

The impact of index-based insurance on informal risk-sharing arrangements*

Steve Boucher[†] and Matthieu Delpierre[‡]

December 1, 2014

Abstract

Moral hazard and adverse selection impede the development of formal crop insurance markets in developing countries. Besides, the risk mitigation provided by informal risk-sharing arrangements is restricted by their inability to protect against covariate shocks. In this context, index-based insurance is seen as a promising scheme as it is immune to moral hazard and adverse selection and may offer effective protection against covariate shocks. It would thus seem that the two institutions are ideal complements. Unfortunately, this intuition ignores the potential effects on incentives and behavior generated by the interaction between both schemes. This paper explores this interaction in a model with moral hazard and shows that the formal contract may crowd out informal risk-sharing if it is offered to individuals. Second, we find that both risk-taking and welfare may be reduced by the introduction of index insurance if the premium is set too high. If the formal insurance is offered to the group instead of the individual, the impact on moral hazard is internalized and welfare increases.

JEL Codes: 012; 013; D81; G22

*Acknowledgements: The authors are grateful to Catherine Guirkingner, Marcel Fafchamps, Daniel Clarke, François Bourguignon, Jean-Philippe Platteau, Bertrand Verheyden and Kathryn Vasilaky for useful comments and suggestions. The paper has also benefited from presentations at the CSAE Conference in Oxford, CEPS/INSTEAD, the GDN Global Development Conference in Manila, the Agricultural and Applied Economics Association annual Meeting in Washington and at the Universidad de Los Andes in Bogota.

[†]University of California - Davis

[‡]CEPS/INSTEAD and Earth and Life Institute, Université catholique de Louvain. Corresponding author: MatthieuDelpierreG@gmail.com.

1 Introduction

The near absence of formal crop insurance markets implies that rural households in many developing countries depend primarily on their own, potentially costly, autarkic strategies for income and consumption smoothing and the strength of their informal risk sharing networks to mitigate the myriad sources of risk they face (Rosenzweig & Binswanger (1993); Townsend (1994); Morduch (1995); Ligon et al. (2002)). This bleak risk management landscape for agricultural households may be changing thanks to the recent re-discovery of index insurance by researchers and development institutions. The indemnity in an index insurance contract is based on an external index, such as rainfall, directly measured average yield, or satellite-based predictions of average yield, which is correlated with the individual insured farmer’s yield, but independent of his/her isolated action and behavior. As such, the contracts are relatively immune to both the moral hazard and adverse selection problems that plague conventional, named peril contracts. This conceptual promise has spurred a number of research initiatives that explore the optimal design of index contracts and put in place and evaluate pilot index insurance initiatives (Barnett et al. (2008); Miranda & Farrin (2012)). The jury is still out on index insurance as significant challenges – including high basis risk, low farmer financial literacy, and high coordination costs across insurers, re-insurers and regulators – remain unresolved. If, however, these pilot projects are deemed successful and scaled-up, rural households in developing countries may enjoy expanded access to formal insurance markets via index insurance.

The goal of this paper is to consider how this expansion of formal crop insurance may play out. Specifically, we develop a theoretical model that explores how the introduction of a formal index insurance market may affect farmers’ risk taking behavior and the degree of risk sharing in existing informal risk sharing arrangements (IRSAs). At first glance, the separate risk domains of the two types of institutions suggest that IRSAs will not be affected, leading to unambiguously positive efficiency and welfare impacts. IRSAs are information intensive and thus tend to be limited to spatially concentrated areas, such as a village. As such, IRSAs are best suited to mitigate idiosyncratic risks, such as those deriving from health or plot-specific pest problems that are relatively independent across households within the village. In contrast, index insurance contracts are aimed at mitigating covariate shocks, such as yield declines due to drought, that tend to simultaneously affect all households in a village.

By removing the covariate risk that IRSAs are unable to address, index insurance would appear to unambiguously increase households’ risk bearing capacity, leading to greater investment and welfare.

Closer consideration of the incentives embedded in IRSAs, however, reveals that this win-win scenario need not obtain. We show that when risk-taking is not contractible by members of the IRSA, the introduction of formal index insurance to individuals will reduce informal risk sharing (crowding out) and can also, under conditions we lay out, reduce risk taking and welfare. The adverse welfare impact of index-insurance is reversed if the index insurance contract is instead offered at the group level.

To understand the intuition behind the crowding out result, we must turn to the tradeoff that exists between incentives and risk sharing within IRSAs of finite size when risk-taking is not contractible. If IRSAs were infinitely large, then members would be able to completely eliminate idiosyncratic risk through full risk pooling, and no tradeoffs would exist. In the real world, however, IRSAs are finite; they are limited to the number of households in a village or the individuals in a household’s extended family. As such, even if members fully pool their idiosyncratic risk, the average realization of the idiosyncratic shock across network members in a given year need not be zero. As a result, IRSA members confront “residual” idiosyncratic

risk. If risk taking is not contractible among members of the IRSA, then this residual idiosyncratic risk becomes a vector of externalities; as I increase my risk taking, I increase the variance of this residual risk and impose a negative externality on others in the group. Informal risk-sharing therefore entails a form of moral hazard¹, which gives rise to excessive risk-taking in equilibrium. To mitigate the adverse consequences of moral hazard, the group finds it optimal to adopt an incomplete rate of risk-sharing. This rate is second best as there is a tradeoff between insurance against idiosyncratic risk and excessive risk-taking.

We now see how the introduction of index insurance may have unintended, adverse consequences as the reduction of covariate risk provides incentives for individuals to increase their risk taking. In order to mitigate the ensuing higher residual idiosyncratic risk, the IRSA may endogenously choose to reduce the amount of idiosyncratic risk pooling, thus providing a counter-incentive in order to decrease risk taking. Our model is thus similar in spirit to that of Arnott & Stiglitz (1991) who show that formal insurers will ration insurance in order to maintain agents' incentives. We then study group-based index insurance contracts and show that they dominate individual-based contracts. The reason thereof is that groups internalize the adverse effects that index-insurance may have on moral hazard.

To our knowledge, Arnott & Stiglitz (1991) were the first to examine the coexistence between formal and informal insurance. In a model with moral hazard, they show that the equilibrium on a competitive formal market entails rationing. Otherwise risk-taking would be excessive and insurers would make losses. Informal risk-sharing is then shown to crowd out formal insurance. If informal risk-sharing equally suffers from moral hazard, then the crowding out effect unambiguously reduces welfare in their model, because the market is more effective than a group of reduced size in absorbing shocks. The standpoint we adopt in this paper is opposite as we explore the impact of index-insurance on pre-existing informal risk-sharing. However, our model is not simply a reversed version of Arnott & Stiglitz (1991)'s analysis. Indeed, while they consider a single source of risk that both formal and informal contracts allow to cover, we analyze a situation where two types of risks coexist, namely idiosyncratic and covariate. Those risks are orthogonal. Moreover, on the one hand, formal index-insurance provides protection against covariate risk only. On the other hand, IRSA's provide protection against idiosyncratic risk only. Hence, the crowding out result we highlight in this paper does not result from the same mechanism.

Attanasio & Rios-Rull (2000) study the impact of compulsory insurance against covariate risk on informal risk-sharing when the latter is impeded by limited commitment. They show that formal insurance crowds out informal risk-sharing and may lead to welfare losses under conditions they lay out. The main difference with our model is that we do not consider enforceability problems regarding informal transfers, but we rather focus on the role played by moral hazard. Furthermore, index-insurance is generally not publicly provided and hence not compulsory. Therefore, we analyze demand for index-insurance under both individual and group subscription cases.

Recent papers are specifically devoted to analyzing the interaction between formal index-insurance and informal risk-sharing. Mobarak & Rosenzweig (2012) and Dercon et al. (2014) highlight the technological complementarity that might exist between them. Their respective theoretical contributions are based on

¹In this paper, we follow Delpierre et al. (2014) in the way the terms moral hazard are used. According to this interpretation, moral hazard refers to the following phenomenon: Risk-sharing entails externalities of risk, which induce agents to adopt risky production choices as they do not support the entire cost of their choice in terms of risk bearing. The classical interpretation of moral hazard in insurance problems involves a costly action that agents can take to reduce the probability of incurring a loss. In this paper, effort takes the form of reduced expected returns as explained below. A discussion of the distinction between this form of moral hazard and the standard one is available in Delpierre et al. (2014).

a similar intuition, which proceeds as follows. Due to the imperfect correlation between the index and a farmer's individual income, farmers face basis risk: they may suffer from an idiosyncratic loss, not captured by the index and that the formal scheme is therefore unable to cover. In those states of the world, the payment of the premium charged by the formal insurance provider generates a high marginal disutility. As informal risk-sharing reduces either the extent or the probability of occurrence of large idiosyncratic losses, it should mitigate basis risk. A farmer's willingness to pay for index-insurance should then be higher if he/she is involved in informal groups, which aim at sharing idiosyncratic risk. In other words, we expect uptake rates to be higher among members of IRSAs. This prediction is tested and confirmed by Mobarak & Rosenzweig (2012) on sub-castes in India. Dercon et al. (2014) conducted experiments among funeral insurance groups in Ethiopia. They exposed group leaders to trainings on index-insurance. The treatment consisted of training sessions that highlighted the potential complementarity between index-insurance and informal risk-sharing, while the basic training was strictly focused on index-insurance per se. They show that agents who express an interest for index-insurance are significantly more engaged in risk-sharing in the treatment group.

So both Mobarak & Rosenzweig (2012) and Dercon et al. (2014) come to the theoretical conclusion that index-insurance and informal risk-sharing are technological complements and to the empirical consequence that individual demand for index-insurance is higher for agents engaged in IRSAs. Regarding theory, Dercon et al. (2014) show that, at the level of an individual, the marginal utility of index-insurance is higher when the rate of informal risk-sharing is high, which implies the opposite, namely that the marginal utility of informal risk-sharing is higher for someone who subscribes to index-insurance. However, this does not imply that index-insurance crowds in informal risk-sharing in the sense that it would increase the rate of risk-sharing. Indeed, in both papers, risk-sharing is exogenously given. Therefore, by construction, those existing models cannot predict how the equilibrium level of risk-sharing is impacted by the introduction of index-insurance. Moreover, the rate of informal risk-sharing does not simply result from isolated individual decisions, but is the outcome of joint decisions taken by IRSA members. Hence, it is not straightforward that the technological complementarity between IRSAs and index-insurance highlighted by Dercon et al. (2014) and Mobarak & Rosenzweig (2012) will give rise to a crowding in effect in equilibrium.

Our paper analyzes the interaction between index-insurance and IRSAs while allowing the rate of risk-sharing, which is decided at the group level, to vary following the introduction of index-insurance. Moreover, we treat as endogenous the risk-taking behavior of agents. Therefore, moral hazard comes into play and we show that it prevents the complementarity at the individual level from generating a crowding in effect at the group level. We highlight that, if we consider that IRSAs are designed cooperatively at the group level and that individual risk-taking decisions are non-cooperatively taken, then the technological complementarity between informal risk-sharing and index-insurance is not incompatible with a crowding out effect at the group level.

de Janvry et al. (2014) also look at subscription to index-insurance by members of formal or informal groups, such as producers' cooperatives or risk-sharing networks. Depending on the type of activity undertaken by the group, they show that either free riding or coordination issues may reduce demand for index-insurance. Their model is very general as it only relies on the fact that agents' interactions within the group make their utility depend not only on their own wealth but on the whole vector of individual wealth levels in the group. In particular, they show that if own wealth and aggregate wealth are substitutes for any agent, then individual subscription to index insurance generates positive externalities, which entail the classical free riding problem. Indeed, as wealth levels are random and agents risk averse, index-insurance

simultaneously reduces the variability of own wealth and of aggregate wealth, which increases the utility of the other group members. They argue that informal risk-sharing creates substitution between own wealth and aggregate wealth. The intuition is that a negative shock at the individual level might be compensated by a high value of the aggregate wealth if the unlucky agent receives transfers from other group members.

As compared to de Janvry et al. (2014)'s paper, the contribution of our model consists in providing more structure to the IRSA in order to capture the mechanism by which subscription to index-insurance and IRSAs interact. This paper is not a particular case of de Janvry et al. (2014)'s model for two reasons.

First, our model does not reproduce the positive externality. The reason is that we assume that only idiosyncratic risks are shared and not income. As a result, the utility function depends on own wealth, or income, and on the aggregate idiosyncratic shock but not on aggregate income. Hence, individual benefits from index-insurance, namely the reduction of the covariate risk, are not transmitted to the group. Assuming idiosyncratic risk-sharing means that shocks are shared but not income per se. This assumption seems relevant. indeed, on the one hand, income sharing violates actuarially fairness and is hence incompatible with the observation that IRSAs satisfy strict reciprocity rules (Platteau (1997), Ligon et al. (2002)). On the other hand, the purpose of IRSAs is generally to insure its members against specific risks that are essentially uncorrelated between agents (Platteau (1997), Dercon et al. (2006), De Weerd & Dercon (2006)). It follows that it makes sense to assume that only idiosyncratic shock are shared. If this is true, then the positive externality à la de Janvry et al. (2014) cannot appear since it relies on a positive impact of an individual covariate risk reduction on the wellbeing of others.

Our second contribution relative to de Janvry et al. (2014)'s paper is an explicit analysis of IRSAs with endogenous risk-taking behaviors.

As already stressed, index-insurance and IRSAs deal with separate risk domains, so that informal risk-sharing may not be a direct vector of externalities that individual subscription to index-insurance would generate. Instead, this paper shows that the impact of index-insurance on IRSAs can result from changes in incentives and behaviors.

Surprisingly enough, the recommendations we formulate are close to those of Dercon et al. (2014) and de Janvry et al. (2014). Indeed, we would also recommend that index-insurance be offered to informal insurance groups, since the latter internalize the moral hazard issue highlighted in this paper. This idea is also supported by experiments conducted in the Dominican Republics (Vasilaky et al. (2014)). One of the originalities of this paper is that it specifically tests whether group subscription induces a higher or a lower uptake rate as compared to the case of individual subscription. It turns out that uptake rates appear significantly lower under group subscription. Our analysis, which shows that individual subscription to index-insurance may amplify moral hazard issues in IRSAs and which suggests that groups may be able to internalize this effect, may provide an explanation for Vasilaky et al. (2014)'s empirical result. Moreover, Vasilaky et al. (2014) randomize social distance within groups and show that groups where individuals are socially closer tend to purchase less formal insurance than others. This is again consistent with the interpretation that groups where effects on moral hazard are the most likely to be internalized seem to ration their formal insurance coverage by themselves.

The paper proceeds as follows. Section 2 introduces our assumptions on technology, risk and risk preferences and characterizes the first best level of risk taking. Section 3 introduces the institutional setup of the IRSA, in which members set the fraction of their idiosyncratic risk that they pool with others. We assume that the level of risk sharing is costlessly enforceable and characterize its equilibrium value. In contrast, we

assume individual risk taking behavior is not contractible within the IRSA and, as described above, this will be the source of moral hazard. We show that for any given level of risk pooling, the equilibrium level of risk taking will be too high relative to the cooperative level. The second main result of Section 3 immediately follows; namely that in order to address the negative risk taking externality, the group will optimally choose incomplete sharing of idiosyncratic risk. Section 4 introduces a stylized index insurance contract in which the farmer chooses the level of coverage. The coverage level, in turn, is calibrated relative to the farmer's level of risk taking so that full coverage, i.e., coverage equal to risk taking, provides full insurance against covariate risk. Section 4 concludes by characterizing index insurance demand. Section 5 delivers the main results of the paper, namely the impact of the introduction of index insurance on informal risk sharing, individual risk taking and welfare under individual versus group subscription to index insurance. Section 6 concludes with a discussion of limitations to and potential extensions of the model and some reflections on the optimal design of index insurance contracts in developing countries.

2 Technology, preferences and the risk environment

We consider a group of farmers belonging to the same community or cooperative with n members, $N = \{1, \dots, n\}$. We assume that group members are homogeneous in terms of technology, endowment and preferences. We begin by describing the technology and the risk environment. An individual farmer's income is stochastic and takes the following form:

$$Y(\sigma_i) = \mu(\sigma_i) + \sigma_i \Theta_i, \forall i \in N, \quad (1)$$

where $\sigma \in \mathbb{R}_+$ is the level of risk-taking chosen by the farmer and corresponds, loosely, to the level of investment, the input intensity in production, or the degree of specialization.² Θ_i is a zero mean random variable with unit variance so that, in the absence of formal insurance or informal risk-sharing, the mean and variance of income are simply given by:

$$E(Y_i; \sigma_i) = \mu(\sigma_i); \text{Var}(Y_i; \sigma_i) = \sigma_i^2.$$

The function μ is assumed twice continuously differentiable with: $\mu'(\sigma) \geq 0$ and $\mu''(\sigma) < 0$. This specification represents the basic tradeoff between risk and return, which is inherent to agricultural production choices in uncertain environments. Such a tradeoff will exist if the farmer's expected income can only increase at the cost of a higher variance. One may argue that some technologies, such as irrigation facilities, allow farmers to simultaneously increase returns while reducing risk. We allow for this possibility as the function $\mu(\sigma)$ can be interpreted as a technological frontier. A point located strictly below the frontier is inefficient as, at this point, it is possible to reduce the variance without decreasing the mean or to increase the mean without increasing the variance. We then assume that, by choosing his/her level of risk-taking σ , the farmer always selects a

²We assume that shocks and expected income are additively separable while multiplicative risks are often encountered in other papers. We argue that our specification adds degrees of freedom to the multiplicative version. Our specification actually encompasses the multiplicative form as a special case: suppose Θ is a standardized random variable. The multiplicative form would be

$$Y(\sigma) = (1 + \Theta) f(\sigma).$$

Hence $E(Y) = f(\sigma)$ and $\text{Var}(Y) = f(\sigma)^2$. The function $f(\sigma)$ can then be normalized as $f(\sigma) = \sigma$, without loss of generality. It is now straightforward to show that this is strictly identical to our specification for $\mu(\sigma) = \sigma$. This is therefore more restrictive than the additive version.

point on the technological frontier $\mu(\sigma)$. In other words, the mean income is assumed to be maximized for any given variance. We will use $\Sigma = (\sigma_1, \dots, \sigma_n) \in \mathbb{R}_+^n$ to denote the risk-taking profile in the group.

Agricultural production is subject to a series of shocks of different natures, from drought and pests to illnesses undermining the farmer's ability to work. We draw an important distinction between *covariate* shocks, such as drought or price shocks, that simultaneously affect all members of the group versus *idiosyncratic* shocks, such as hail and other very localized weather events or non-epidemic illnesses, that are orthogonal across individuals in the group. From a statistical point of view, it is always possible to decompose a given risk between a covariate, or common, component, which is perfectly correlated within the group, and an idiosyncratic component. This is precisely how our risk structure is framed. We specify the stochastic term as the sum of two independent random variables: $\Theta_i = \theta_g + \theta_i$, with $(\theta_g, \theta_i) \in \mathbb{R}^2$. The covariate risk is embodied by $\theta_g \sim G$. This variable is common to all individuals in the group in the sense that there is a single draw and hence a unique value of θ_g at the group level. In addition, there are n i.i.d. random variables $\theta_i \sim F$ representing farmers' idiosyncratic risk. At the group level, the set of all possible vectors of idiosyncratic income shocks is then \mathbb{R}^n . Let $S = (\sigma_1\theta_1, \dots, \sigma_n\theta_n) \in \mathbb{R}^n$ denote an element of this set. The total value of the individual farmer's income shock is then the sum of the covariate shock $\sigma_i\theta_g$ and the idiosyncratic shock $\sigma_i\theta_i$. To maintain our assumptions that the total income shock, Θ_i , has zero mean and unit variance, we further assume:

$$E(\theta_g) = E(\theta_i) = 0,$$

$$Var(\theta_g) = (1 - b); Var(\theta_i) = b,$$

so that, indeed, $E(\theta_g + \theta_i) = 0$ and $Var(\theta_g + \theta_i) = 1$, by independence between θ_g and θ_i . The parameter b measures the - constant - idiosyncratic fraction of aggregate risk. When index-insurance is introduced, this parameter will approach the notion of basis risk. Intuitively, $(1 - b)$ is the fraction of risk that a farmer may be able to insure via index-insurance, while b defines the scope for sharing idiosyncratic risk through the group's IRSA.³

We now turn to agents' preferences. Utility is formed over consumption c . The utility function $u(c)$ is increasing and concave and characterized by constant absolute risk aversion (CARA). Making use of Pratt's approximation of the risk premium, the certainty equivalent of consumption can be written as

$$u^{-1}(Eu(c)) \approx E(c) - \frac{1}{2}\eta Var(c) \equiv \tilde{c}, \quad (2)$$

where η is the coefficient of absolute risk aversion. Our results are not qualitatively affected by the assumption of CARA preferences. In our setting, the main implication of this choice is the following. Since there is a

³At first sight, a constant b may appear restrictive. It requires that production choices impact the total size of a farmer's income risk but not the composition; i.e., the relative importance of idiosyncratic versus covariate risk. While a choice between different types of seeds, or a decision on the intensity of chemicals application, seem to meet this requirement, the allocation of land or changes in crop portfolio, may intuitively violate it. This is not necessarily true. Consider the following example. Suppose an initial situation where farmers allocate half of their land parcels to subsistence crops and the remaining to export crops. What are then the consequences on the risk structure of an increase in the fraction of land devoted to cash crops? Aggregate risk may certainly increase as the farmer's exposure to a single random price is higher and his/her diversification level is reduced. As regards the impact on b , the question is whether the scope for risk-sharing will be modified or, put differently, whether the correlation between the group members' respective incomes will be affected. Therefore, the other farmers' choices will matter. If everyone modifies her allocation the same way, correlations remain unchanged. Correlations will be impacted provided changes are heterogeneous or different activities are available. b can indeed be higher if farmers coordinate in order to specialize on uncorrelated activities. Since our focus is on moral hazard, we do not allow for this possibility.

tradeoff between risk and return, risk-taking σ increases both the expected value and variance of income. Under CARA, those two effects can be readily identified. Under decreasing absolute risk aversion (DARA), a third positive effect would appear, namely a change in the farmer's subjective perception of risk which decreases the marginal cost of risk taking.⁴ CARA allows a simplified exposition as it eliminates this second order effect, which is clearly dominated by the increase in objective risk in any optimal decision on σ .⁵

3 Informal risk-sharing with moral hazard

Due to information asymmetries and high transaction costs, traditional indemnity-based (i.e., named peril) insurance products are unavailable to most farmers in developing countries. Consistent with this observation, we assume that formal insurance and capital markets are missing. Since a fraction b of income variability is uncorrelated across individuals within the group, farmers have an incentive to pool idiosyncratic risk in order to smooth consumption. As amply documented, however, informal risk-sharing is subject to several enforceability constraints and is generally incomplete (Townsend (1994); Jalan & Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002); Morduch (1995)). In this section, we provide micro-foundations for this incompleteness in a model with moral hazard in risk taking in production.⁶ In the next section, index-insurance will be introduced to assess its impact on the, initially imperfect, functioning of informal risk-sharing.

3.1 The informal risk-sharing setup

We assume that group composition is exogenously given. This is the case if informal risk-sharing takes place at the level of a producers' cooperative, an extended family or a rural community with stable membership. In addition, we rule out limited commitment as a source of incomplete risk-sharing by assuming that group members can fully commit to make transfers to each other ex post. Instead, we assume that the group is unable to enforce risk-taking σ . This will generate the moral hazard problem on which we want to focus.

Since the covariate shock is perfectly correlated across group members, the IRSA provides insurance for the idiosyncratic component of risk only. The IRSA specifies income transfers that are contingent on the realization of shocks S , as follows:

A transfer scheme is a vector of transfers $T = (t_1, \dots, t_n) \in \mathbb{R}^n$, where t_i stands for the net transfer received by farmer i . Let α_i denote the fraction of his/her own idiosyncratic income shock $\sigma_i \theta_i$ that farmer i transfers to the group. The pool of idiosyncratic shocks that are shared within the group $\sum_{j \in N} \alpha_j \sigma_j \theta_j$ is noted $P(S; \Sigma)$. The net transfer received t_i is defined as the difference between the share of the pool of idiosyncratic shocks that farmer i commits to support t_i^I and the share of his/her own shock that he/she

⁴In autarky, for instance, the impact of a marginal change in risk-taking would be

$$\frac{\partial \tilde{Y}}{\partial \sigma} = \mu'(\sigma) \left(1 - \frac{1}{2} \eta' \sigma^2\right) - \eta \sigma,$$

under DARA, while it is simply given by

$$\frac{\partial \tilde{Y}}{\partial \sigma} = \mu'(\sigma) - \eta \sigma,$$

under CARA.

⁵Moreover, as shown by Clarke (2011), the shape of demand for index-insurance as a function of risk aversion is similar in the two cases, namely CARA and DARA. In the former case, it does not seem abusive to think of the potential heterogeneity in the degree of absolute risk aversion as a good proxy for wealth inequalities.

⁶The model is based on the analysis developed in Delpierre et al. (2014).

transfers to the group t_i^O :

$$\begin{aligned} t_i^I &= \gamma_i P(S, \Sigma), \\ t_i^O &= \alpha_i \sigma_i \theta_i. \end{aligned}$$

We impose two restrictions on the transfer scheme T . On the one hand, we impose anonymity in the sense that the transfer t_i should not depend on the identity of farmers. Because group members are homogeneous, the transfer t_i will only depend on the realization of shocks S . As a consequence, $\gamma_i = \gamma_j$ and $\alpha_i = \alpha_j, \forall \{i, j\} \subset N$. On the other hand, since formal insurance and capital markets are missing, external markets cannot absorb a deficit of the IRSA, which must therefore satisfy a budget constraint. In addition, we do not allow the group to form savings, so that this budget constraint must hold with equality for all state of the world: $\sum_{j \in N} t_j = 0, \forall S \in \mathbb{R}^n$. The transfer scheme T satisfies anonymity and the budget constraint if and only if $\gamma_i = \gamma_j = 1/n$, so that $P(S; \Sigma)$ is entirely distributed among group members. As a result, the transfer writes

$$t_i(S) = \alpha \left(-\sigma_i \theta_i + \frac{1}{n} \sum_{j \in N} \sigma_j \theta_j \right), \forall i \in N. \quad (3)$$

As can be seen from this expression, a transfer scheme T that satisfies anonymity and the budget constraint has only one degree of freedom, $\alpha \in [0, 1]$, which determines the rate of risk-sharing. Equation (3) can be interpreted in the following way: The net transfer received by farmer i is positive provided his/her idiosyncratic shock is lower than the average shock at the group level. Hence, a farmer facing a negative shock cannot always benefit from a transfer. If, for instance, the idiosyncratic shock faced by group members is on average negative, farmer i will be a net contributor if his/her idiosyncratic shock is negative but lower than average in absolute value. Notice also that, by the Law of Large Numbers, the average realization of idiosyncratic shocks converges to zero as the size of the group increases, so that the capacity to absorb idiosyncratic risk is increasing in group size.

Farmer i 's consumption level c is equal to his/her post transfer income, which we find by using equations (1) and (3):

$$\begin{aligned} c_i(\alpha, \Sigma, S) &\equiv Y(\sigma_i) + t_i(S) = \mu(\sigma_i) + \sigma_i(\theta_g + \theta_i) + t_i(S) \\ &= \mu(\sigma_i) + \sigma_i \theta_g + \sigma_i \left[(1 - \alpha) + \alpha \frac{1}{n} \right] \theta_i + \alpha \frac{1}{n} \sum_{j \in N \setminus \{i\}} \sigma_j \theta_j. \end{aligned} \quad (4)$$

By independence between the different random variables that appear in (4), we can write the mean and the variance of consumption as:

$$E(c_i; \sigma_i) = \mu(\sigma_i), \quad (5)$$

$$Var(c_i; \alpha, \Sigma) = \sigma_i^2 (1 - b) + \sigma_i^2 \left[(1 - \alpha) + \alpha \frac{1}{n} \right]^2 b + \left(\alpha \frac{1}{n} \right)^2 \sum_{j \in N \setminus \{i\}} \sigma_j^2 b. \quad (6)$$

Recall that, in autarky, consumption variance is simply equal to σ_i^2 . Inspection of expression (6) highlights the distinction between covariate and idiosyncratic risks and the impact of informal risk-sharing on the latter. The covariate fraction of the consumption variance corresponds to the first term on the right hand side of (6) and remains unchanged, while the idiosyncratic risk is shared and then reduced. Indeed, the second term pertains to the idiosyncratic fraction of consumption variance and illustrates this effect. The higher is the

level of risk-sharing α , the lower is this term. The third term shows, however, that farmer i 's consumption variance is now affected by the risk-taking behavior of other farmers in the group through $\sum_{j \in N \setminus \{i\}} \sigma_j^2$. Risk-sharing therefore entails risk externalities that will generate a moral hazard issue.

In order to highlight this moral hazard issue, we characterize here the first best allocation as a benchmark. The first best allocation is defined as the rate of risk-sharing α and the risk-taking profile Σ that maximize welfare in the absence of formal insurance markets. Suppose a social planner is able to enforce α and Σ . The planner maximizes a social welfare function $W = \sum_{i \in N} u(\tilde{c}_i(\alpha, \Sigma))$, which aggregates the farmers' levels of welfare.⁷

Proposition 1 *First best*: *In the absence of formal insurance markets, the first best allocation is characterized by full risk-sharing $\alpha^{FB} = 1$ and a homogeneous risk-taking profile $\Sigma^{FB} = (\sigma^{FB}, \dots, \sigma^{FB})$, where the level of risk-taking σ^{FB} satisfies:*

$$\mu'(\sigma^{FB}) - \sigma^{FB} \eta \left[(1-b) + \frac{1}{n} b \right] = 0.$$

At the first best, the consumption variance is equal to

$$\text{Var}(c_i; 1, \Sigma^{FB}) = (\sigma^{FB})^2 (1-b) + (\sigma^{FB})^2 \frac{1}{n} b. \quad (7)$$

Proof. The maximization program of the planner is as follows

$$\text{Max}_{\alpha, \Sigma} W = \sum_{i \in N} u(\tilde{c}_i(\alpha, \Sigma)),$$

where $\tilde{c}_i(\alpha, \Sigma)$ can be found by substituting the mean (5) and variance (6) of consumption in the expression of the certainty equivalent (2).

The first order condition with respect to any given σ_i imposes that

$$\frac{\partial W}{\partial \sigma_i} = 0 \iff \sum_{j \in N} u'(\tilde{c}_j) \frac{\partial \tilde{c}_j(\alpha, \Sigma)}{\partial \sigma_i} = 0$$

where

$$\begin{aligned} \frac{\partial \tilde{c}_j(\alpha, \Sigma)}{\partial \sigma_i} &= -\eta \left(\alpha \frac{1}{n} \right)^2 \sigma_i b, \forall i \neq j, \\ &= \mu'(\sigma_i) - \eta \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b \right] \sigma_i, \text{ for } j = i. \end{aligned}$$

This gives an implicit function $\sigma_i^C(\alpha)$. Because farmers are homogeneous in terms of preferences (η) and technology ($\mu(\sigma)$), this function is identical for all $i \in N$. As a result, $\sigma_i^C(\alpha) = \sigma^C(\alpha)$ and $\tilde{c}_i = \tilde{c}_j, \forall i \in N$. Hence, $\sigma^C(\alpha)$ is defined by

$$\sum_{j \in N} \frac{\partial \tilde{c}_j(\alpha, \Sigma)}{\partial \sigma_i} = 0 \iff \mu'(\sigma^C) = \eta \sigma^C \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b + \left(\alpha \frac{1}{n} \right)^2 (n-1) b \right]. \quad (8)$$

Let us substitute $\sigma^C(\alpha)$ to the σ_i 's in the planner's objective function. In particular, a farmer's consumption variance (6) becomes

$$\text{Var}(c_i; \alpha, \sigma^C(\alpha)) = \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b + \left(\alpha \frac{1}{n} \right)^2 (n-1) b \right] \sigma^C(\alpha), \forall i \in N.$$

⁷Notice that, because farmers are homogeneous, we do not attribute different Pareto weights to farmers.

The first order condition with respect to α imposes that

$$\frac{dW(\alpha, \sigma^C(\alpha))}{d\alpha} = \frac{\partial W}{\partial \alpha} + \frac{\partial W}{\partial \sigma} \frac{\partial \sigma^C}{\partial \alpha} = 0,$$

where $\partial W / \partial \sigma = 0$, by the envelope theorem. Hence,

$$\frac{dW(\alpha, \sigma^C(\alpha))}{d\alpha} = 0 \iff \frac{\partial \text{Var}(c_i; \alpha, \sigma^C)}{\partial \alpha} = 0 \iff \alpha^{FB} = 1.$$

Therefore, $\sigma^{FB} = \sigma^C(1)$, which gives the condition for σ^{FB} . ■

The first best can be reached if the risk-taking profile Σ is enforceable. The group is then able to internalize the risk externalities highlighted in equation (6). In the absence of moral hazard, the IRSA allows members to fully pool idiosyncratic risk. Yet, even under complete risk-sharing, the idiosyncratic fraction of the consumption variance is equal to $(\sigma^2 b) / n$ (see equation 7), remaining strictly positive and vanishing only asymptotically with group size. Insurance remains imperfect for two reasons: (1) The group has a finite size and (2) The IRSA cannot provide insurance against covariate risk. The covariate fraction of the consumption variance remains equal to $\sigma^2(1 - b)$. As a result, risk-taking is increasing in group size n and in b , which denotes the extent of the risk that can be covered by the IRSA. Finally, as expected, risk-taking is negatively related to the coefficient of absolute risk aversion η . The condition for first best risk-taking tells us that the marginal benefit of risk-taking, namely the increase in expected returns ($\mu'(\sigma^{FB})$) must be equal to the social marginal cost, which is the increase in the risk premium when risk is efficiently shared within the group: $\sigma^{FB} \eta [(1 - b) + \frac{1}{n} b]$.

3.2 Informal risk-sharing under moral hazard

Assume now that the risk-taking profile Σ is unenforceable by the group. As highlighted in the expression of the consumption variance (6), the IRSA generates risk externalities. Decisions on Σ are therefore taken individually and in a context of strategic interaction. Under unenforceable risk-taking, the timing of the game is as follows:

1. Group members agree on a rate of risk-sharing α .⁸
2. Group members simultaneously and non-cooperatively choose their own level of risk-taking, σ_i .

The outcome of this game will give us the second best value of α .

Using backward induction, we first solve the second stage of the game. The following Lemma gives the risk-taking profile at the Nash equilibrium $\Sigma^N(\alpha)$ as a function of the level of risk-sharing α . The Lemma also compares the Nash level of risk-taking to the cooperative level so as to highlight the moral hazard problem.

Lemma 1 Moral hazard: *For any given level of risk-sharing α ,*

1. *the risk-taking profile at the Nash equilibrium is homogeneous: $\Sigma^N(\alpha) = (\sigma^N(\alpha), \dots, \sigma^N(\alpha))$, where $\sigma^N(\alpha)$ is given by*

$$\mu'(\sigma^N) = \eta \sigma^N \left[(1 - b) + \left[(1 - \alpha) + \alpha \frac{1}{n} \right]^2 b \right],$$

⁸As group members are homogeneous, any cooperative solution concept gives us the same value of α .

2. the Nash level of risk-taking is always larger than the cooperative level and is therefore always inefficiently high: $\sigma^C(\alpha) < \sigma^N(\alpha), \forall \alpha \in [0, 1]$.

Proof. In order to find the risk-taking profile at the Nash equilibrium Σ^N , we derive farmer i 's best response function. Let $\Sigma^{-i} = (\sigma_1, \dots, \sigma_{i-1}, \sigma_{i+1}, \dots, \sigma_n) \in \mathbb{R}^{n-1}$ denote the vector collecting the risk-taking levels adopted by all group members with the exception of agent i . Farmer i 's optimization problem is as follows

$$Max_{\sigma_i} \tilde{c}_i(\alpha, \sigma_i, \Sigma^{-i}) = \mu(\sigma_i) - \frac{1}{2}\eta \left[\sigma_i^2(1-b) + \sigma_i^2 \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b + \left(\alpha \frac{1}{n} \right)^2 \sum_{j \in N \setminus \{i\}} \sigma_j^2 b \right]. \quad (9)$$

The first order condition of this problem gives us

$$\frac{\partial \tilde{c}_i(\alpha, \sigma_i, \Sigma^{-i})}{\partial \sigma_i} = 0 \iff \mu'(\sigma^N) = \eta \sigma^N \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b \right], \forall i \in N \quad (10)$$

By homogeneity, the optimization problem is identical for all agents. As a result, equation (10) directly defines the Nash level of risk-taking $\sigma^N(\alpha)$.

The cooperative level of risk-taking $\sigma^C(\alpha)$ is the level that would choose the planner for a given level of risk-sharing α and therefore corresponds to the function defined in the proof of Proposition 1 (equation 8).

A simple comparison of conditions (8) and (10) confirms that $\sigma^C(\alpha) < \sigma^N(\alpha)$. Indeed, on the left hand side of both equations, we have the same decreasing function of σ ($\mu'' < 0$), while the right hand side is linearly increasing in σ , with a higher slope in the case of equation (8). ■

The intuition behind this result is simply that risk externalities are not internalized by farmers when they choose σ non-cooperatively. The individual marginal cost of risk-taking, which is given by the right hand side of equation (10) is strictly lower than the social marginal cost (the right hand side of 8). This gives rise to excessive risk-taking.

In order to find the rate of risk-sharing α that prevails at the second best, we turn to the first stage of the game. In stage 1, farmers cooperatively set the level of risk-sharing itself, anticipating that the risk-taking profile will be non-cooperatively determined in stage 2. As stated in the following proposition, under moral hazard, risk-sharing is incomplete.

Proposition 2 Second best: *In the absence of formal insurance markets and if the risk-taking profile Σ is unenforceable by the group, then risk-sharing is incomplete $\alpha^{SB} < 1$ and the level of risk-taking is homogeneous $\Sigma^{SB} = (\sigma^{SB}, \dots, \sigma^{SB})$, where $\sigma^{SB} = \sigma^N(\alpha^{SB})$, which satisfies*

$$\mu'(\sigma^{SB}) - \eta \sigma^{SB} \left[(1-b) + \left[(1-\alpha^{SB}) + \alpha^{SB} \frac{1}{n} \right]^2 b \right] = 0.$$

Proof. The optimization program is

$$Max_{\alpha} W = \sum_{i \in N} u(\tilde{c}_i(\alpha, \Sigma^N(\alpha))) = nu(\tilde{c}(\alpha, \sigma^N(\alpha))),$$

by Lemma 1. The first order condition of this problem has the following form:

$$\frac{dW(\alpha, \Sigma^N(\alpha))}{d\alpha} = 0 \iff \frac{\partial \tilde{c}}{\partial \alpha} + \frac{\partial \tilde{c}(\sigma^N)}{\partial \sigma} \frac{\partial \sigma^N}{\partial \alpha} = 0, \quad (11)$$

where

$$\begin{aligned}\frac{\partial \tilde{c}}{\partial \alpha} &= \eta \sigma^{N2} (1 - \alpha) \frac{n-1}{n} b > 0 \iff \alpha < 1, \\ \frac{\partial \tilde{c}(\sigma^N)}{\partial \sigma} &= -\eta \sigma^N \alpha^2 \frac{n-1}{n^2} b < 0.\end{aligned}$$

Those expressions are obtained by using equations (2), (5) and (6). Since we evaluate $\partial \tilde{c} / \partial \sigma$ at $\sigma = \sigma^N(\alpha)$, we can use condition (10), which leads to the above expression. Finally, applying the implicit function theorem on equation (10), we find that

$$\frac{\partial \sigma^N}{\partial \alpha} = -\frac{2\eta \sigma^N [(1 - \alpha) + \alpha \frac{1}{n}] \frac{n-1}{n} b}{\mu''(\sigma) - \eta [(1 - b) + [(1 - \alpha) + \alpha \frac{1}{n}]^2 b]} > 0.$$

The second term on the right hand side of (11) is therefore strictly negative. This implies that the first term must be strictly positive for (11) to be satisfied, which is only possible if α is strictly lower than one.

Σ^{SB} is then found by applying Lemma 1. ■

Proposition 1 tells us that, at the second best, the rate of risk-sharing is lower than one. As highlighted in Lemma 1, for any given rate of risk-sharing, moral hazard generates excessive risk-taking. The main intuition behind the result of Proposition 1 is that incomplete risk-sharing mitigates moral hazard. More precisely, the optimal rate of risk-sharing solves the tradeoff between a reduction in idiosyncratic risk and the resulting increase in risk externalities that excessive risk-taking generates. This is what condition (11) reveals: the first term on the right hand side gives the direct partial effect of risk-sharing on \tilde{c} and is positive as long as α is lower than one. In other words, holding risk-taking behaviors constant, the expected marginal utility of risk-sharing is positive as it reduces idiosyncratic risk. However, the non-cooperative level of risk-taking increases with α . Because of the moral hazard issue, risk-taking is always excessive. This indirect partial effect of risk-sharing is then negative. A parallel can be drawn between our setup and the Arnott & Stiglitz (1991)'s model. Indeed, in both cases, the equilibrium level of insurance is inferior to the first best level because of the presence of moral hazard. In Arnott & Stiglitz (1991)'s model, the partial insurance takes the form of quantity rationing on the formal market. In both cases, the partial insurance provides the agents with minimal incentives to behave cautiously.

A last point pertaining to the equilibrium level of risk-taking deserves a brief discussion here. An argument based on excessive risk-taking might appear at odds with empirical observations, as farmers in developing countries do not seem to take excessive levels of risk. In fact, the level of risk-taking has to be evaluated while allowing for the rate of insurance coverage. In other words, $\sigma^N(\alpha^{SB})$ may be low in absolute value, namely lower its first best level $\sigma^{FB} = \sigma^C(1)$, and at the same time inefficiently high given the insurance coverage offered by α^{SB} . On the one hand, the IRSA does not provide insurance against covariate risk, on the other hand, $\alpha^{SB} < 1$ under moral hazard. The global level of insurance coverage is therefore relatively low. This allows us to argue that our result is compatible with the observation that farmers in poor countries tend to adopt low levels of risk in absolute terms.

4 Index insurance: Formal insurance against covariate risk

This section describes the index-insurance contract. We adopt a very stylized contract, with the intention to capture the main purpose of index insurance, which is to offer coverage against covariate risk. We argue that

the interaction between index-insurance and informal risk-sharing highlighted in this paper entirely relies on the ability of formal insurance to reduce covariate risk, not on the particular form taken by the contract. We therefore define the simplest and most efficient way of reducing the covariate variance. More precisely, we make the two following simplifications.

4.1 The index

First, we assume that the formal insurance provider can observe the realization of θ_g , which will therefore be the index. The main simplification that this assumption implies consists in ruling out basis risk at the group level. In other words, the index perfectly coincides with the realization of the covariate shock of the group. To fix ideas, one can imagine that θ_g is simply rainfall, that rainfall is uniformly distributed across space, at least within the area where the group is located, and that there is a weather station in this area. Due to the existence of idiosyncratic risk, θ_i , there is basis risk at the individual level, since individual yields are only imperfectly correlated with θ_g . However there is no basis risk at the group level.⁹

4.2 The payout function

Second, we assume the payout, or indemnity, function takes the following form:

$$P_i(\theta_g) = -\beta_i \theta_g, \quad (12)$$

where $\beta_i \in R_+$ represents the level of coverage. In the following sections, we will explore two alternative cases: (1) the case of individual subscription, where β is a farmer's choice variable and (2) the case of group subscription, where the decision on β is taken by the group. As shown by equation (12), the payout, or indemnity, P is positive in case of an adverse shock ($\theta_g < 0$) and negative otherwise. Full coverage against covariate risk is then obtained for $\beta_i = \sigma_i$, in which case the payout perfectly offsets the covariate shock $\sigma_i \theta_g$. In addition, we assume that the farmer pays a linear premium, $\pi \beta_i$, in every state of the world. Since the expected payout is zero ($E(P) = 0$), the contract will be actuarially fair if and only if $\pi = 0$. While such a contractual form is the most efficient in terms of variance reduction, it is not observed in practice. Instead, the payout function of index insurance contracts typically takes the following form:

$$P_i(\theta_g) = \begin{cases} \beta_i (\tilde{\theta}_g - \theta_g) & \text{if } \theta_g \leq \tilde{\theta}_g \\ 0 & \text{otherwise,} \end{cases}$$

where $\tilde{\theta}_g$ is a strike point, generally negative, below which an indemnity payment is made. A premium is then paid independently of θ_g . For this scheme, the actuarially fair premium is equal to the expected payout:

$$E(P_i) = \beta_i \int_{-\infty}^{\tilde{\theta}_g} (\tilde{\theta}_g - \theta_g) dG(\theta_g) = \beta_i G(\tilde{\theta}_g) \left[\tilde{\theta}_g - E(\theta_g | \theta_g < \tilde{\theta}_g) \right].$$

⁹ To make our framework closer to reality, we could have assumed the existence of a third source of risk. Let us briefly describe this alternative setting. Suppose that the index is measured at a larger geographical scale. Maintaining the assumption that the index perfectly matches the variable θ_g , we need a new random variable to capture the divergence between the covariate shock of the group and the index. To this end, a random variable θ_c , independent to the $(n + 1)$ others, could have denoted a group specific shock. With our assumption, we then abstract from this third source of risk (basis risk at the group level). Neglecting it does not affect our results. Indeed, perfect income smoothing cannot be achieved, even in our simplified setting. Complete coverage against covariate risk is technically feasible, but the finite size of the group will prevent farmers from being perfectly insured against idiosyncratic risk.

Our specification instead implies that: (1) the strike point, θ_g , is equal to zero; (2) the premium is paid only in cases of positive covariate shocks and; (3) the premium is proportional to the size of the shock (the premium is $\beta_i \theta_g$). Our specification therefore completely smooths income over both negative and positive values of θ_g and is thus more efficient in terms of variance reduction. In practice, however, transaction costs and limited enforceability limit the feasibility of this type of contract. The existence of fixed costs associated with indemnity payments imply that there is an interest to reduce the frequency of payouts by restricting them to cases when they are most valuable to the farmer, namely to relatively large adverse shocks ($\tilde{\theta}_g < 0$). Enforceability problems relate to the timing of the payment of premiums and indemnities. In our scheme, the value of the premium paid by the farmer (cases where $\theta_g > 0$) depends on the realization of the index, and its payment can therefore only take place ex post. While ex ante, before uncertainty is realized, farmers should be willing to pay the premium; ex post, the payment will be subject to commitment problems (ex post moral hazard). With these caveats, we assume that the payout function defined by equation (12) is enforceable so that we can concentrate on the covariate variance reduction that index-insurance may provide.

Taken together, our two assumptions, on the index and the payout function, tend to overestimate the performance of index-insurance as a protection against risk at the group level. The interaction effects highlighted in this paper are therefore, themselves, overestimated as compared to real-life situations.

4.3 Demand for index insurance

Let us now analyze the demand for the index insurance contract described above. First, consider how a farmer's level of consumption is affected by the introduction of this contract. For a given level of coverage $\beta = \tilde{\beta}$ and for a given level of risk-sharing α and risk-taking profile Σ , consumption becomes

$$\begin{aligned} c_i^{II}(\alpha, \tilde{\beta}, \Sigma, S) &= c_i + (P - \pi \tilde{\beta}) \\ &= \mu(\sigma_i) - \pi \tilde{\beta} + (\sigma_i - \tilde{\beta}) \theta_g + \sigma_i (1 - \alpha) \theta_i + \alpha \frac{1}{n} \sum_{j \in N \setminus \{i\}} \sigma_j \theta_j, \end{aligned}$$

where the superscript *II* stands for *Index-Insurance*. Consumption with index insurance then has the following mean and variance:

$$E(c_i^{II}; \alpha, \tilde{\beta}, \sigma_i) = \mu(\sigma_i) - \pi \tilde{\beta}, \quad (13)$$

$$Var(c_i^{II}; \alpha, \tilde{\beta}, \Sigma) = (\sigma_i - \tilde{\beta})^2 (1 - b) + \sigma_i^2 \left[(1 - \alpha) + \alpha \frac{1}{n} \right]^2 b + \left(\alpha \frac{1}{n} \right)^2 \sum_{j \in N \setminus \{i\}} \sigma_j^2 b. \quad (14)$$

In the absence of index-insurance, the covariate variance was simply $\sigma_i^2 (1 - b)$. Equation (14) shows that it becomes $(\sigma_i - \tilde{\beta})^2 (1 - b)$ with index-insurance. Let $\Delta(\sigma_i)$ denote the variance reduction from index insurance. It is straightforward to show that:¹⁰

$$\Delta(\sigma_i) = Var(c_i) - Var(c_i^{II}) > 0 \iff \sigma_i > \frac{\tilde{\beta}}{2}.$$

This condition states that index-insurance reduces the farmer's income variance provided his/her level of risk taking σ_i is sufficiently high. This result is consistent with the work of Miranda (1991) on area-yield crop insurance. To understand this condition, it is useful to re-write equation (14) as

$$Var(c_i^{II}) = Var(c_i) + Var(P) + 2Cov(c_i, P).$$

¹⁰See Appendix 1.

This implies that

$$\Delta(\sigma_i) = -[\text{Var}(P) + 2\text{Cov}(c_i, P)],$$

where¹¹

$$\begin{aligned}\text{Var}(P) &= \tilde{\beta}^2(1-b), \\ \text{Cov}(c_i, P) &= -\sigma_i\tilde{\beta}(1-b).\end{aligned}$$

This decomposition of $\Delta(\sigma_i)$ highlights that the payout is itself stochastic with a variance of $\tilde{\beta}^2(1-b)$. As a result, index-insurance is an additional source of risk. This risk is useful as it countervails the variability of income. The covariance between the farmer's income c_i (income in the absence of the index insurance payout) and the payout is indeed negative. The benefit derived from this risk countervailing effect is higher than the cost imposed by the additional risk only provided σ_i is sufficiently high. It can be seen that the covariance is linearly increasing in σ_i . The demand for index-insurance will therefore be increasing in the farmer's own level of risk taking, σ_i .

The following proposition describes the demand for index insurance in the case of individual subscription. Let σ_{II} denote the equilibrium level of risk-taking when demand for index-insurance is interior.

Lemma 2 *Under individual subscription, the demand for index insurance is given by:*

$$\beta_i^* = \text{Max}\{0, \beta_i^o(\sigma_i)\}, \forall i \in N, \quad (15)$$

with

$$\beta_i^o = \sigma_{II} - \frac{\pi}{\eta(1-b)}, \quad (16)$$

where σ_{II} is the equilibrium level of risk-taking when demand for index-insurance is interior.

1. Under cooperative risk-taking, $\sigma_{II} = \sigma_{II}^C(\alpha)$, which is given by

$$\mu'(\sigma_{II}^C) - \pi = \eta\sigma_{II}^C \left[\left[(1-\alpha) + \alpha\frac{1}{n} \right]^2 + \left(\alpha\frac{1}{n} \right)^2 (n-1) \right] b. \quad (17)$$

2. Under non-cooperative risk-taking, $\sigma_{II} = \sigma_{II}^N(\alpha)$, which is given by

$$\mu'(\sigma_{II}^N) - \pi = \sigma_{II}^N \eta \left[(1-\alpha) + \alpha\frac{1}{n} \right]^2 b. \quad (18)$$

Proof. Provided in Appendix 2. ■

Proposition 3 *Under individual subscription, the demand for index insurance, β_i^* , is:*

1. decreasing in the premium π : $d\beta_i^*/d\pi \leq 0$,
2. decreasing in the fraction of idiosyncratic risk over total risk b : $d\beta_i^*/db \leq 0$,
3. hump-shaped in the coefficient of absolute risk-aversion η , with

$$\lim_{\eta \rightarrow 0^+} \beta_i^*(\eta) = \lim_{\eta \rightarrow +\infty} \beta_i^*(\eta) = 0, \forall \pi > 0.$$

¹¹See Appendix 1.

Proof. Those results derive from Lemma 2 and the application of the implicit function theorem on equations (17) and (18). ■

As expected, demand for index-insurance decreases with the premium π charged by the provider. Also, consistently with the work of Clarke (2011): (1) demand decreases with basis risk b . The latter parameter describes indeed the fraction of idiosyncratic risk over total risk, which the index is unable to capture. (2) Demand is hump-shaped in the coefficient of absolute risk aversion η . Moreover, we observe that risk-neutral and infinitely risk-averse agents do not subscribe to index-insurance. The intuition behind this result is given by the two following effects of risk-aversion: When subscription is interior,

$$\frac{d\beta^*}{d\eta} = \frac{\partial\beta^*}{\partial\eta} + \frac{\partial\beta^*}{\partial\sigma} \frac{\partial\sigma_{II}}{\partial\eta},$$

where

$$\frac{\partial\beta^*}{\partial\eta} > 0; \frac{\partial\beta^*}{\partial\sigma} = 1; \frac{\partial\sigma_{II}}{\partial\eta} < 0.$$

On one hand, the direct partial effect of risk-aversion on demand for index-insurance is positive, as intuition would suggest. Indeed, other things being equal, willingness to pay for insurance increases with risk-aversion. On the other hand, as noted above (see also Miranda (1991)), the benefit of index-insurance is increasing in the agent's level of risk-taking σ . Since risk-taking is itself decreasing in risk-aversion, infinitely risk-averse agents opt for $\sigma = 0$ and do not subscribe. With exogenous risk-taking, demand would have been monotonically increasing in risk-aversion, which would have violated the observation that adoption of index-insurance is hump-shaped in absolute risk aversion. This shows the importance of considering risk-taking as endogenous.

5 The impact of index-insurance on informal risk-sharing

We are now set to explore the effects that the introduction of a formal index-insurance contract might have on a pre-existing risk-sharing network. Our focus will be twofold. First, we analyze whether index-insurance has the potential to either crowd in or crowd out informal risk-sharing. In terms of our framework, where the composition of the informal risk sharing group is exogenous, we will focus on the potential impact of the introduction of index insurance on the equilibrium rate of risk-sharing, α . Second, we examine welfare implications.

5.1 The impact of index-insurance on first best informal risk-sharing

Before turning to the case of moral hazard, we present here the impact of index-insurance on the first best IRSA as a benchmark. The following proposition characterizes the impact of index-insurance on risk-sharing, risk-taking and welfare in the absence of moral hazard. The initial situation (absence of index insurance) is therefore given by Proposition 1. In addition, the social planner is assumed to be able to enforce individual demand for index-insurance β_i . The first best level of risk-sharing and risk-taking when index-insurance is available to farmers are denoted by α_{II}^{FB} and σ_{II}^{FB} , respectively.

Proposition 4 *The impact of index-insurance on first best IRSA: If risk-taking σ is enforceable by the group, then:*

1. The rate of risk-sharing is unaffected by the presence of index-insurance: $\alpha_{II}^{FB} = \alpha^{FB} = 1$.

2. Individual demand for index-insurance is homogeneous $\beta_i^{FB} = \beta^{FB}, \forall i \in N$, with

(a) If the premium π is relatively high: $\pi > \sigma^{FB}\eta(1-b)$, where σ^{FB} is defined in Proposition 1, then demand for index-insurance is zero $\beta^{FB} = 0$ and the initial level of risk-taking remains unchanged $\sigma_{II}^{FB} = \sigma^{FB}$.

(b) If the premium π is relatively low: $\pi \leq \sigma^{FB}\eta(1-b)$, then demand for index-insurance is interior $\beta^{FB} = \beta^o$ and the equilibrium level of risk-taking is $\sigma_{II}^{FB} = \sigma_{II}^C(1)$, which is defined by equation (17) with $\alpha = 1$. Moreover, risk-taking is higher when the premium decreases: $\partial\sigma_{II}^{FB}/\partial\pi < 0$.

3. Following the introduction of index-insurance, welfare increases. Farmers' level of expected utility is a decreasing function of the premium: $du\left(\tilde{c}^{II}\left(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB}\right)\right)/d\pi < 0$.

Proof. Provided in Appendix 3. ■

At the first best IRSA, risk-sharing is complete ($\alpha^{FB} = 1$). This proposition first points out that this remains true with index-insurance. Moral hazard is therefore the source of the crowding out effect that we highlight in the case of individual subscription.

Second, we see that, in the absence of moral hazard, index-insurance cannot reduce welfare. Moreover, the impact on welfare is strictly positive, provided the insurance premium is low enough. This benchmark case can be directly related to Arnott & Stiglitz (1991)'s paper, where a formal insurance market can be supplemented by informal risk-sharing. They show that if agents can perfectly monitor each other to enforce the cooperative level of risk-taking, then informal risk-sharing improves welfare. This statement simply needs to be reversed in this paper; namely, in our case, it is the introduction of formal insurance that increases welfare if informal risk-sharing is not impeded by moral hazard.

Finally, it should be noted that the first best can be achieved even if β is unenforceable. In other words, as soon as the risk-taking profile Σ is enforceable by the group, non-cooperative subscription to index-insurance does not entail efficiency losses.¹²

The remainder of this section is devoted to the case of moral hazard under individual versus group contracts. The following subsection tackles the case of individual subscription to index-insurance, while the subsequent one explores the case of group subscription.

5.2 The impact of index-insurance on second best informal risk-sharing: The case of individual subscription

We now analyze the impact of the introduction of index-insurance under moral hazard. Assume then that the risk-taking profile Σ is unenforceable by the group and that an index-insurance contract is offered to farmers on an individual basis. We assume that all farmers in the group have access to the index-insurance contract. In order to highlight the main effect of this model, we assume from now on that $\mu'''(\sigma) = 0$.¹³

Since we are dealing with the moral hazard case, the initial equilibrium, in the absence of index insurance, is described by Proposition 2. Recall, in particular, that equilibrium risk-sharing is, in this case, incomplete ($\alpha^{SB} < 1$). The timing of the game is as follows:

¹²As shown in Appendix 3, the planner solution is identical to the individual decision on β as described by Lemma 2.

¹³Assuming that $\mu'''(\sigma) = 0$ amounts to making a second order approximation of the relationship between risk and return. This allows to get rid of higher order effects and to focus on the main result of this paper. The interested reader can refer to Appendix 5, where this assumption is relaxed.

1. The group members agree on a rate of risk-sharing α .
2. Each member simultaneously and non-cooperatively chooses his/her risk-taking level, σ_i , and his/her level of index-insurance coverage, β_i .

We denote by α_{II}^{SB} the second best level of risk-sharing when index-insurance is available to farmers. The following proposition describes the effect of index-insurance on risk-sharing under moral hazard.

Proposition 5 *The impact of index-insurance on second best IRSA under individual subscription: Crowding out.* *If risk-taking σ is unenforceable by the group, then index-insurance crowds out informal risk-sharing. Three cases need to be distinguished depending on the value of the premium π :*

1. *Case 1: $\pi \in [\tilde{\pi}_2, +\infty)$: In this case, risk-sharing is unaffected by the availability of index-insurance $\alpha_{II}^{SB} = \alpha^{SB}$ and the demand for index-insurance is equal to zero: $\beta_i^* = 0, \forall i \in N$,*
2. *Case 2: $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$: In this case, the level of risk-sharing decreases as compared to the case where index-insurance is unavailable: $\alpha_{II}^{SB} = \tilde{\alpha}(\pi)$, such that*

$$\tilde{\alpha}(\tilde{\pi}_1) = \alpha_{II}^*; \tilde{\alpha}(\tilde{\pi}_2) = \alpha^{SB}; \frac{\partial \tilde{\alpha}}{\partial \pi} \geq 0;$$

and the demand for index-insurance is equal to zero: $\beta_i^ = 0, \forall i \in N$,*

3. *Case 3: $\pi \in [0, \tilde{\pi}_1)$: In this case, the level of risk-sharing decreases as compared to the case where index-insurance is unavailable: $\alpha_{II}^{SB} = \alpha_{II}^* < \alpha^{SB}$, with $\partial \alpha_{II}^* / \partial \pi = 0$ and the demand for index-insurance is interior: $\beta_i^* = \beta^\circ(\sigma_{II}^N(\alpha_{II}^*)) > 0, \forall i \in N$,*

where

$$\begin{aligned} \tilde{\pi}_1 &= \sigma_{II}^N(\alpha_{II}^*) \eta(1-b), \\ &< \tilde{\pi}_2 = \sigma^N(\alpha^{SB}) \eta(1-b), \end{aligned}$$

and where α_{II}^* solves

$$(1 - \alpha_{II}^*) + 2\alpha_{II}^{*2} \frac{n-1}{n^2} \frac{\eta \left[(1 - \alpha_{II}^*) + \alpha_{II}^* \frac{1}{n} \right] b^2}{\mu'' - \eta \left[(1 - \alpha_{II}^*) + \alpha_{II}^* \frac{1}{n} \right]^2 b} = 0.$$

Proof. Provided in Appendix 4. ■

This proposition states that the rate of informal risk-sharing is negatively impacted by the introduction of an index-insurance contract if the latter is offered at the individual level. The extent of the crowding out effect depends on the value of the index-insurance premium π and hence on the resulting level of subscription adopted by the group members.

Proposition 5 highlights three potential situations. We start by providing intuitions for cases 1 and 3, where the crowding out effect is either absent (1) or complete (3) and turn to the intermediate case (2) afterwards.

Case 1 pertains to a range of prices at which the rate of informal risk-sharing is unaffected and where there is no demand for index-insurance. The intuition behind this case is simply that the availability of index-insurance at prohibitive prices is conceptually equivalent to the absence of index-insurance. The second best allocation without index-insurance therefore prevails in this case.

Case 3 is of particular interest as it describes situations where informal risk-sharing coexists with a positive subscription to formal index-insurance. Under moral hazard, we show that when subscription to index-insurance becomes positive, risk-sharing necessarily drops to a rate of α_{II}^* , which is strictly below the initial rate of α^{SB} . At first sight, the interaction between formal and informal insurance is unclear. Indeed, index-insurance offers protection against covariate risk, while informal risk-sharing only pertains to idiosyncratic risk. As covariate and idiosyncratic risks are independent, adverse effects of index-insurance on IRSA's are unexpected. The mechanism proceeds as follows. If the index-insurance coverage is freely chosen by farmers, then they can adapt it to their risk-taking level. Recall that the marginal cost of risk-taking consists of the increase in covariate and in idiosyncratic risks. Lemma 2 shows that the demand for index-insurance β increases linearly with σ . More precisely, when σ increases by one unit, β also increases by one unit.¹⁴ It follows that, at the margin, farmers are fully covered against covariate risk, even when subscription is incomplete ($\beta^* < \sigma_{II}^N(\alpha_{II}^*)$). In other words, the fraction of the marginal cost of risk-taking pertaining to covariate risk vanishes. This has an influence not only on the equilibrium level of risk-taking but also on the responsiveness of risk-taking to risk-sharing, which is at the heart of the moral hazard problem. Appendix 4 shows that risk-taking is more responsive to risk-sharing when the demand for index-insurance is interior: $\partial\sigma_{II}^N/\partial\alpha > \partial\sigma^N/\partial\alpha$. The optimality condition for risk-sharing has the following form (see Appendix 4):¹⁵

$$\frac{d\tilde{c}^{II}}{d\alpha} = \frac{\partial\tilde{c}^{II}}{\partial\alpha} + \frac{\partial\tilde{c}^{II}}{\partial\sigma} \frac{\partial\sigma^*}{\partial\alpha} = 0.$$

This condition states that the marginal benefit of risk-sharing ($\partial\tilde{c}^{II}/\partial\alpha$) should offset its marginal cost which consists of an increase in the mutual externalities of risk. The externalities are of a larger extent because a higher rate of risk-sharing induces agents to take more risk ($\partial\sigma^*/\partial\alpha > 0$). Fundamentally, the presence of index-insurance increases farmers' responsiveness to risk-sharing, thereby increasing the marginal cost of risk-sharing at the group level. The moral hazard issue is more severe and the second best rate of risk-sharing is consequently lower. It is important to note that this crowding out effect is totally independent of the extent of index insurance coverage. Put differently, index-insurance crowds out informal risk-sharing, even if β^* is arbitrarily close to zero. The reduction in α is indeed not due to a higher level of risk-taking. Instead, effects on incentives are acting at the margin. As attested by the following proposition, risk-taking might even decrease after the introduction of index-insurance. It should also be noted that, if π is strictly lower than $\tilde{\pi}_2$, then the index coverage is positive but close to zero and α falls. The overall level of protection against risk is, in this case, unambiguously lower. Implications for welfare are analyzed below.

Finally Case 2 is remarkable and deserves further explanations. If the value of π is intermediate, namely if it belongs to the interval $[\tilde{\pi}_1, \tilde{\pi}_2)$, then the availability of index-insurance causes a reduction in informal risk-sharing ($\tilde{\alpha} < \alpha^{SB}$), even though β is equal to zero. We can provide an intuition for this case as follows. Recall that α is determined cooperatively at the first stage of the game. The group therefore chooses α so as to maximize social welfare. Suppose that α is initially equal to its second best level without index-insurance α^{SB} . Suppose further that index-insurance is offered to farmers at a price $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$. If α remained unchanged, then farmers' subscription to index-insurance would be interior since $\tilde{\pi}_2$ is the farmers'

¹⁴This linearity is due to our assumption of constant absolute risk-aversion. Relaxing this assumption would complexify the relationship between σ and β , but would not alter our results. Indeed, even if $\partial\beta^*/\partial\sigma < 1$, it would still be positive (higher risk-taking should always entail higher subscription to index-insurance). It follows that the covariate part of the marginal cost of risk-taking is reduced. Risk-taking is then more responsive to risk-sharing and the moral hazard problem worsens.

¹⁵The function $\sigma^*(\alpha)$ is formally defined in Appendix 4. Fundamentally, $\sigma^*(\alpha)$ is given by $\sigma^N(\alpha)$, when β is at a corner and by $\sigma_{II}^N(\alpha)$ when β is interior.

willingness to pay for the first unit of β when $\alpha = \alpha^{SB}$. However, this situation would not be optimal. Indeed, as we saw in case 3, the group should reduce its level of risk-sharing when β is interior. If it did so and set it equal to $\alpha_{II}^* < \alpha^{SB}$, then β would be at a corner because the farmers' willingness to pay for the first unit of β when $\alpha = \alpha_{II}^*$ is only $\tilde{\pi}_1$, the reason being that the equilibrium level of risk-taking is lower under $\alpha = \alpha_{II}^*$ than under $\alpha = \alpha^{SB}$. This situation would not be optimal either because α would be too low for a situation where β equals zero. The group therefore adopts the level of risk-sharing such that β is interior, but just equal to zero.

We now turn to the analysis of the impact of index-insurance on risk-taking.

Proposition 6 *The impact of index-insurance on risk-taking under individual subscription: If risk-taking σ is unenforceable by the group, three cases may occur depending on the value of the premium π .*

1. *Case 1: $\pi \in [\tilde{\pi}_2, +\infty)$: In this case, risk-taking is unaffected by the availability of index-insurance: $\sigma_{II}^{SB} = \sigma^{SB}$ and $\partial\sigma_{II}^{SB}/\partial\pi = 0$.*
2. *Case 2: $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$: In this case, the level of risk-taking decreases as compared to the case where index-insurance is unavailable: $\sigma_{II}^{SB} = \sigma_{II}^N(\pi, \tilde{\alpha}(\pi)) < \sigma^{SB}$, where the function $\sigma_{II}^N(\cdot)$ is defined by equation (18) and $\tilde{\alpha}(\pi)$ is given in Proposition 5 and $d\sigma_{II}^{SB}/d\pi \geq 0$.*
3. *Case 3: $\pi \in [0, \tilde{\pi}_1)$: In this case, $\sigma_{II}^{SB} = \sigma_{II}^N(\pi, \alpha_{II}^*)$ and $d\sigma_{II}^{SB}/d\pi < 0$.*
4. *Therefore, σ_{II}^{SB} reaches a minimum at $\pi = \tilde{\pi}_1$.*

Proof. In case 1, Proposition 5 tells us that β is at a corner. The second best allocation is then unaffected by index-insurance.

In case 2, we know by Proposition 5 that $\beta = 0$ and that α_{II}^{SB} is lower than its second best level without index-insurance α^{SB} . Risk-taking increases with both α and β , which provide insurance and decrease the marginal cost of risk-sharing. Therefore, in this case, $\sigma_{II}^{SB} < \sigma^{SB}$. The impact of π on the equilibrium level of risk-taking writes

$$\frac{d\sigma_{II}^{SB}}{d\pi} = \frac{d\sigma_{II}^N(\alpha_{II}^{SB}, \beta^*)}{d\pi} = \frac{\partial\sigma_{II}^N}{\partial\alpha} \frac{\partial\alpha_{II}^{SB}}{\partial\pi} + \frac{\partial\sigma_{II}^N}{\partial\beta} \frac{\partial\beta^*}{\partial\pi},$$

where the function $\sigma_{II}^N(\alpha_{II}^{SB}, \beta^*)$ is given by equation (24) (see Appendix 2). As shown in Proposition 5, α is chosen by the group such that $\beta^* = 0$ for all $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$. Hence $\partial\beta^*/\partial\pi = 0$ if $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$. Besides, in this case, $\partial\alpha_{II}^{SB}/\partial\pi \geq 0$. Therefore, $d\sigma_{II}^{SB}/d\pi \geq 0$.

In case 3, on the one hand, the rate of risk-sharing remains constant over the range $[0, \tilde{\pi}_1)$, hence $\partial\alpha_{II}^{SB}/\partial\pi = 0$. On the other hand, β is interior and $\partial\beta^*/\partial\pi < 0$ (see equation 16). As a result, $d\sigma_{II}^{SB}/d\pi < 0$.

Finally, point 4 is a direct corollary of points 2 and 3. ■

Proposition 6 tells us that the equilibrium level of risk-taking reacts non-monotonically to the value of the premium π . If index-insurance is prohibitively costly, namely when $\pi \in [\tilde{\pi}_2, +\infty)$, risk-taking is unaffected by the availability of the formal contract. When π is lower than $\tilde{\pi}_2$, two types of effects are at play. On the one hand, the crowding out effect reduces the rate of risk-sharing, thereby decreasing the level of risk taken by farmers. On the other hand, when $\pi \in [0, \tilde{\pi}_1)$, subscription to index-insurance becomes interior and starts increasing as π decreases. This provides farmers with coverage against covariate shocks and leads them to increase risk-taking. The main lesson to be drawn from this proposition is that risk-taking reaches a minimum precisely at the point where β becomes interior. Therefore, if index-insurance is offered to farmers

with the aim of increasing their expected profits by stimulating risk-taking, attention has to be paid to the pricing policy: If offered on an individual basis, index-insurance can only stimulate risk-taking if subscription is large enough so as to compensate for the crowding out effect.

Welfare effects follow the same logic as the next proposition shows.

Proposition 7 *The impact of index-insurance on welfare under individual subscription: If risk-taking σ is unenforceable by the group, then the introduction of index-insurance decreases welfare if the premium π is too high:*

$$u(\tilde{c}^{II}(\alpha_{II}^{SB}, \sigma_{II}^{SB}, \beta^*)) < u(\tilde{c}(\alpha^{SB}, \sigma^{SB})) \iff \pi \in [\tilde{\pi}_0, \tilde{\pi}_2),$$

where $\tilde{\pi}_0 < \tilde{\pi}_1$.

Proof. Provided in Appendix 6. ■

The introduction of index-insurance might reduce welfare. We highlight two simple conditions for this unintended outcome to occur. First, index insurance must be offered on an individual basis so that individual farmers, instead of the group, decide on the level of coverage. Second, the premium must be sufficiently high (i.e., actuarially unfair). To illustrate this point, consider the situation in which the index coverage β^* is initially interior, but just equal to zero ($\pi = \tilde{\pi}_1$). We then show that a marginal decrease in π reduces expected utility. As the following equation illustrates, the marginal impact of π on expected utility is twofold:

$$\frac{d\tilde{c}^{II}}{d\pi} = \frac{\partial\tilde{c}^{II}}{\partial\pi} + \frac{\partial\tilde{c}^{II}}{\partial\sigma} \frac{\partial\sigma_{II}^{SB}}{\partial\pi},$$

where

$$\frac{\partial\tilde{c}^{II}}{\partial\pi} = -\beta^*; \frac{\partial\tilde{c}^{II}}{\partial\sigma} < 0; \frac{\partial\sigma_{II}^{SB}}{\partial\pi} < 0.$$

On the one hand, the direct partial effect of π on \tilde{c}^{II} is negative (positive) and simply corresponds to the increase (decrease) in the price paid for each unit of coverage. On the other hand, the equilibrium level of risk-taking is a decreasing function of the premium π .¹⁶ Also, because of moral hazard, we know that the equilibrium level of risk-taking is excessive: $\partial\tilde{c}^{II}/\partial\sigma < 0$. If π decreases at the margin, then the direct effect on welfare is positive and the indirect effect (through σ_{II}^{SB}) negative. One can immediately see that the former effect will be dominated by the latter if β^* is small, namely when π is only slightly lower than $\tilde{\pi}_1$.

5.3 The impact of index-insurance on second best informal risk-sharing: The case of group subscription

Finally, we turn to the case of group subscription. Suppose that the index-insurance contract is offered to the group. The timing of the game has to be modified as follows:

1. The group members agree on a rate of risk-sharing α and on a coverage level β .
2. They simultaneously and non-cooperatively decide on their risk-taking level σ_i .

Let the superscript G stand for group subscription.

¹⁶By the envelop theorem, there is no indirect effect through a change in the optimal coverage β^* .

Proposition 8 *The impact of index-insurance on second best IRSA under group subscription. If risk-taking σ is unenforceable by the group, then*

1. *Unless offered at actuarially favorable prices, group subscription to index-insurance is always incomplete in equilibrium:*

$$\beta^G \in \left[0, \sigma^N \left(\alpha^G, \beta^G\right)\right], \forall \pi \geq 0.$$

2. *The introduction of index-insurance is always welfare improving under group subscription.*

Proof. Provided in Appendix 7. ■

Subscription by the group is always incomplete but also always welfare enhancing. The intuition behind those two key results proceeds as follows: in the case of group subscription, the moral hazard effect of index-insurance is internalized by the group. In order to mitigate moral hazard, the group adopts a lower coverage level as compared to what an individual would have chosen, for any given level of the premium. In particular, if the contract is actuarially fair, group subscription remains incomplete. This result may contribute to explaining the low take-ups that are generally observed empirically. With group subscription, index-insurance is welfare-enhancing, whatever the level of the premium. To understand the latter result, one should simply notice that the coverage level is chosen by the group in stage 1 so as to maximize social welfare. As explained above, by opting for incomplete subscription, the group takes into account the moral hazard effect of insurance. The optimization on β follows therefore the same logic than the choice of α and both the index-insurance coverage and the rate of informal risk-sharing are incomplete and affected by the same tradeoff between the provision of insurance coverage and the production of reciprocal risk externalities through the IRSA.

6 Concluding Remarks

Farmers in developing countries are exposed to a series of shocks of different natures. With missing credit and insurance markets, rural households mainly rely on informal risk-sharing arrangements (IRSA's) to smooth consumption. The ability of IRSA's to mitigate risk is limited, however, by two constraints. First, IRSA's are themselves subject to important information and commitment constraints that reduce their capacity to mitigate idiosyncratic risk. Second, they do not offer any protection against common, or covariate, shocks. In this context, the development of index-insurance programs, whose primary target is precisely the types of covariate risks originating in certain types of weather shocks such as droughts, is held up as a highly promising means of enhancing farmers' welfare. In addition, since the types of shocks that IRSA's and index-insurance are designed to protect against are orthogonal, we might expect minimal interaction between the two types of insurance schemes.

We have shown, however, that if informal risk-sharing suffers from moral hazard, then the introduction of index-insurance contracts at the individual level may indeed introduce an interaction between the two schemes that results in the crowding out of informal risk-sharing. The reason is that index-insurance reduces the marginal cost of risk-taking, and thereby exacerbates the moral hazard problem faced by the IRSA. In response, the group will endogenously reduce the amount of idiosyncratic risk sharing and, for certain ranges of the index insurance premium, this may actually reduce farmer welfare relative to a no-insurance world. This moral hazard effect will be offset if index-insurance proves sufficiently beneficial in terms of coverage.

This will be the case if the premium charged by the provider is sufficiently low. Fortunately, adverse effects on welfare are not entirely robust to changes in the contractual form. In particular, if index-insurance is offered at the group level, then its impact on risk-taking is properly internalized by the group and welfare effects are always positive.

Finally, we identify limitations of and potential extension to this model. First, the crowding out result depends critically on the presence of moral hazard in the IRSA. If the imperfection instead originates from commitment constraints, contrasting predictions can be derived. For example, suppose that risk-taking is contractible (no moral hazard) but that informal insurance transfers are subject to ex-post commitment constraints. Suppose also that the group can sanction defaulters by excluding them from future participation in the IRSA. In this context, group members evaluate the cost of the insurance transfer they are supposed to make against the benefit of continued group membership. In the case of a cooperative, this benefit not only includes informal insurance but also other benefits associated with membership such as access to credit and preferential input and output prices. Assume now that index-insurance is offered at the group level. In such a context, index-insurance may crowd in informal risk-sharing as the value of group membership is enhanced, which tends to relax the incentive compatibility condition and may increase the rate of informal risk-sharing. However, risk-taking seems difficult to enforce in practice and moral hazard may still prove relevant, even if combined with limited commitment in reality.

Second, the moral hazard effect of index-insurance that our model highlights is actually attributable to the fact that agents are allowed to choose their subscription level at the margin. This is indeed what generates the effect on the marginal cost of risk-taking. Therefore, a take it or leave it insurance offer would solve the issue. However, information asymmetries may prevent policy-makers from finding the appropriate rate of coverage. The solution would then be to delegate this task to agents that are better informed, such as the group itself. This observation offers another interpretation of our recommendation that group subscription should be favored.

Third and finally, an important limitation of our framework is that only homogeneous groups are considered. While we do not expect important changes to occur in the case of individual subscription, the impact of group heterogeneity under group subscription deserves attention. Indeed, if a single subscription level is adopted at the group level, it is unlikely to be optimal for every member. Welfare effects should then be carefully analyzed. A final issue that our paper does not address is: what if the two groups, namely the informal risk-sharing network and the set of agents to whom index insurance is offered, do not coincide? Those two considerations remain for future research.

References

- Arnott, R. & Stiglitz, J. E. (1991). Moral hazard and nonmarket institutions: Dysfunctional crowding out or peer monitoring? *American Economic Review*, 81(1), 179–90.
- Attanasio, O. & Rios-Rull, J.-V. (2000). Consumption smoothing in island economies: Can public insurance reduce welfare? *European Economic Review*, 44(7), 1225–1258.
- Barnett, B. J., Barrett, C. B., & Skees, J. R. (2008). Poverty traps and index-based risk transfer products. *World Development*, 36(10), 1766–1785.

- Clarke, D. J. (2011). *A Theory of Rational Demand for Index Insurance*. Economics Series Working Papers 572, University of Oxford, Department of Economics.
- de Janvry, A., Dequiedt, V., & Sadoulet, E. (2014). The demand for insurance against common shocks. *Journal of Development Economics*, 106(C), 227–238.
- De Weerd, J. & Dercon, S. (2006). Risk-sharing networks and insurance against illness. *Journal of Development Economics*, 81(2), 337–356.
- Delpierre, M., Verheyden, B., & Weynants, S. (2014). *Do the poor benefit less from informal risk-sharing? Risk externalities and moral hazard in decentralized insurance arrangements*. CEPS/INSTEAD Working Paper Series 2014-08, CEPS/INSTEAD.
- Dercon, S., De Weerd, J., Bold, T., & Pankhurst, A. (2006). Group-based funeral insurance in ethiopia and tanzania. *World Development*, 34(4), 685–703.
- Dercon, S., Hill, R. V., Clarke, D., Outes-Leon, I., & Seyoum Taffesse, A. (2014). Offering rainfall insurance to informal insurance groups: Evidence from a field experiment in ethiopia. *Journal of Development Economics*, 106(C), 132–143.
- Hoogeveen, H. (2002). Evidence on informal insurance in rural zimabwe. *Journal of African Economies*, 11(2), 249–278.
- Jalan, J. & Ravallion, M. (1999). Are the poor less well insured? evidence on vulnerability to income risk in rural china. *Journal of Development Economics*, 58, 61–81.
- Ligon, E., Thomas, J. P., & Worrall, T. (2002). Informal insurance arrangements with limited commitment: Theory and evidence from village economies. *Review of Economic Studies*, 69(1), 209–44.
- Miranda, M. J. (1991). Area-yield crop insurance reconsidered. *American Journal of Agricultural Economics*, 73(2), 233–242.
- Miranda, M. J. & Farrin, K. (2012). Index insurance for developing countries. *Applied Economic Perspectives and Policy*, 34(3), 391–427.
- Mobarak, A. M. & Rosenzweig, M. (2012). *Selling Formal Insurance to the Informally Insured*. Working Papers 1007, Economic Growth Center, Yale University.
- Morduch, J. (1995). Income smoothing and consumption smoothing. *Journal of Economic Perspectives*, 9(3), 103–114.
- Murgai, R., Winters, P., Sadoulet, E., & de Janvry, A. (2002). Localized and incomplete mutual insurance. *Journal of Development Economics*, 67, 245–274.
- Platteau, J.-P. (1997). Mutual insurance as an elusive concept in traditional rural communities. *Journal of Development Studies*, 33(6), 764–796.
- Rosenzweig, M. R. & Binswanger, H. P. (1993). Wealth, weather risk and the composition and profitability of agricultural investments. *Economic Journal*, 103(416), 56–78.

Townsend, R. M. (1994). Risk and insurance in village india. *Econometrica*, 62(3), 539–91.

Vasilaky, K., Osgood, D., Martinez, S., & Stanimirova, R. (2014). *Informal Network within Index Insurance: Randomizing Distance in Group Insurance*. Technical report, Columbia University.

7 Appendix 1: Index-insurance and the income variance

The consumption variance with index insurance can be written as

$$Var(c_i^{II}) = Var(c_i) + Var(P) + 2Cov(c_i, P).$$

Therefore,

$$\Delta(\sigma_i) = -[Var(P) + 2Cov(c_i, P)]. \quad (19)$$

Let us reproduce here the expression of consumption without index insurance:

$$c_i = \mu(\sigma_i) + \sigma_i\theta_g + \sigma_i(1 - \alpha)\theta_i + \alpha\frac{1}{n}\sum_{j \in N} \sigma_j\theta_j.$$

Since $E(P) = 0$, the covariance between Y_i and the P is equal to $E(Y_i, P)$. Denoting by F_j the distribution function of $\theta_j, \forall j \in N$ and by G the distribution function of θ_g and making use of the expression of the payout (12), one obtains

$$E(c_i, P) = - \int_{R^{n+1}} \tilde{\beta}\theta_g \left[\mu(\sigma_i) + \sigma_i\theta_g + \sigma_i(1 - \alpha)\theta_i + \alpha\frac{1}{n}\sum_{j \in N} \sigma_j\theta_j \right] dF_1(\theta_1) \dots dF_n(\theta_n) dG(\theta_g).$$

By independence between θ_g and the θ_j 's and the fact that $E(\theta_g) = 0$, this expression reduces to

$$E(c_i, P) = -\tilde{\beta}\sigma_i \int_R \theta_g^2 dG(\theta_g).$$

Hence

$$Cov(c_i, P) = -\sigma_i\tilde{\beta}(1 - b). \quad (20)$$

Therefore, substituting for $Var(P) = \tilde{\beta}^2(1 - b)$ and $Cov(c_i, P)$ (equation 20) in (19), we find that $\Delta(\sigma_i) > 0$ if and only if $\sigma_i > \tilde{\beta}/2$.

8 Appendix 2: Proof of Lemma 2

The individual demand for index-insurance can be found by solving the following optimization problem:

$$Max_{\beta} \tilde{c}_i^{II} \approx \mu(\sigma_i) - \pi\beta_i - \frac{\eta}{2}Var(c_i^{II}), \quad (21)$$

where

$$Var(c_i^{II}) = (\sigma_i - \beta_i)^2(1 - b) + \sigma_i^2 \left[(1 - \alpha) + \alpha\frac{1}{n} \right]^2 b + \left(\alpha\frac{1}{n} \right)^2 \sum_{j \in N \setminus \{i\}} \sigma_j^2 b. \quad (22)$$

The first order condition with respect to β writes

$$\frac{\partial \tilde{c}_i^{II}}{\partial \beta_i} \leq 0 \iff -\pi + \eta(\sigma_i - \beta_i^*)(1 - b) \leq 0. \quad (23)$$

Rearranging, we obtain expressions (15) and (16).

We now need to find the level of risk-taking σ_{II} that agents adopt when they benefit from index-insurance under cooperative and non-cooperative risk-taking, respectively.

Under cooperative risk-taking, Proposition 1 applies and the level of risk-taking is homogeneous: $\sigma_i = \sigma, \forall i \in N$. We maximize \tilde{c}_i^{II} (equation 21) with respect to σ . The first order condition imposes that

$$\frac{\partial \tilde{c}_i^{II}}{\partial \sigma} = 0 \iff \mu'(\sigma) - \eta \left[(\sigma - \beta_i)(1 - b) + \sigma \left[\left((1 - \alpha) + \alpha \frac{1}{n} \right)^2 + \left(\alpha \frac{1}{n} \right)^2 (n - 1) \right] b \right] = 0.$$

substituting for β^o (16) leads to equation (17).

Under non-cooperative risk-taking, agents maximize \tilde{c}_i^{II} (equation 21) with respect to their own level of risk-taking σ_i , while considering the others' ($\sigma_j, j \neq i$) as given. Taking the first order condition with respect to σ_i , we find

$$\frac{\partial \tilde{c}_i^{II}}{\partial \sigma_i} = 0 \iff \mu'(\sigma_i) - \eta \left[(\sigma_i - \beta_i)(1 - b) + \sigma_i \left[\left((1 - \alpha) + \alpha \frac{1}{n} \right)^2 b \right] \right] = 0. \quad (24)$$

Substituting for β^o (16), we end up with equation (18).

9 Appendix 3: Proof of Proposition 4

When σ and β are enforceable and because the group is homogeneous, the planner chooses homogeneous levels of risk-taking σ and of index-insurance coverage β and a rate of risk-sharing α so as to maximize $u(\tilde{c}^{II})$ and hence \tilde{c}^{II} (equation 21). As compared to the first best in the absence of index-insurance, the first order condition with respect to α is unchanged and imposes that

$$\frac{\partial Var(\tilde{c}^{II}; \alpha, \sigma, \beta)}{\partial \alpha} = 0 \iff \alpha_{II}^{FB} = 1.$$

The envelope theorem allows us to integrate this result in the objective function before maximizing \tilde{c}^{II} with respect to β and σ :

$$Max_{\sigma, \beta} \tilde{c}^{II}(1, \sigma, \beta) = \mu(\sigma) - \pi\beta - \frac{\eta}{2} \left[(\sigma - \beta)^2 (1 - b) + \sigma^2 \frac{1}{n} \right].$$

The first order condition with respect to β is then identical to condition (23). The index-insurance coverage is therefore interior if and only if

$$\frac{\partial \tilde{c}^{II}(1, \sigma, 0)}{\partial \beta} \geq 0 \iff \pi \leq \tilde{\pi}(\sigma) = \eta\sigma(1 - b), \quad (25)$$

otherwise it is at a corner.

In case of corner index-insurance coverage, the first best level of risk-taking is given by $\sigma^C(1) = \sigma^{FB}$ (See the proof of Proposition 1).

In case of an interior index-insurance coverage, the first best level of risk-taking is equal to $\sigma_{II}^C(1)$ (see Appendix 2). The first best level of index-insurance coverage is then $\beta^o(\sigma_{II}^C(1))$.

Because the threshold value $\tilde{\pi}(\sigma)$ depends itself on risk-taking, we need to show that this threshold is unique. In other words, given the threshold, risk-taking is either σ^{FB} or $\sigma_{II}^C(1)$. For the threshold to be

unique, we need that $\sigma^{FB} = \sigma_{II}^C(1)$ in the neighborhood of $\tilde{\pi}$. Making use of equation (8), $\sigma^{FB} = \sigma^C(1)$ is defined by

$$\mu'(\sigma^C) = \eta\sigma^C \left[(1-b) + \frac{1}{n}b \right]. \quad (26)$$

Making use of equation (17), we have that $\sigma_{II}^C(1)$ satisfies

$$\mu'(\sigma_{II}^C) - \pi = \eta\sigma_{II}^C \frac{1}{n}b. \quad (27)$$

In the neighborhood of the threshold, condition (25) is satisfied with equality. Substituting for $\pi = \tilde{\pi} = \eta\sigma(1-b)$ in the latter equation (27), we find that equations (26) and (27) are identical. There is therefore no discontinuity in risk-taking, so that the threshold is unique.

This lead to the conclusion that if $\pi > \eta\sigma^{FB}(1-b)$, then $\beta = 0$ and $\sigma_{II}^{FB} = \sigma^{FB}$; if $\pi \leq \eta\sigma^{FB}(1-b)$, then $\beta = \beta^o(\sigma_{II}^C(1))$ and $\sigma_{II}^{FB} = \sigma_{II}^C(1)$.

Point 2.b requires us to show that $\partial\sigma_{II}^{FB}/\partial\pi < 0$. To this end, one just needs to apply the implicit function theorem on (27), which directly leads to the result.

Finally, point 3 states that $du(\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB}))/d\pi < 0$. To prove this result, it is useful to write the total effect of π on \tilde{c}^{II} as

$$\frac{d\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{d\pi} = \frac{\partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{\partial\pi} + \frac{\partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{\partial\sigma} \frac{\partial\sigma_{II}^{FB}}{\partial\pi} + \frac{\partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{\partial\beta} \frac{\partial\beta^{FB}}{\partial\pi}.$$

By the envelope theorem, we have that $\partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})/\partial\sigma = \partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})/\partial\beta = 0$. Making use of equations (21) and (22), we obtain

$$\frac{d\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{d\pi} = \frac{\partial\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})}{\partial\pi} = -\sigma_{II}^{FB} + \frac{\pi}{\eta(1-b)}.$$

When demand for index-insurance is interior, we know, by (15), that $\sigma_{II}^{FB} - \pi/[\eta(1-b)] > 0$, which implies that $d\tilde{c}^{II}(\alpha_{II}^{FB}, \sigma_{II}^{FB}, \beta^{FB})/d\pi < 0$.

10 Appendix 4: Proof of Proposition 5

10.1 Solving the game with index-insurance

In order to show the crowding out result, we solve the game in the case where index-insurance is available. The timing is as follows. In stage (1), the rate of risk-sharing α is defined cooperatively at the group level. In stage (2), agents choose simultaneously and non-cooperatively σ and β .

Solving stage 2:

Using backward induction, we solve the second stage for any given level of α . The demand for index-insurance and the equilibrium level of risk-taking are given by Lemma 2. β^* can be either at a corner or interior, depending on the value of α . To see this, we reproduce here equation (15), which tells us that $\beta^* = \text{Max}\{0, \beta^o\}$, where

$$\beta^o = \sigma_{II}^N(\alpha) - \frac{\pi}{\eta(1-b)}.$$

By an application of the implicit function theorem on equation (18), we know that $\partial\sigma_{II}^N/\partial\alpha > 0$. Therefore there exists a threshold value of α , which we denote by $\tilde{\alpha}$ such that $\beta^o(\tilde{\alpha}) = 0$. If alpha is higher than $\tilde{\alpha}$,

then β^* is interior and equilibrium risk-taking is given by $\sigma_{II}^N(\alpha)$ as defined by equation (18), otherwise β^* is at a corner and equilibrium risk-taking equals $\sigma^N(\alpha)$, which is given by equation (10). For notational convenience, let us define $\sigma^*(\alpha)$ as follows

$$\begin{aligned}\sigma^*(\alpha) &= \sigma^N(\alpha), \text{ if } \alpha \in [0, \tilde{\alpha}), \\ &= \sigma_{II}^N(\alpha), \text{ if } \alpha \in [\tilde{\alpha}, 1].\end{aligned}$$

Solving stage 1:

Turning to stage (1), we solve the collective decision-making problem and select the optimal value of α . The objective function is given by

$$\tilde{c}^{II}(\alpha, \sigma^*(\alpha), \beta^*(\alpha)) \approx \mu(\sigma^*) - \pi\beta^* - \frac{\eta}{2} \text{Var}(c^{II}), \quad (28)$$

where

$$\text{Var}(c^{II}) = (\sigma^* - \beta^*)^2(1-b) + \sigma^{*2} \left[\left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 + \left(\alpha \frac{1}{n} \right)^2 (n-1) \right] b.$$

Before analyzing the first order condition of this problem, we need to highlight that the non-negativity constraint on β creates a discontinuity of the first derivative of \tilde{c}^{II} with respect to α . The generic expression of this first derivative is as follows

$$\frac{d\tilde{c}^{II}}{d\alpha} = \frac{\partial \tilde{c}^{II}}{\partial \alpha} + \frac{\partial \tilde{c}^{II}}{\partial \sigma} \frac{\partial \sigma^*}{\partial \alpha} + \frac{\partial \tilde{c}^{II}}{\partial \beta} \frac{\partial \beta^*}{\partial \alpha}. \quad (29)$$

where

$$\begin{aligned}\frac{\partial \tilde{c}^{II}}{\partial \alpha} &= \eta \frac{n-1}{n} \sigma^{*2} (1-\alpha) b, \\ \frac{\partial \tilde{c}^{II}(\sigma^*)}{\partial \sigma} &= -\eta \sigma^* \left(\alpha \frac{1}{n} \right)^2 (n-1) b,\end{aligned}$$

by equation (24). The third term of (29) is always equal to zero. Indeed, for $\alpha \in [0, \tilde{\alpha})$, β^* is at a corner and $\partial \beta^*/\partial \alpha = 0$. For $\alpha \in [\tilde{\alpha}, 1]$, β^* is interior. The envelope theorem therefore applies and $\partial \tilde{c}^{II}/\partial \beta = 0$ (see equation 23).¹⁷ Combining those terms, equation (29) can be rewritten as

$$\frac{d\tilde{c}^{II}}{d\alpha} = \eta \sigma^* \frac{n-1}{n} \left[\sigma^* (1-\alpha) - \alpha^2 \frac{1}{n} \frac{\partial \sigma^*}{\partial \alpha} \right] b. \quad (30)$$

To find the expression of $\partial \sigma^*/\partial \alpha$, we need to distinguish between two cases: $\alpha \in [0, \tilde{\alpha})$ and $\alpha \in [\tilde{\alpha}, 1]$. Applying the implicit function theorem on (10) and (18), respectively, we find that

$$\begin{aligned}\frac{\partial \sigma^*}{\partial \alpha} &= -2 \frac{n-1}{n} \frac{\eta \sigma^* [(1-\alpha) + \alpha \frac{1}{n}] b}{\mu'' - \eta [(1-b) + [(1-\alpha) + \alpha \frac{1}{n}]^2 b]}, \text{ if } \alpha \in [0, \tilde{\alpha}), \\ &= -2 \frac{n-1}{n} \frac{\eta \sigma^* [(1-\alpha) + \alpha \frac{1}{n}] b}{\mu'' - \eta [(1-\alpha) + \alpha \frac{1}{n}]^2 b}, \text{ if } \alpha \in [\tilde{\alpha}, 1].\end{aligned} \quad (31)$$

Hence, $\tilde{\alpha}$ is a point of discontinuity of the first derivative of \tilde{c}^{II} with respect to α . Indeed, the latter equations show that

$$\lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{\partial \sigma^*(\alpha)}{\partial \alpha} < \frac{\partial \sigma^*(\tilde{\alpha})}{\partial \alpha},$$

¹⁷The same argument does not apply to σ because σ generates externalities while c does not.

which implies that

$$\lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{d\tilde{c}^{II}(\alpha)}{d\alpha} > \frac{d\tilde{c}^{II}(\tilde{\alpha})}{d\alpha}.$$

As a consequence of this discontinuity, three cases need to be distinguished for optimization.

Case 1: Consider first the case where $d\tilde{c}^{II}(\tilde{\alpha})/d\alpha > 0$. In this case, $\arg \max_{\alpha} \tilde{c}^{II} \in [\tilde{\alpha}, 1]$, which implies that β^* is interior and given by equation (23), holding with equality, and that $\sigma^* = \sigma_{II}^N(\alpha)$. More precisely, making use of (30) and (31), we have that $\arg \max_{\alpha} \tilde{c}^{II} = \alpha_{II}^*$, where α_{II}^* is such that the following condition is satisfied:

$$\frac{d\tilde{c}^{II}}{d\alpha} = 0 \iff \Lambda_1(\alpha) = (1 - \alpha_{II}^*) + 2\alpha_{II}^{*2} \frac{n-1}{n^2} \frac{\eta [(1 - \alpha_{II}^*) + \alpha_{II}^* \frac{1}{n}] b^2}{\mu'' - \eta [(1 - \alpha_{II}^*) + \alpha_{II}^* \frac{1}{n}]^2 b} = 0. \quad (32)$$

Case 2: If $\lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{d\tilde{c}^{II}(\alpha)}{d\alpha} > 0$ and $\frac{d\tilde{c}^{II}(\tilde{\alpha})}{d\alpha} \leq 0$, then $\arg \max_{\alpha} \tilde{c}^{II} = \tilde{\alpha}$, β^* is equal to zero and that $\sigma^* = \sigma^N(\alpha)$.

Case 3: If $\lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{d\tilde{c}^{II}(\alpha)}{d\alpha} \leq 0$, then $\arg \max_{\alpha} \tilde{c}^{II} \in [0, \tilde{\alpha})$. In this case, β^* is at a corner while $\sigma^* = \sigma^N(\alpha)$. Combining (30) and (31), we obtain that $\arg \max_{\alpha} \tilde{c}^{II} = \alpha^*$, where α^* is such that

$$\frac{d\tilde{c}^{II}}{d\alpha} = 0 \iff \Lambda_2(\alpha) = (1 - \alpha^*) + 2\alpha^{*2} \frac{n-1}{n^2} \frac{\eta [(1 - \alpha^*) + \alpha^* \frac{1}{n}] b^2}{\mu'' - \eta [(1 - b) + [(1 - \alpha^*) + \alpha^* \frac{1}{n}]^2 b]} = 0. \quad (33)$$

Notice that $\alpha^* = \alpha^{SB}$, which is given by Proposition 2.

We also have that $\alpha_{II}^* < \alpha^*$. This can be seen by comparing equations (32) and (33). Indeed, one can rewrite condition (33) as

$$\frac{d\tilde{c}^{II}}{d\alpha} = 0 \iff (1 - \alpha) + 2\alpha^2 \frac{n-1}{n^2} \frac{\eta [(1 - \alpha) + \alpha \frac{1}{n}] b^2}{\mu'' - \eta [(1 - \alpha) + \alpha \frac{1}{n}]^2 b - x} = 0, \quad (34)$$

where $x = \eta(1 - b)$. Let us denote by $\alpha^\circ(x)$ the value of α that solves this condition for any given x . We have that $\alpha^\circ(\eta(1 - b)) = \alpha^*$ and $\alpha^\circ(0) = \alpha_{II}^*$ (see equation 32). Applying the implicit function theorem on condition (34), we see that $\partial\alpha^\circ/\partial x > 0$.¹⁸ It follows that $\alpha_{II}^* < \alpha^*$.

The optimal value of α therefore depends on the value of the threshold $\tilde{\alpha}$:

$$\begin{aligned} \alpha &= \alpha_{II}^*, \text{ if } \tilde{\alpha} \in [0, \alpha_{II}^*), \\ &= \tilde{\alpha}, \text{ if } \tilde{\alpha} \in [\alpha_{II}^*, \alpha^*), \\ &= \alpha^*, \text{ if } \tilde{\alpha} \in [\alpha^*, 1). \end{aligned}$$

To see this, notice that, by (30) and (31),

$$\begin{aligned} \text{sign} \left\{ \frac{d\tilde{c}^{II}}{d\alpha} \right\} &= \text{sign} \{ \Lambda_2(\alpha) \}, \text{ if } \tilde{c}^{II} \in [0, \tilde{\alpha}), \\ &= \text{sign} \{ \Lambda_1(\alpha) \}, \text{ if } \tilde{c}^{II} \in [\tilde{\alpha}, 1], \end{aligned}$$

and that both Λ_1 and Λ_2 are decreasing functions of α , by the second order condition.

¹⁸Indeed,

$$\frac{\partial\alpha^\circ}{\partial x} = -\frac{\partial\Lambda}{\partial x} \left(\frac{\partial\Lambda}{\partial\alpha} \right)^{-1},$$

where $\partial\Lambda/\partial x > 0$ and $\partial\Lambda/\partial\alpha < 0$ at any interior solution, by the second order condition with respect to α .

Consider the first case where $\tilde{\alpha} \in [0, \alpha_{II}^*)$. This corresponds to case 1. Indeed, $\text{sign} \left\{ \frac{d\tilde{c}^{II}(\tilde{\alpha})}{d\alpha} \right\} = \text{sign} \{ \Lambda_1(\tilde{\alpha}) \} > 0$, because $\tilde{\alpha} < \alpha_{II}^*$, the point at which $\Lambda_1 = 0$.

The second case where $\tilde{\alpha} \in [\alpha_{II}^*, \alpha^*)$ corresponds to case 2. Indeed, on the one hand, $\text{sign} \left\{ \frac{d\tilde{c}^{II}(\tilde{\alpha})}{d\alpha} \right\} = \text{sign} \{ \Lambda_1(\tilde{\alpha}) \} \leq 0$, because $\tilde{\alpha} \geq \alpha_{II}^*$. On the other hand, $\text{sign} \left\{ \lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{d\tilde{c}^{II}(\alpha)}{d\alpha} \right\} = \text{sign} \{ \Lambda_2(\tilde{\alpha}) \} > 0$, because $\tilde{\alpha} < \alpha^*$, the point at which $\Lambda_2 = 0$.

Finally, when $\tilde{\alpha} \in [\alpha^*, 1)$, we have case 3. Indeed, $\text{sign} \left\{ \lim_{\alpha \rightarrow \tilde{\alpha}^-} \frac{d\tilde{c}^{II}(\alpha)}{d\alpha} \right\} = \text{sign} \{ \Lambda_2(\alpha) \} \leq 0$, because $\tilde{\alpha} \geq \alpha^*$.

10.2 Comparative statics with respect to π

We now analyze the impact of the insurance premium π on the rate of risk-sharing α . As a preliminary remark, notice that α^* (condition 33) and α_{II}^* (condition 32) do not depend on π , neither directly, nor indirectly through σ^* , which does not appear in (32) and (33) because $\mu''' = 0$, by assumption.

However, the threshold value $\tilde{\alpha}$ is affected by π in the following way: by definition of $\tilde{\alpha}$,

$$\beta^o = \sigma^N(\tilde{\alpha}) - \frac{\pi}{\eta(1-b)} = 0.$$

By an application of the implicit function theorem on this equation, we have that $\partial\tilde{\alpha}/\partial\pi > 0$, since $\partial\sigma^*/\partial\alpha > 0$. It follows that

$$\begin{aligned} \alpha_{II}^{SB} &= \alpha_{II}^*, \text{ if } \pi \in [0, \sigma_{II}^N(\alpha_{II}^*)\eta(1-b)), \\ &= \tilde{\alpha}, \text{ if } \pi \in [\sigma_{II}^N(\alpha_{II}^*)\eta(1-b), \sigma^N(\alpha^*)\eta(1-b)), \\ &= \alpha^*, \text{ if } \pi \in [\sigma^N(\alpha^*)\eta(1-b), +\infty), \end{aligned}$$

with $\alpha^* = \alpha^{SB}$.

11 Appendix 5: Relaxing the assumption that $\mu''' = 0$

If we relax the assumption that $\mu''' = 0$, we obtain an additional effect by which the rate of risk-sharing decreases when π decreases. This additional effect takes place when β^* is interior. In this situation, α_{II}^{SB} is given by condition 32. The comparative statics of α_{II}^{SB} with respect to π is given by

$$\frac{d\alpha_{II}^*}{d\pi} = \frac{\partial\alpha_{II}^*}{\partial\sigma} \frac{\partial\sigma_{II}^N}{\partial\pi},$$

where

$$\frac{\partial\sigma_{II}^N}{\partial\pi} = \frac{1}{\mu''(\sigma_{II}^N) - \eta \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b} < 0,$$

by an application of the implicit function theorem on equation (18). Similarly, making use of condition 32, we find that

$$\frac{\partial\alpha_{II}^*}{\partial\sigma} < 0 \iff -2\alpha_{II}^{*2} \frac{n-1}{n^2} \frac{\eta \left[(1-\alpha_{II}^*) + \alpha_{II}^* \frac{1}{n} \right] b^2}{\left[\mu''(\sigma) - \eta \left[(1-\alpha_{II}^*) + \alpha_{II}^* \frac{1}{n} \right]^2 b \right]^2} \mu'''(\sigma) \leq 0 \iff \mu'''(\sigma) \geq 0.$$

Therefore,

$$\frac{d\alpha_{II}^*}{d\pi} > 0 \iff \mu'''(\sigma) \geq 0.$$

The crowding out effect would be reinforced under this condition.

12 Appendix 6: Proof of Proposition 7

The function $\tilde{c}^{II}(\alpha_{II}^{SB}, \sigma_{II}^{SB}, \beta^*)$ is given by equation (28) and evaluated at the second best allocation with index-insurance. Notice that $\tilde{c}^{II}(\alpha, \beta, 0) = \tilde{c}(\alpha, \beta)$ for any given (α, β) . Let us consider separately the three usual cases, while making use of Propositions 5 and 6:

1. Case 1: $\pi \in [\tilde{\pi}_2, +\infty)$: In this case, $\alpha_{II}^{SB} = \alpha^{SB}$; $\beta^* = 0$ and $\sigma_{II}^{SB} = \sigma^{SB}$. Therefore $\tilde{c}^{II}(\alpha_{II}^{SB}, \sigma_{II}^{SB}, \beta^*) = \tilde{c}(\alpha^{SB}, \sigma^{SB})$.
2. Case 2: $\pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$: In this case, there is crowding out, $\alpha_{II}^{SB} = \tilde{\alpha}(\pi) \in [\alpha_{II}^*, \alpha^{SB})$, and demand for index-insurance is equal to zero, $\beta_i^* = 0$. As shown in Appendix 4, we know that, at this point, $\lim_{\alpha \rightarrow \tilde{\alpha}^-} d\tilde{c}^{II}(\alpha)/d\alpha > 0$ and $d\tilde{c}^{II}(\tilde{\alpha})/d\alpha \leq 0$. Recall from Appendix 4 that

$$\begin{aligned}\tilde{c}^{II}(\alpha) &= \tilde{c}^{II}(\alpha, \sigma^N(\alpha), 0), \text{ if } \alpha \in [0, \tilde{\alpha}), \\ &= \tilde{c}^{II}(\alpha, \sigma_{II}^N(\alpha), \beta^o(\alpha)), \text{ if } \alpha \in [\tilde{\alpha}, 1].\end{aligned}$$

For $\alpha < \tilde{\alpha}$, β is at a corner and the function $\tilde{c}^{II}(\alpha, \sigma^N(\alpha), 0)$ coincides with the function $\tilde{c}(\alpha, \sigma^N(\alpha))$. Hence, $d\tilde{c}(\tilde{\alpha}, \sigma^N(\tilde{\alpha}))/d\alpha = \lim_{\alpha \rightarrow \tilde{\alpha}^-} d\tilde{c}^{II}(\alpha)/d\alpha > 0$. We can conclude that $\tilde{c}^{II}(\alpha_{II}^{SB}, \sigma_{II}^{SB}, \beta^*) = \tilde{c}(\tilde{\alpha}, \sigma^N(\tilde{\alpha})) < \tilde{c}(\alpha^{SB}, \sigma^{SB})$, $\forall \pi \in [\tilde{\pi}_1, \tilde{\pi}_2)$.

3. Case 3: $\pi \in [0, \tilde{\pi}_1)$: In this case $\alpha_{II}^{SB} = \alpha_{II}^* < \alpha^{SB}$ and β is interior: $\beta^* = \beta^o > 0$. As shown in the text, the impact of π on welfare is given by

$$\frac{d\tilde{c}^{II}}{d\pi} = \frac{\partial \tilde{c}^{II}}{\partial \pi} + \frac{\partial \tilde{c}^{II}}{\partial \sigma} \frac{\partial \sigma_{II}^{SB}}{\partial \pi},$$

with

$$\frac{\partial \tilde{c}^{II}}{\partial \pi} = -\beta^*; \quad \frac{\partial \tilde{c}^{II}}{\partial \sigma} < 0; \quad \frac{\partial \sigma_{II}^{SB}}{\partial \pi} < 0.$$

The two effects go in opposite directions. However, $d\tilde{c}^{II}/d\pi$ is unambiguously positive if β^* is close to zero. This is the case when π is only slightly lower than $\tilde{\pi}_1$. Therefore, for $\pi \in [0, \tilde{\pi}_1)$, but close to $\tilde{\pi}_1$, we know that $\tilde{c}^{II}(\alpha_{II}^{SB}, \sigma_{II}^{SB}, \beta^*) < \tilde{c}(\alpha^{SB}, \sigma^{SB})$. Indeed, for $\pi = \tilde{\pi}_1$ (case 2), we have $\tilde{c}^{II} < \tilde{c}$. Starting from this point, a reduction of π causes an additional reduction of \tilde{c}^{II} . A fortiori, the result that $\tilde{c}^{II} < \tilde{c}$ is maintained for $\pi \in [0, \tilde{\pi}_1)$, but close to $\tilde{\pi}_1$. If subscription is large enough, then the negative effect of π on \tilde{c}^{II} outweighs the positive effect and $d\tilde{c}^{II}/d\pi < 0$. At some point $\tilde{\pi}_0 < \tilde{\pi}_1$, expected utility recovers its level in the absence of index-insurance. For lower premiums, the inequality is reversed, $\tilde{c}^{II} > \tilde{c}$, meaning that index-insurance improves welfare.

13 Appendix 7: Proof of Proposition 8

We solve the game in the case where subscription to index-insurance β is chosen by the group along with the rate of risk-sharing α . The timing is as follows. In stage (1), α and β are defined cooperatively at the group level. In stage (2), agents choose simultaneously and non-cooperatively σ .

Solving stage 2:

Using backward induction, we solve the second stage for any given level of α and β . The Nash level of risk-taking in the case of group subscription σ_G^N can be found by solving the following optimization problem:

Individual farmers maximize the certainty equivalent of consumption (2), where the expressions of the mean and variance of consumption are given by (13) and (14), respectively. The first order condition of this problem writes

$$\mu'(\sigma_G^N) - \eta \left[(\sigma_G^N - \beta)(1-b) + \sigma_G^N \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b \right] = 0,$$

and provides an implicit definition of the Nash level of risk-taking as a function of α and β : $\sigma_G^N(\alpha, \beta)$. An application of the implicit function theorem on this equation allows to show that σ_G^N is an increasing function of both α and β :

$$\begin{aligned} \frac{\partial \sigma_G^N}{\partial \alpha} &= -\frac{2\eta\sigma_G^N \left[(1-\alpha) + \alpha \frac{1}{n} \right] \frac{n-1}{n} b}{\mu'' - \eta \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b \right]} > 0, \\ \frac{\partial \sigma_G^N}{\partial \beta} &= -\frac{\eta(1-b)}{\mu'' - \eta \left[(1-b) + \left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 b \right]} > 0. \end{aligned}$$

Solving stage 1:

Turning to stage (1), we solve the collective decision-making problem and select the optimal values of α and β . The objective function is given by

$$\tilde{c}(\alpha, \beta, \sigma_G^N(\alpha, \beta)) \approx \mu(\sigma_G^N) - \pi\beta - \frac{\eta}{2} \text{Var}(c),$$

where

$$\text{Var}(c) = (\sigma_G^N - \beta)^2 (1-b) + \sigma_G^{N2} \left[\left[(1-\alpha) + \alpha \frac{1}{n} \right]^2 + \left(\alpha \frac{1}{n} \right)^2 (n-1) \right] b.$$

The first order conditions of this problem are given by:

$$\begin{aligned} \frac{d\tilde{c}}{d\alpha} &= \frac{\partial \tilde{c}}{\partial \alpha} + \frac{\partial \tilde{c}(\sigma_G^N)}{\partial \sigma} \frac{\partial \sigma_G^N}{\partial \alpha} = 0, \\ \frac{d\tilde{c}}{d\beta} &= \frac{\partial \tilde{c}}{\partial \beta} + \frac{\partial \tilde{c}(\sigma_G^N)}{\partial \sigma} \frac{\partial \sigma_G^N}{\partial \beta} = 0. \end{aligned}$$

Under moral hazard, we know that the non-cooperative level of risk-taking is excessive (see Lemma 1). It follows that $\partial \tilde{c}(\sigma_G^N) / \partial \sigma$. Therefore, both $\partial \tilde{c} / \partial \alpha$ and $\partial \tilde{c} / \partial \beta$ are strictly positive in equilibrium. For risk-sharing, it means that $\alpha_G^{SB} < 1$, as shown in the proof of Proposition 2. As far as index-insurance is concerned,

$$\frac{\partial \tilde{c}}{\partial \beta} = -\pi + \eta(\sigma_G^N - \beta)(1-b) > 0 \iff \beta < \sigma_G^N - \frac{\pi}{\eta(1-b)}.$$

In particular, if index-insurance is actuarially fair ($\pi = 0$), then subscription remains incomplete: $\beta < \sigma_G^N$.

The second point of the proposition states that index-insurance is always welfare enhancing under group subscription. This is simply due to the structure of the game where both α and β are selected by the group in period 1 so as to maximize social welfare, by definition.