor lifts. These machines generate lower profits than skid steers, both absolutely (i.e. in terms of lower profits per machine), and in relative terms (i.e. lower ratios of profits to OEC). There are 3 makes of scissor lifts owned by the firms in our data set. Once again we compare the profitability of these three makes of machines for the companies using data on one of their locations in region 6, but we exclude company D because we have no data on scissor lifts from any of its locations in region 6. From table we see that company F earns the greatest profits on all makes of scissor lifts, followed by company C. Company E has the least profitable scissor lifts. Looking across makes of scissor lift, make 2s earn the most profits on average for each of the 5 companies in table and make 3s earn the least.

Table shows that the percentage profit gains from switching from the status quo replacement policy to the optimal policy are generally very big, ranging from a 23% gain for make 3s owned by company B, to a 1777% gain (i.e. a nearly 19-fold increase in profits per machine) for make 3s owned by company E. How does the optimal replacement policy achieve such dramatic gains?
Table 11: Gain in discounted profits from optimal replacement policy, scissor lifts

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<td>Make 1</td>
<td>78.5%</td>
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<td>46.9%</td>
<td>624.2%</td>
<td>70.9%</td>
</tr>
<tr>
<td>Make 2</td>
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<td>27.1%</td>
<td>49.4%</td>
<td>407.1%</td>
<td>74.8%</td>
</tr>
<tr>
<td>Make 3</td>
<td>79.3%</td>
<td>23.2%</td>
<td>45.2%</td>
<td>1777.0%</td>
<td>69.8%</td>
</tr>
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</table>

We find the greatest percentage increase in profits for company E, but this company has negative rates of return. So the extremely high percentage gains for company E is caused by the fact that its expected discounted profits from scissor lifts are so low that any gain in profits from switching to the optimal policy results in high percentage increases in profits as well. Overall we generally find the highest percentage profit gains for scissor lifts out of the five machine types in our data set.

Figure 20 gives us further insight into how the optimal replacement policy improves profitability of these companies relative to their status quo replacement policies by comparing them for company E for its make 3 scissor lifts. From the middle panel it is immediately evident that firm E keeps its make 3s far longer before replacing them relative to what is optimal. The top panel of figure 20 shows that optimal policy entails “early replacements” (approximately at age 20) in boom and normal months, but much delayed investment in bust months, so that only machines that are 120 months or older are generally replaced in a bust month (with October and December being exceptions where replacements occur for machines that are over 90 and 40 months in those two months, respectively).

The dashed lines representing the status quo replacement policy for make 3s by company E in the middle panel of figure 20 make it obvious that this company keeps these machines far longer before replacing them. The median age of replacement of a make 3 in a normal month is predicted to be about 220 months. Consulting the gross monthly profit prediction in figure 12, we see that company E is keeping its make 3s beyond what we called the “breakeven age” at which expected rental revenue equals expected maintenance costs. The breakeven age in that figure is about 80 months, and the breakeven ages for make 3s owned by company E range from 70 to 80 months. It follows that for some reason company E is keeping its make 3s well beyond this breakeven age, and so our econometric model predicts that the company is incurring current losses on many of its make 3 scissor lifts.

The last panel of figure 20 shows that indeed, the expected value of profits for make 3 scissor lifts of company E are negative for all but a brand new machines, whereas the expected discounted profits of
Figure 20: Optimal replacement policy for scissor lifts owned by company E, region 6 location
Table 12: Gain in undiscounted profits: scissor lifts simulated for 60 months (20 replications)

<table>
<thead>
<tr>
<th>Make/Company</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>118.4%</td>
<td>29.5%</td>
<td>56.0%</td>
<td>270.9%</td>
<td>105.4%</td>
</tr>
<tr>
<td>Make 2</td>
<td>120.0%</td>
<td>31.1%</td>
<td>63.2%</td>
<td>119.8%</td>
<td>97.7%</td>
</tr>
<tr>
<td>Make 3</td>
<td>203.1%</td>
<td>26.7%</td>
<td>53.9%</td>
<td>221.8%</td>
<td>119.3%</td>
</tr>
</tbody>
</table>

these machines under the optimal policy is positive starting from over $40,000 for a brand new unit, and decreasing to about $30,000 when the machine is 120 months old, which is an age it would be likely to be replaced. The mean simulated age of replacement under the optimal policy ranges from a low of 19 months for a make 1 owned by company A, to a high of 24 months for a make 3 owned by company B.

However the mean replacement age under the status quo was far larger, ranging from 47 months for make 3s owned by company B to a high of 95 for make 2s owned by company A. But the most striking case is company E, which, in our simulations of its replacements of make 1 and 2 scissor lifts there were no replacements in its fleet of 50 machines over the 60 month horizon in any of the 20 simulation replications. For make 3s there were replacements and the mean replacement age for company E was 114 months.

Table 12 reports the average undiscounted per machine profits earned by the machines in our simulations under the optimal replacement policy versus the status quo. We can see from the table that we get large percentage gains in profits from adopting the optimal replacement policy in virtually all cases we considered, from a low of 26.7% gain in profits for make 3s owned by company B to a high of a 271% gain in profits for make 1s owned by company E. In general we found the largest percentage gains for firms E, A and F.

What could be the explanation for the apparently very “poor” performance and profitability of several companies in their policy for replacing scissor lifts? One possibility is that our results are spurious — due to some data problem that leads us to underpredict rental revenues for scissor lifts. In particular our data set has a large number of scissor lift rental records that show zero rental revenues for a series of months in succession. They are particularly frequent for company E. This could potentially be an indication of a strategy of refurbishment by company E of its scissor lifts that our dynamic programming model is not giving it credit for. However we double checked with the company that provided us the data and they had no record of any refurbishments of these machines and they could not provide any explanation for the large number of zero utilization months for these machines.

75
It is possible to extend the DP model to allow for decision to refurbish (versus to sell) an older piece of equipment. These refurbishments are then properly considered as investments that result in the machine being out of service for several months in succession, and not simply regarded as a series of months where the company is not earning any revenue with no other benefit beyond that. The refurbishment may serve to partially renew or regenerate an older machine in a way that is more cost-effective than replacing the machine with a brand new unit. For this reason we have to qualify our findings, and if there were actually refurbishments that were not recorded in our data set, then the poor performance we impute to company E would be incorrect, and instead the refurbishments may be part of a very inspired strategy on its part to save on replacement costs by using refurbishment as an alternative to replacement. Thus, this emphasizes that the conclusions from our analysis rely critically on the quality of the data.

6.5 Telescopic Booms

We complete our discussion of results by describing our findings for telescopic booms. Table 13 presents the expected discounted values of profits for the three makes of booms in our data for all six companies in their southwest locations. We see that telescopic booms generate about as much profit per machine as skid steers do, ranging from $70,000 to about $112,000 in discounted profit depending on the make and company. Actually our valuations of the profits from skid steers are slightly less than for booms, ranging from about $50,000 to about $95,000 per skid steer, depending on the make and company that owns them.

Compared to scissor lifts, the percentage gains form adopting the optimal replacement policy are calculated to be much more moderate for telescopic booms: ranging from a low of an 8.2% gain in profits for make 2s owned by company F to a high of a 25.9% gain for make 1s owned by company E. Company C’s southwest locations appear to generate the highest profits for all makes of telescopic booms, and under both the status quo and optimal replacement policies. We also see that make 3s generate the highest per machine profits across all six companies in our data set, with the exception of company E which our model predicts earns the highest profits from it make 1 telescopic booms.

Figure 21 illustrates the optimal replacement policy for make 1 telescopic booms at company A’s southwest locations. Here we clearly see that the optimal replacement policy is countercyclical. For example the first panel shows the replacement thresholds by month for the three key cases, i.e. where the month is bust month (blue line), normal month (black line) or boom month (red line). Thus, if June is a bust month, the optimal replacement rule entails replacing a make 1 telescopic boom that is older than
Table 13: Gain in discounted profits from optimal replacement policy, telescopic booms

<table>
<thead>
<tr>
<th>Make/Company</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make 1</td>
<td>14.2%</td>
<td>14.7%</td>
<td>13.1%</td>
<td>14.8%</td>
<td>25.9%</td>
<td>10.7%</td>
</tr>
<tr>
<td>Make 2</td>
<td>10.4%</td>
<td>10.0%</td>
<td>8.9%</td>
<td>10.6%</td>
<td>18.9%</td>
<td>8.2%</td>
</tr>
<tr>
<td>Make 3</td>
<td>10.3%</td>
<td>9.9%</td>
<td>8.9%</td>
<td>10.4%</td>
<td>17.6%</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

about 38 months. However if June is a normal month, then the company should not replace any booms unless they are over 140 months old, and if the economy is in a boom, it should not replace any machine unless it is over 158 months old.

When we compare this replacement policy to what firm A actually does (the dashed S-curves in the middle panel of figure 21) we see that company A follows a pro cyclical replacement strategy. That is, it is significantly more likely to replace an older telescopic boom in a boom month relative to a normal or bust month. For example, the median age of replacement in a boom month under company A’s status quo replacement policy is predicted to be about 110 months old, versus the 158 month threshold under the optimal policy. In a normal or bust month, the median age of replacement under the status quo is about 138 months, which is just slightly less than the 140 month replacement threshold under the optimal policy.

Thus, in a normal month the company replaces its make 1s at about the same median age as the replacement threshold calculated for the optimal replacement policy, but in a boom month, the optimal policy implies keeping machines much longer than what the company does, and in a bust month the optimal policy prescribes the company to replace its make 1s when they are relatively new (i.e. only 38 months old) whereas the under the status quo the company does not start to replace its make 1s until they are at least 60 months old and even for machines that are 140 months old, there is only a 60% probability they will be replaced by the company under its status quo replacement policy.

We can see the countercyclical nature of the optimal replacement policy clearly in figure 22, which shows a single simulation replication of a fleet of 50 make 1 telescopic booms owned by company A in one of its locations in region 6. The top panel shows that there are two major replacement spikes at 14 and 50 months into the simulation. These spikes were triggered by the fact that the economy went into a bust period, as indicated by the dips in the dashed line indicating that the macro shock reached its lowest possible value ($s = 1$, indicating a bust month) in the 14th and 50th months of the simulation. These were relatively short bust periods of only 3 to 4 months in each case, and we also see small subsequent
Figure 21: Optimal replacement policy for make 1 telescopic booms at company A, region 6 location
Figure 22: Simulation of fleet of make 1 telescopic booms of company A, region 6

Simulated replacements in a fleet of 50 Telescopic Booms
Make 1 owned by company A in region 6

Mean age of fleet (months)
Simulated mean age of a fleet of 50 Telescopic Booms
Make 1 owned by company A in region 6
replacement spikes at 16 months and 53 months, respectively where some additional machines were re-
placed under the optimal policy. Otherwise the economy had been only in normal and boom months, and
so the optimal replacement policy for company A entailed no replacements in those other months of the
simulation. However there were a small number of replacements driven by idiosyncratic shocks under the
status quo replacement policy throughout the 60 month simulation horizon.

The middle panel of figure [22] shows that the average age of the fleet increases linearly until month 14, whereas the average age of the fleet under the status quo increases less rapidly due to the rejuvenating effect of the replacements that occured under the status quo policy. But by the time that the bust hits in month 14, the average age of the fleet under the optimal replacement policy is nearly 50 months old, and this exceeds the 38 month optimal replacement threshold in a bust month, so the firm ends up replacing 21 machines in its fleet that are over 38 months old in month 14 of the simulation.

The economy recovers from the short three month bust period and so there are no further replacements under the optimal policy until month 50 of the simulation when the macro state returns to the bust state. This motivates the firm to again replace all make 1 telescopic booms that are over 38 months old, and it replaces another 2 machines in month 53 of the simulation, bringing the average age of the fleet down to 20 months old, considerably younger than the average age of the fleet under the status quo which is about 45 months at that point.

We conclude our analysis of results with table [14] which presents the average undiscounted profits earned under the status quo versus the optimal policy over the 20 replications run over the 60 month simulation horizon. Here we find that the average gains from adopting the optimal replacement policy range from -2.1% (i.e. a loss) for make 2 booms owned by company F to a high of a 47% gain for make 1 booms owned by company E. Looking across makes of machines we find that the make 3 booms are the most profitable of the three makes under the optimal policy, and looking across companies we find that company C earns higher profits for all three makes of telescopic booms than the other companies.
7 Conclusions

This paper has developed an exploratory framework for evaluating the profitability of rental machine replacement decisions that firms in our data set have actually made by comparing them with counterfactual “optimal” replacement decisions obtained from a mathematical model of optimal replacement decisions where the objective is to find a replacement strategy that maximizes expected discounted gross profits (net cash flows) from the replacement of existing machines by new machines of the same make.

We emphasize that there are many simplifying assumptions and abstractions from reality that we made to solve our mathematical model, and some of these assumptions may not be sufficiently good approximations to the very much more complex reality that actual firms face on a day to day basis.

Thus there is a distinct possibility that we are analyzing an overly simplified model of the real problems that rental companies are facing and for this reason the predictions of our model should be taken with a grain of salt until further explorations determine whether our model is a “sufficiently good approximation to reality” — or not.

We also acknowledged that the predictions of our model depend critically on the quality and accuracy of the data we were provided, and on the econometric models and assumptions we made to predict the rental revenues, maintenance costs, new OEC prices and used machine resale values that firms can expect to realize in their day to day operations. If the data are not the best, or our econometric model is providing misleading forecasts due to failure to use the best econometric techniques, then the conclusions we draw from our analysis are suspect from the standpoint of providing reliable advice to rental companies on the replacement policies that will enable them to earn the highest possible profits.

We have recognized that the overall problem that rental companies face is a very difficult, high dimensional problem that we do not have the data or the mental/computational resources to try to “crack” at the present time. However we followed a common sense approach known as decomposition in the computer science literature. That is, we decomposed the overall profit maximization problem that rental companies face and focused on the replacement subproblem. We argued that we can take other aspects of firm behavior/strategy as “given” (such as their policies over how they set rental rates and allocate lot space and capital to ownership of different types, makes, and models of machines) and focus on a simpler subproblem that we believe these companies need to solve, at least for their set of “core machines”. We argued that firms can alter the solution to their replacement subproblems without significantly constraining or altering the other aspects of their fleet management and rental rate policies, especially because we have found that
rental rates are flat and thus changing the mean age of replacement (at least when the changes are not sufficiently different from when firms currently replace machines) should not affect rental rates, though it does affect rental revenues, maintenance costs, and replacement costs.

We have shown that in some cases the optimal replacement policies calculated by our dynamic programming algorithm were not able to result in a significant improvement in profit relative to the firms’ status quo replacement policies (such as for certain makes of excavators by certain companies in specific locations). In these cases the optimal replacement policies we calculated are similar to the replacement policies these firms are already using, so our analysis does not suggest any concern about the potential suboptimality of company decision making in these specific cases.

However in the majority of cases we studied, including most makes of excavators at most of the companies and locations we analyzed, our calculations and simulations suggested that firms could significantly improve their profitability by altering their replacement policies, and a major share of the discounted increases in profits can be achieved over a relatively short horizon, i.e. within 5 years. Further, the optimal policies that we calculated numerically were significantly different from the policies these firms are currently using. For excavators, we found that in many cases firms could significantly increase their profitability by keeping their excavators longer than they do under the status quo, whereas for the other four types of machines we studied, we generally found significant profit gains from replacing the machines significantly sooner than these companies replace them under the status quo. Indeed, for one of the machine types and companies — scissor lifts at the southwest locations of company E — we found huge gains from replacing scissor lifts much sooner than this company is currently doing (such as after only 23 months for a make 3, compared to 114 months under the status quo). Indeed our calculations suggest that company E is past the breakeven age in keeping its scissor lifts for so long — that is, beyond the age where expected maintenance costs exceed expected rental revenue, so the firm is actually incurring an expected loss for every month beyond the breakeven age that it keeps these older machines.

The other major difference between our calculated optimal replacement policies and the ones these companies are using are that the optimal replacement strategies are more sensitive to macro shocks. For some machines such as excavators, we found that due to the way OEC and resale prices shift over the business cycle, replacement costs are lower in booms and thus the profit-maximizing replacement strategy involves doing most replacement investment in boom periods, so the optimal replacement policy is procyclical for excavators. However for telescopic booms we found that OEC is significantly lower in bust
periods, and thus it is significantly cheaper to replace telescopic booms in bust periods relative to normal or boom periods. Since replacement costs are such a significant part of the costs of the rental business, this implies that the optimal replacement policy for telescopic booms is counter-cyclical.

We will need to get more data and extend our model and subject it to further “stress tests” before we are willing to conclude that these preliminary findings are robust, generalizable findings. Further, we have ignored a very important consideration: state-dependent borrowing constraints. If many rental companies have lower ability to borrow or if the capital/retained earnings available to finance replacement investment “dries up” or becomes significantly more expensive during bust periods, then this is a consideration that we ignored in our analysis that could explain why the companies in our sample follow a pro-cyclical rather than counter-cyclical replacement policy for telescopic booms and the other four machine types we analyzed. These firms may realize there is a profit opportunity by following a “contrarian” investment strategy, but once they take the borrowing constraints they face into account, they may conclude that it is infeasible for them to exploit this particular profit opportunity.

So we also need to further examine and try to relax some of the simplifying assumptions we made in this analysis, such as our assumption that firms in our sample did not face borrowing constraints, particularly a drying up of capital for investment during bust periods. A related issue is the opportunity cost of capital. A countercyclical replacement investment strategy amplified the procyclicality of the cash flow stream rental companies, since it causes big cash flow outlays during bust periods, in return for cash inflows in normal and boom periods. Capital markets may strongly reward rental companies whose dividend payouts are more countercyclical, making their stocks better hedges for portfolio purposes, which could in turn lower their cost of capital. It may be important to try to account for the cyclical patterns of cash flows and the way stock markets (for publicly owned firms) or personal financial needs (for privately owned firms) lead firms to prefer cash flow streams that are less pro-cyclical and thus serve as a better hedge to overall comovements between the stock market and the business cycle.

If these “capital market” considerations are important, it suggests that different firms may have different optimal decision rules for similar assets. It may be that publicly traded firms where investors can hedge and diversify on their own and which have better access to capital markets can more easily pursue counter-cyclical strategies if the pro-cyclicality of the induced cash flow and dividend streams are not heavily penalized in the stock market. However capital constraints may be a real problem for smaller privately held firms. If so, the risk and “dividend smoothing” preferences of each firm might need to be
accounted for in the model so that decision rules reflect those preferences. We believe it may be possible to take these capital market considerations into account and develop customized decision rules that reflect firms’ practices, financial constraints, and preferences. If we can accurately capture these features in future versions of our models, the asset replacement rules that we derive from them will likely provide better guidance to firms.

A final area for future extensions is to allow for a wider range of actions besides replacing an existing machine with a new one. This could include replacing existing machines with other used machines acquired at auction, or strategies that involve asset rejuvenation or rebuilds as a substitute to outright replacement. Rejuvenation or rebuilding adds to the useful life of the asset but at a cost. Therefore, there is a decision to be made about whether to rejuvenate an asset or simply dispose of the asset in favor of a new one or a different asset. There is also the question of how extensive the rejuvenation should be. The effect of rejuvenation on useful life and on reliability, or the probability of failure can also be included in the model and help produce more relevant, realistic and profitable decision rules.

References


