

Price Co-Movements and Investment Funds

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Abstract

This paper discusses price co-movements between fundamentally independent financial markets populated by risk neutral global funds and specialized funds. The investment decisions are delegated to risk neutral fund managers who are informed or uninformed of the state of the markets and have reputational concerns. We show that in any equilibrium of the model, prices of the risky assets co-move with each other following any shock to ex-ante probabilities of default. The mechanism that generates this co-movement relies on two sources: the information asymmetry between fund managers and the reputational concerns of uninformed fund managers facing the threat of dismissal. The reputational channel reinforces the co-movement but it is not necessary to generate it. Information asymmetry induces co-movement even in the absence of reputational concerns.

Keywords: Price Co-Movement, Global Funds, Specialized Funds.

JEL classification: G11, G12 .

1 Introduction

According to New York Stock Exchange Factbook, in 2003 institutional investors held almost 50% of corporate equity in NYSE. In 1950, this number was only 7%. Investors reward fund managers according to some measure of their success in generating returns and withdraw their funds if they deem

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the manager incompetent and unsuccessful. So the managers incentives are two fold; they want to maximize the return on their portfolio and build up a reputation for themselves as competent managers. There is a growing literature on the general equilibrium implications of institutional trading that discusses the price distortions generated by the incentives of fund managers.¹

Alongside the shift from individual investors to institutional investors, there have been episodes of the spread of financial crisis between emerging markets that had no common fundamentals. A good example is the 1998 Russian Flu that spread to Brazil. The common notion in the literature about these episodes has been multiplicity of equilibrium due to financial vulnerability and market incompleteness. Interestingly, the affected markets were all populated by institutional investors such as global hedge funds. At the same time, there has also been a rise in interdependence and co-movement between stock prices all over the world that is not explained by common fundamentals, global shocks and changes in volatility.²

Our aim is to address the equilibrium consequences of having specialized and global investment funds, delegating the investment decision to reputationally motivated managers for price co-movement between fundamentally independent markets. Our key assumption is the asymmetric information among managers. Managers can be *informed* of the true state of the assets or *uninformed*. We show that in any equilibrium of the model, prices co-move with each other following any shock to the prior beliefs about the markets. Our model builds on Sami and Brusco (2014) and Guerrieri and Kondor (2012). There are two fundamentally independent risky assets and a risk less bond. We have three types of funds; specialized in market one, specialized in market two, and global. Risk neutral fund managers are either informed or uninformed and are hired to invest the money of risk neutral investors. Types of funds are observable but types of managers are private information. Also, there are independent masses of liquidity traders at each risky asset market. Managers are paid a fixed share of return they have made and are retained by the funds if they have made the highest possible return feasible for them. This means that specialized funds retain the manager if he has bought the repaying asset or the risk-less bond when the asset

¹For example, Cuoco and Kaniel (2006), Dasgupta and Prat (2008), Basak and Pavlova (2011)

²Forbes (2012) surveys empirical and theoretical literature on contagion and documents significant rise in co-movement between stocks within advanced countries, Euro region and all over the world controlling for global shocks and changes in volatility. Anton and Polk (2014) identifies a significant increase in the return co-movement of the stocks held by the same mutual funds.

defaults. However, global funds retain their managers if they have bought the repaying risky asset with the lowest price or risk free bond when both assets default.

As in Sami and Brusco (2014), we consider partially revealing rational expectation equilibria. We focus on equilibria in which if p^* is an equilibrium price at a certain value of the liquidity and return shock realization, and at p^* the excess demand is identical for another shock realization, then the equilibrium price must be the same.

If asset i repays, p_i reveals it's repaying when it is equal to $\frac{1}{R}$, where R is the return on riskless bond with a price normalized to 1. If asset i defaults, p_i reveals the default if it is less than or equal to \bar{p}_i which clears the market with the demands of liquidity traders³. In Sami and Brusco (2014), we showed that there is no equilibrium at which prices don't reveal any information about the true state of the assets. Besides, when all the funds are global, there is no partially revealing equilibrium with different unrevealing prices, i.e., in any equilibrium, unrevealing prices must be equal. In this paper, I first prove that as long as there are global funds in the market, prices are interdependent in any equilibrium. Consequently, interdependent prices co-move with each other following any shock to the priors on the assets. The result is obtained despite the fact that all agents are risk-neutral. This co-movement is magnified by reputational concerns of managers but does not go away if there is no reputational concern. Moreover, we show that when there are heterogeneous funds, we have both types of equilibria, equilibria with equal and unequal unrevealing prices. The analysis in Sami and Brusco (2014) was only limited to one type of equilibria-*simple equilibria*- while in this paper we characterize both simple and non-simple equilibria.⁴

The mechanism that generates the interdependence and co-movement relies on two sources, the information asymmetry between fund managers and the reputational concerns of uninformed fund managers facing the threat of dismissal by funds. Informed managers are perfectly informed and have strict demands for the repaying asset. I also assume that there are very few informed managers and a lot of uninformed managers in the market so that the demands of informed managers can't clear the market so uninformed managers must have positive demand for the assets for the market to get cleared. Now, imagine that the risky assets are Russian bond and Brazilian bond. Suppose that Russian bond defaults and Brazilian bond repays. Global informed managers know this and they all demand Brazilian

³We will later on solve for \bar{p}_i in equilibrium.

⁴An equilibrium is simple if only one unrevealing price vector occurs in equilibrium.

bond. But then the probability that uninformed managers receive Brazilian bond is less than receiving Russian bond simply because all the informed managers demand Brazilian bond. This shows a couple of things. First uninformed managers face an adverse selection problem, with higher probability they receive the defaulting bond (Russian bond). Second, the probability of receiving Brazilian bond depends on the state of the Russian bond and decreases with the increase in the default probability of Russian bond. So uninformed managers demand the bonds if prices compensate them for this adverse selection problem. Also, prices must co-move with each other following any change in ex-ante default probability of one of the bonds. To see this, suppose that the ex-ante default probability of the Russian bond increases. Clearly, price of Russian bond suffers. But at the same time, uninformed managers would rationally believe that if Brazilian bond has repaid, the probability of receiving it is now even less. So to compensate the uninformed for the rise in the risk of not receiving the repaying Brazilian bond, the price of Brazilian bond must also go down.

Since I assume that the total mass of informed managers and liquidity traders is never enough to clear the markets, unrevealing prices are clearing the markets only if there is a positive demand from uninformed managers. Uninformed managers have positive demand for risky assets if prices compensate them for the risk of being dismissed. This means that they ask for premia over the return of the risk free bond which are not independent of each other. This premia increases the price co-movements, however, the co-movement doesn't disappear if there is no reputational concern and the premium is zero. In other words, if instead of delegating the investment, investors directly invest in the markets, uninformed traders face the same signal extraction problem of uninformed managers. They have to learn the signals of informed traders from prices. Since prices reflect the signals of informed global traders, they are interdependent and co-move with each other following any change in the ex-ante probabilities of the default of any asset.

Literature Review. This paper is an extension of Guerrieri and Kondor (2012). In a model with only one risky asset, one risk free bond and one type of investors, they show that the reputationally concerned managers distort the price of the risky asset by asking a premium over the risk free bond that compensates them for the risk of getting fired and makes the price more volatile. Our paper contributes to the literature on general equilibrium models of contagion with information asymmetry and delegation. The closest models to ours are the models that discuss information channels of contagion and the contagion due to delegation. Calvo (1999) has a rational expectation model at which uninformed traders see the actions of informed

traders but face a signal extraction problem; when informed traders don't buy an asset, uninformed traders don't know if this is because of a negative idiosyncratic shock to their demand or it is because of a negative shock to the valuation of the assets. Thus, when the volatility of the returns in emerging markets are relatively higher than the volatility of the idiosyncratic shocks, following a negative shock to one market uninformed traders attach higher probability to the low return for other market as well. The main difference between Calvo (1999) and us is the pricing mechanism; at his model uniformed traders first observe the actions of informed ones and then choose to buy or sell emerging markets. In our model, all the traders move simultaneously and it's only the price that reveals information to the market.

Chakravorti and Lall (2005) have a general equilibrium model of delegated portfolio management. They have dedicated and opportunist managers. Dedicated managers only invest in emerging markets and are compensated based on the excess return that they make over a benchmark index of emerging markets. Opportunist managers are allowed to short sell and are payed a fixed share of the total return made on the portfolio. They show that price co-movement between emerging markets is the result of the portfolio re-balancing by managers following a shock to one market. Our model differs from them in having asymmetric information as the main source of price co-movement. Dasgupta and Prat (2008) is a sequential trading model with one risky asset that extends Glosten and Milgrom (1985) model by introducing career concerned traders. They show that managers with reputational concerns distort the price so that it never reveals the true state of the asset. Kodres and Pritsker (2002) has a rational expectations model of contagion with asymmetric information where fundamentally unrelated markets can experience contagion due to the cross-market re-balancing. There is no contagion in their model when fundamentals and liquidity shocks are uncorrelated. Finally, our model is related to the big literature on contagion due to herding. In Scharfstein and Stein (1990) managers follow each other to avoid being regarded dumb and share the blame if the things go wrong. In a more recent paper, Wagner (2012) shows that the threat of dismissal by investors induces the managers to fire sales and run when they suspect others would do the same to avoid selling the assets at lower prices later evenif they are not going to be evaluated in the future.

The rest of this paper is organized as follows. Next section presents the model. In section 3, we characterize the equilibrium. Section 4 contains concluding remarks.

2 Model

There are two risky assets and one risk free bond paying $R > 1$. The return on risky asset i at time t is determined by the realization of a random variable $\tilde{\chi}_{i,t}$ which takes values in the set $\{0, 1\}$. The realization of $\tilde{\chi}_t = (\tilde{\chi}_{1,t}, \tilde{\chi}_{2,t})$ is denoted $\chi_t = (\chi_{1,t}, \chi_{2,t})$. If $\chi_{i,t} = 0$ then the asset repays an amount of 1, while if $\chi_{i,t} = 1$ the asset defaults and pays zero. The random variables $\{\tilde{\chi}_{i,t}\}_{t=0}^{\infty}$ are all independent and identically distributed, with $\Pr(\tilde{\chi}_{i,t} = 1) = q_i$ and $q_2 > q_1$. Furthermore, each $\tilde{\chi}_{i,t}$ is independent of all variables $\{\tilde{\chi}_{j,\tau}\}_{\tau=0}^{\infty}$ with $j \neq i$.

Risky assets are sold at prices $p_i \leq \frac{1}{R}$. They are supplied in fixed inelastic amounts of b_1 and b_2 . Let $\mathbf{b} = (b_1, b_2)$ be the vector of supply. There is also a perfectly elastic supply of risk free bonds at price $\frac{1}{R}$.

We have three kinds of agents; investors, fund managers and liquidity traders. Investors are endowed with one unit of capital but they can't invest it themselves and have to hire fund managers. Investors are of three types, only investing in asset 1 and bond, I_1 , investing in asset 2 and bond, I_2 , or investing in both assets and bond, I_3 . We assume that the mass of I_j investors is also I_j . We can think of each type of investor as a type of fund. Fund managers are also of two types; informed (I) and uninformed (U). Informed managers observe the realizations of $\tilde{\chi}_{i,t}$ for $i = 1, 2$. Uninformed managers only observe prices of the assets. The types of investors are observable while the types of managers are private information.

The mass of informed managers (M^I) is less than the mass of any fund I_j . Liquidity traders are only demanding risky assets for random reasons. Let y_1 and y_2 be the masses of liquidity traders at each asset market. We assume that y_i s are independently and identically distributed according to the uniform distribution over $[y, \bar{y}]$.

At the beginning of each day, funds with no manager are randomly matched with a manager in the unemployment pool. We assume that funds looking for a manager are not observing the previous history of any employment of the managers in the unemployment pool. Funds offer the matched manager a contract that pays a fixed share of return γ , and retains him only if the manager has made the highest possible return. We will discuss the asset and labor markets in detail later but before that we present the time line of the model.

2.1 Timing

The timeline of the model is as follows;

- In the morning
 - Unemployed managers decide to pay the search cost κ and enter the unemployment pool or stay out of market.
 - Funds with no manager randomly pick a fund manager from unemployment pool.
 - Informed managers observe the realization of return shocks $\chi_{\mathbf{t}}$.
 - Managers choose their demand of the assets and the bond.
 - Equilibrium prices $p_t = (p_{1t}, p_{2t})$ are determined and the assets are allocated.

- In the evening,
 - $\chi_{\mathbf{t}}$ is publicly observed and the investments of the managers are realized by their investors.
 - Managers receive a share γ of the returns.
 - Any fund receives an exogenous binary signal, σ_t^l , about the type of manager l . If the manager is informed, then σ_t^l is always zero. Otherwise, $\sigma_t^l = 0$ with probability ω and $\sigma_t^l = 1$ with probability $1 - \omega$.
 - Funds decide to fire or retain their managers.
 - With probability $1 - \delta$ any manager is exogenously separated from the job.

2.2 Labor Market

To hire a manager each fund randomly picks a manager from the pool of unemployed managers. Let $Z_t = Z_t^I + Z_t^U$ be the total mass of unemployed managers of both types and A_t the mass of funds looking for a manager at any time t . Also define μ_t as the probability of matching. Since funds and managers are matched randomly the probability that a manager is matched is:

$$\mu_t = \frac{\min\{A_t, Z_t\}}{Z_t} \quad (1)$$

Clearly, funds decision to fire or retain any manager after observing managers returns depends on the matching probability μ_t , the fraction of informed unemployed managers out of all unemployed managers $\frac{Z_t^I}{Z_t}$, and their updated probability about the managers competence. Let N_j^i be the set of

the managers of type $i = I, U$ hired by the funds of type I_j . Let also $\phi_j(\theta_j^q, \sigma^q, p_j, \chi_j) \in \{0, 1\}$ denote the retention decision of fund I_j after observing the investment decision θ_j^q of the manager q , the exogenous separation signal σ^q , equilibrium price(s) and the true value of the asset(s) χ_j . By the same argument, let $\phi_3(\theta_3^q, \sigma^q, \mathbf{p}, \chi) \in \{0, 1\}$ be the firing decision for I_3 funds. Then, $\phi_j = 0$ if the manager is retained and $\phi_j = 1$, otherwise.

The investments of managers in $I_1(I_2)$ funds are successful if they buy risky asset 1(2) when it repays and buy risk free bond when it defaults. For managers in I_3 funds, the investment is successful whenever they buy risk free bond when both assets default or they buy the cheapest asset that is repaying.

2.3 Asset Markets

Each manager submits a demand schedule. Managers in I_1 and I_2 funds can demand risk free bond, the risky asset the fund specializes in, or state indifference between them. Managers hired by I_3 funds can demand each of the risky assets, risk free bond or be indifferent for a subset of assets. The auctioneer collects the demand schedules, sets the market clearing prices and allocates the assets to managers and liquidity traders. Given the submitted demands of managers, the auctioneer first assigns the managers with the strict demand of asset 1, asset 2 or risk free bond and then assigns to the managers stating indifference between the investment opportunities at prices that clear markets.

N_1^I and N_2^I managers submit the demand schedules $d_j^I(p_j|\chi_j) : [0, \frac{1}{R}] \times \{0, 1\} \rightarrow \{0, 1\}^2$, $j = 1, 2$ to the auctioneer. If $d_1^I = (0, 1)$ for some χ_1 and p_1 , then the manager demands no bond and $1/p_1$ units of risky asset 1 while $d_1^I = (1, 1)$ means that the manager is indifferent between 1 unit of bond or $1/p_1$ units of risky asset. Given $\chi = (\chi_1, \chi_2)$ and $\mathbf{p} = (p_1, p_2)$, N_3^I managers submit $d_3^I(\mathbf{p}|\chi) : [0, \frac{1}{R}]^2 \times \{0, 1\}^2 \rightarrow \{0, 1\}^3$ to the auctioneer. Finally, uninformed managers hired at I_j funds, N_j^U , have no private signal so when hired by I_1 or I_2 funds, their demand schedules are given by $d_j^U(p_j) : [0, \frac{1}{R}] \rightarrow \{0, 1\}^3$ where $d_{jk}^U = 0$ for $k \notin \{0, j\}$. If hired by I_3 funds, uninformed managers demand is given by $d_3^U(p_1, p_2) : [0, \frac{1}{R}]^2 \rightarrow \{0, 1\}^3$. Like managers, liquidity traders are also endowed one unit of capital that they invest it entirely on a risky asset. At any price p_{it} liquidity traders in market i buy $1/p_i$ units of asset i . Throughout the paper, we assume that $b_i > \bar{y}$ so there is always sufficient supply to cover the demands of liquidity traders. Now, assume that asset i is expected to default and the only agents that

still demand the auctioneer to assign them asset i are liquidity traders. The auctioneer clears the market by assigning the entire b_i units of asset i to liquidity traders at $p_{it}(y_{it}) = \frac{y_{it}}{b_i}$. Note that $p_{it}(y_{it}) \in [\frac{\underline{y}}{b_i}, \frac{\bar{y}}{b_i}]$. This means that in equilibrium, any price below $\frac{\bar{y}}{b_i}$ automatically reveals that the asset is defaulting. From this point on, let $\bar{p}_i = \frac{\bar{y}}{b_i}$ and $\bar{p} = \max\{\bar{p}_1, \bar{p}_2\}$.

Define W_j^U , $j = 1, 2, 3$, as the continuation payoff for an uniformed manager of being employed at fund I_j . Also define $v_j^U(k, p_j)$ as the expected payoff of N_j^U manager, $j = 1, 2$, buying asset $k = 0, j$. We have,

$$v_j^U(k, p) = E[\gamma e_j + (1 - \phi_j(\theta_j^q, \sigma^q, p_j, \chi_j))\beta W_j^U | \mathbf{p}^e = (p_1, p_2)] \quad (2)$$

where

$$e_j = \begin{cases} R & \text{if } k = 0 \\ \frac{1 - \chi_j}{p_j} & \text{if } k = j \end{cases}$$

Now let $v_3^U(k, p_1, p_2)$ be the payoff of N_3^U manager buying asset $k = 0, 1, 2$. Then,

$$v_3^U(k, p_1, p_2) = E[\gamma e_3 + (1 - \phi_3(\theta_3^q, \sigma^q, \mathbf{p}, \chi))W_3^U | \mathbf{p}^e = (p_1, p_2)] \quad (3)$$

where,

$$e_3 = \begin{cases} R & \text{if } k = 0 \\ \frac{1 - \chi_1}{p_1} & \text{if } k = 1 \\ \frac{1 - \chi_2}{p_2} & \text{if } k = 2 \end{cases}$$

The payoffs for informed managers are defined the same as (2) and (3) but note that N_1^I and N_2^I managers receive single perfect signals χ_j and their payoff is $v_j^I(p_j | \chi_j)$. N_3^I managers observe $\chi = (\chi_1, \chi_2)$ and hence their payoff is denoted by $v_3^I(\mathbf{p} | \chi)$ and is given by

$$v_3^I(k, \mathbf{p}, \chi) = E(\gamma r_3 + (1 - \phi_3(\theta, \sigma, p, \chi))\beta W_3^I | \chi) \quad (4)$$

Define the set of all possible demand vectors as

$$\Delta = \{(d_0, d_1, d_2) | \sum_{i=0}^2 d_i \geq 1\} \quad (5)$$

Let $A(p) = (A_1(p), A_2(p))$ be the aggregated demand vector at price p , that is

$$A(p) = \int_{q \in \mathcal{N}} d^q(p) dq \quad (6)$$

where \mathcal{N} is the set of all traders. Let $x_k(d^q; A_k) : \Delta \rightarrow [0, 1]$ where $\sum_{k=0}^2 x_k d^q = 1$ denotes the feasible allocation to a manager with demand d^q .

We have now defined all the elements of the equilibrium and are ready to define the equilibrium.

Definition 1. *Given any collection of (N_j^I, N_j^U, W_j^U) , $j = 1, 2, 3$, the rational expectations equilibrium consists an equilibrium price mapping $\mathbf{p} : \{0, 1\}^2 \times [\underline{y}, \bar{y}]^2 \rightarrow [\frac{\underline{y}}{b_1}, \frac{1}{R}] \times [\frac{\underline{y}}{b_2}, \frac{1}{R}]$; equilibrium demand schedules d_j^i , for $i = I, U$ and $j = 1, 2, 3$; and feasible allocation mapping $x_k(d_j^i) \in [0, 1]$, for each asset k such that,*

(A1). *the price vector $p(\chi, y) = (p_1(\chi, y), p_2(\chi, y))$ clears the markets. That is, for asset $k = 1, 2$,*

$$\begin{aligned} & \int_{q \in N_k^I} x_k(\hat{d}^q) \hat{d}_k^q dq + \int_{q \in N_3^I} x_k(\hat{d}^q) \hat{d}_k^q dq + \int_{q \in N_k^U} x_k(\hat{d}^q) \hat{d}_k^q dq \\ & \quad + \int_{q \in N_3^U} x_k(\hat{d}^q) \hat{d}_k^q dq = p_k b_k - y_k \end{aligned} \quad (7)$$

where $\hat{d}^q = d_j^i(p, \cdot)$.

(A2). *the demand schedules of N_j^U managers are optimal given $p(\chi, y)$. That is, if $d_{kj}^i = 1$ then $v_j^U(k, p) \geq v_j^U(k', p)$ for all $k' = k$.*

(A3). *the demand schedules of N_j^I , $j = 1, 2$, and N_3^I managers are optimal given $p(\chi, y)$, i.e., $d_j^I(p_j | \chi_j) = 1$ and $d_{j3}^I(p | \chi) = 1$ for $\chi_j = 0$, and $d_j^I(p_j | \chi_j) = 0$ and $d_{j3}^I(p | \chi_j) = 0$ for $\chi_j = 1$; $j = 1, 2$.*

Let $D_k(p)$ be the set of all the traders with strict demands for asset k at price p . That is, for $q \in D_k(p)$, $d_k^q = 1$ and $d_j^q = 0$ for $j = k$. Let $Z_k(\chi, \mathbf{y})$ be the mass of all d^q , $q \in D_k(p)$ at (χ, \mathbf{y}) That is,

$$Z_k(\chi, \mathbf{y}) = \int_{q \in D_k(p)} d_k^q dq \quad (8)$$

The equilibrium that we construct satisfies the following *belief consistency* condition.

Definition 2. *Let \mathbf{p}^e be a rational expectations equilibrium price mapping and $\mathbf{p}^e(\chi, \mathbf{y})$ be the equilibrium price vector at (χ, \mathbf{y}) . Also assume that there exists (χ', \mathbf{y}') such that $\mathbf{Z}(\chi, \mathbf{y}) = \mathbf{Z}(\chi', \mathbf{y}')$. Then, \mathbf{p}^e is **belief consistent** if $\mathbf{p}^e(\chi, \mathbf{y}) = \mathbf{p}^e(\chi', \mathbf{y}')$.*

Definition 2 restricts the set of equilibria to the partially revealing equilibria. If the equilibrium price vector is belief consistent there are some values of \mathbf{y} that price vector is not revealing χ . Hence, the equilibrium price vector that always reveals the repay or default of the assets is not belief consistent.

Before moving on to the next section, we introduce another feature of our equilibrium.

Definition 3. *An equilibrium price mapping $p^e(\chi, y)$ is **simple** if there is at most one pair (p_1, p_2) with $p_i \in (\bar{p}, \frac{1}{R})$, $i = 1, 2$, such that $p^e(\chi, y) = (p_1, p_2)$.*

An equilibrium is *non-simple* if the equilibrium price mapping $p^e(\chi, y)$ takes more than one value in $(\bar{p}, \frac{1}{R})^2$.

3 Equilibrium

We construct a class of stationary equilibria at which $N_{jt}^i = N_j^i$, $\mu_{jt} = \mu_j$, and $W_{jt}^U = W_j^U$. In none of these equilibria prices are fully revealing, so funds have higher expected payoff if they have an informed manager. This suggests that reputation is valuable for uninformed managers as well, because any mistake leads to dismissal and loss of W_j^U . Hence, from this point on we take $(\{N_j^I, N_j^U, W_j^U, \mu_j\}_{j=1,2,3})$ as given and discuss the existence and properties of the rational expectations equilibrium at asset markets. We also assume that it is optimal for the funds to fire any manager who hasn't made the highest possible return. Later on we solve for equilibrium N_j^i and W_j^U and prove the optimality of the firing rule.

The existence of this class of equilibria is guaranteed under the following assumptions;

$$M^I < \min\{\bar{y}, \bar{y} - \underline{y}\} \quad , \quad M^I + \bar{y} < \underline{C} \quad , \quad \frac{\max\{b_1, b_2\}}{R} < \underline{y} + \min\{I_1, I_2, I_3\} \quad (9)$$

where \underline{C} is given in the Appendix. The first part ensures that the mass of informed managers is small relative to noise traders, making the equilibrium not always fully revealing. The second part ensures that the total investments of informed managers and noise traders are never enough to clear the markets so there is always some amount of each asset that is allocated to uninformed managers. However, by the third part the supply is never enough to allocate risky assets to all uninformed managers. Thus, in equilibrium uninformed managers are always indifferent between risky asset(s) and riskfree bond.

- $\omega > \frac{1}{1+\delta}$

This assumption is identical to the assumption made by Guerrieri and Kondor (2012). After observing a right decision by a fund manager at the end of each day, funds attach a higher probability to the event that the manager is informed than uninformed. This assumption ensures that in equilibrium, the beliefs of funds about successful managers grows at the high enough speed so that they are retained after a right decision.

- $\kappa < \gamma R$

This assumption ensures that the search cost is not more than the expected payoff of getting hired for uninformed managers. It excludes equilibria which all unemployed uninformed managers are matched with probability 1.

Given these assumptions, we discuss how information spreads into the market from the demands of informed managers. Before characterizing the equilibria we need to introduce the concept of marginal traders that is going to play a central role in the characterization of each equilibrium.

3.1 Marginal Traders

Definition 4. Suppose $p^e(\chi, y)$ is an equilibrium price mapping with $p^e(\chi, y) = (p_1, p_2) \in (\bar{p}, \frac{1}{R})^2$ for some (χ, y) . Also assume $f_j = \Pr(\chi_j = 1 \mid p^e = (p_1, p_2))$. Then, N_j^U managers $j = 1, 2, 3$ are **marginal traders** at $p = (p_1, p_2)$ if

$$v_j^U(j, p) = v_j^U(0, p)$$

By Definition 4, if N_j^U $j = 1, 2, 3$, are marginal traders at $p = (p_1, p_2)$, their expected payoff of buying asset j is equal to the expected payoff of buying risk free bond. This means that N_j^U are indifferent between asset j and risk free bond at p_j . This also implies that p_j is the maximum price that marginal traders are willing to pay for asset j . At any price above p_j they never demand asset j . Rewriting the condition in Definition 4 for $j = 1, 2$, we have

$$(1 - f_j)\left(\frac{\gamma}{p_j} + \delta\omega\beta W_j^U\right) = \gamma R + \delta\omega f_j \beta W_j^U \quad (10)$$

The right hand side of (10) is the expected payoff of buying asset j . Recall that uninformed managers in specialized funds are paid γ share of the return and are only retained if they buy risky asset when it repays and riskless bond when risky asset defaults. Thus, their expected payoff of buying asset

j is the expected retrun on asset j , $\frac{\gamma(1-f_j)}{p_j}$, plus the expected payoff of being retained. But the probability of being retained for N_j^U when buying asset j is the probability of the repay of asset j , $1 - f_j$, times the probability that he is not exogenously separated, δ , times the probability that his type is not revealed ω . The left hand side of (10) is the expected payoff of buying the safe asset. Now, the manager buying riskless asset is retained only if risky asset j has defaulted, hence the expected payoff of being retained is $\delta\omega f_j \beta W_j^U$.

Now suppose N_3^U managers are marginal traders at (p_1, p_2) and $p_1 > p_2$. This means that

$$(1 - f_1)\left(\frac{\gamma}{p_1} + \delta\omega f_2 \beta W_3^U\right) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \quad (11)$$

$$(1 - f_2)\left(\frac{\gamma}{p_2} + \delta\omega \beta W_3^U\right) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \quad (12)$$

Note that when $p_1 > p_2$, N_3^U managers buying asset 1 are only retained when asset 1 repays and asset 2 defaults, because if asset 2 repays, the return on asset 2 is higher than the return on asset 1 and N_3^U are only retained when they buy the asset that pays the highest return. Therefore, the probability of the retainment for a manager buying asset 1 is $\delta\omega(1 - f_1)f_2$. However, the manager that buys asset 2 is retained whenever this asset repays irrespective of the default or repay of asset 1 and his probability of retainment is equal to $\delta\omega(1 - f_2)$. Any N_3^U manager who buys risk free bond is only retained if both assets default. This means that the probability of the retainment is $\delta\omega f_1 f_2$. Note that if $p_1 = p_2$, both assets are paying the same return if they repay. Thus, the indifference conditions for N_3^U managers are

$$(1 - f_j)\left(\frac{\gamma}{p_j} + \delta\omega \beta W_3^U\right) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \quad (13)$$

where $j = 1, 2$.

Note that, N_3^U managers continuation payoff of being employed, W_3^U , is less than the continuation payoff of uninformed managers of specialized funds. This is because if both risky assets repay, N_3^U managers who buy the more expensive asset are fired. Nevertheless, N_j^U , $j = 1, 2$, managers buying the repaying risky asset are always retained. Notice that if $W_3^U \leq W_j^U$,

$$\gamma R + \delta\omega f_j \beta W_j^U > \gamma R + \delta + \omega f_1 f_2 \beta W_3^U \quad (14)$$

If $p = (p_1, p_2)$, $p_1 > p_2$, occurs in equilibrium and marginal traders are N_j^U managers, (14) and indifference conditions (11)-(12) imply that at $p =$

(p_1, p_2) the payoff to N_3^U managers of buying riskless bond is less than the payoff of buying asset j . When N_j^U are marginal traders at $p = (p_1, p_2)$ the maximum price that N_3^U are willing to pay for asset j is always higher than p_j . Therefore N_3^U managers are not marginal traders at $p = (p_1, p_2)$ and strictly demand the cheapest risky asset.

Let P_{jj}^U and P_{j3}^U denote the maximum prices that N_j^U and N_3^U pay for asset j . This means that P_{jj}^U and P_{j3}^U are solved from (10) and (11)-(12) and are given as

$$P_{jj}^U = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \quad (15)$$

$$P_{j3}^U = \frac{\gamma(1 - f_j)}{\gamma R + (f_1 f_2 - f_j - 1)\delta\omega\beta W_3^U} \quad (16)$$

3.2 Information Revelation in Asset Market Equilibrium

In equilibrium, price is a mapping from the space of stochastic shocks $(\chi_1, \chi_2) \times (y_1, y_2)$ to the interval $[\frac{y}{b_1}, \frac{1}{R}] \times [\frac{y}{b_2}, \frac{1}{R}]$. Clearly, the inverse mapping $(p^e)^{-1}(p)$ at any $p \in [\frac{y}{b_1}, \frac{1}{R}] \times [\frac{y}{b_2}, \frac{1}{R}]$ is a subset of $\{0, 1\}^2 \times [\underline{y}, \bar{y}]^2$. So any $p = p^e(\chi, y)$ is in principle revealing information about (χ, y) . Now the question is, how much information is revealed at a belief consistent equilibrium? Is there any belief consistent equilibrium at which price does not reveal any information, that is $p^e(\chi, y) = p$ for all (χ, y) ? Is there any equilibrium that is fully revealing, i.e. $p_j^e(\chi_j, y) \in \{\frac{1}{R}, \underline{p}_j\}$ for any (χ_j, y_j) ? Is there any equilibrium that is revealing for some (χ, \mathbf{y}) and unrevealing for other values of (χ, \mathbf{y}) ? Before answering these questions, let us first state the following proposition about the properties of the equilibrium price mappings.

Proposition 1. *Any rational expectations equilibrium price mapping $\mathbf{p}^e(\chi, \mathbf{y})$ satisfies the following conditions;*

- (i). *If $p_i^e = \frac{1}{R}$, then $\chi_i = 0$.*
- (ii). *If $p_i^e \in [\frac{y}{b_i}, \frac{\bar{y}}{b_i}]$, then $\chi_i = 1$.*

The proof of the above result is very simple and is omitted. If there is any equilibrium at which $p_i^e = \frac{1}{R}$ when $\chi_i = 1$, there would be no demand from informed or uninformed managers to buy asset i and only noise traders demand asset i at $\frac{1}{R}$. But then to clear the market p_i^e must be $\frac{y_i}{b_i}$ and not $\frac{1}{R}$.

Proposition 2. *Under assumption (9),*

(A1). *There is no belief consistent unrevealing equilibrium.*

(A2). *Suppose $\frac{\max\{b_1, b_2\}}{R} < \underline{y} + \min\{I_1, I_2\}$. There exists a revealing equilibrium. This equilibrium is not belief consistent.*

These results are similar to the results in Sami and Brusco (2014) and their proof is presented in the on-line appendix of the paper.

We know by the above proposition that none of the two extremes, no revelation and full revelation are possible or plausible. So the equilibrium must always be partially revealing; something is always leaked to the market. Indeed, this is the case in the base line model of Guerrieri and Kondor (2012). We show that there exist simple and non-simple partially revealing equilibria with a common property; when prices are not fully revealing they are *interdependent*.

Definition 5. *Suppose $\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^e(\chi, \mathbf{y}), p_2^e(\chi, \mathbf{y}))$ is an equilibrium price mapping. Then $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ are **interdependent** if there is at least one pair $(p_1, p_2) \in (\bar{p}, \frac{1}{R})^2$; $p_i^e(\chi, \mathbf{y}) = p_i$ for some (χ, \mathbf{y}) such that $Pr(p_1^e = p_1, p_2^e = p_2 \mid \chi_1, \chi_2) = Pr(p_1^e = p_1 \mid \chi_1)Pr(p_2^e = p_2 \mid \chi_2)$.*

We call $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ independent if they aren't interdependent. To understand Definition 5, suppose $\mathbf{p}^e(\chi, \mathbf{y})$ is a simple equilibrium price function where $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ are independent. Also assume that all the funds are global and there is no specialized fund. Let N^I be the mass of informed managers. Suppose $\mathbf{p}^e(\chi, \mathbf{y}) = (p_1, p_2) \in (\bar{p}, \frac{1}{R})^2$ for the following values of (χ, \mathbf{y}) ;

- $(\chi_1, \chi_2) = (0, 0)$ and $(y_1, y_2) \in [\underline{y}, \bar{y} - N^I]^2$
- $(\chi_1, \chi_2) = (0, 1)$ and $(y_1, y_2) \in [\underline{y}, \bar{y} - N^I] \times [\underline{y} + N^I, \bar{y}]$.
- $(\chi_1, \chi_2) = (1, 0)$ and $(y_1, y_2) \in [\underline{y} + N^I, \bar{y}] \times [\underline{y}, \bar{y} - N^I]$.
- $(\chi_1, \chi_2) = (1, 1)$ and $(y_1, y_2) \in [\underline{y} + N^I, \bar{y}]^2$

Suppose, $(\chi_1, \chi_2) = (0, 1)$. The probability of $\mathbf{p}^e(\chi, \mathbf{y}) = (p_1, p_2)$ is equivalent to the probability that $(y_1, y_2) \in [\underline{y}, \bar{y} - N^I] \times [\underline{y} + N^I, \bar{y}]$ and is equal to $(1 - \frac{N^I}{\bar{y} - \underline{y}})^2$. But note that $Pr p_1^e(\chi, \mathbf{y}) = p_1, p_2^e(\chi, \mathbf{y}) = p_2 \mid \chi_1, \chi_2$ is equal to $(1 - \frac{N^I}{\bar{y} - \underline{y}})^2$ for any (χ_1, χ_2) . Furthermore, $Pr p_i^e(\chi, \mathbf{y}) = p_i \mid \chi_i, \chi_j, p_j^e = p_j = 1 - \frac{N^I}{\bar{y} - \underline{y}}$ for any value of χ_i, χ_j and $p_j \in (\bar{p}, \frac{1}{R})$. Hence, by Definition 5, p_1^e and p_2^e are independent of each other.

When prices are independent, conditional on χ_1 the probability of p_1^e being unrevealing is independent of p_2^e and χ_2 . Therefore, if p_1^e is unrevealing, uninformed managers at all funds know that price of asset 2 reveals nothing about the state of asset 1.

Observing $p_1^e(\chi, \mathbf{y}) = p_1$ and $p_2^e(\chi, \mathbf{y}) = p_2$, uninformed managers at **all** funds try to figure out the probability of the repayment of the assets by learning the actions of informed managers of **both** specialized and global funds. As long as the mass of I_3 funds is non-zero, uninformed managers form their posteriors about asset i taking into account both p_i and p_j . Recall that $1 - f_j(p_1, p_2) \equiv Pr(\chi_j = 0 | \underline{p} < p_1 < \frac{1}{R}, \underline{p} < p_2 < \frac{1}{R})$, i.e., $1 - f_j(p_1, p_2)$ is the posterior of the uninformed managers funds after observing a price pair (p_1, p_2) . Next result shows that prices are interdependent in any equilibrium.

Proposition 3. *As long as there are some global funds in the market, prices are interdependent in any equilibrium.*

To understand the intuition behind this result, suppose asset 2 defaults and p_2 is unrevealing. When asset 1 repays, informed managers of global funds and informed managers of funds specializing in market 1 are all demanding asset 1. However, informed managers of global funds can demand either asset 1 or asset 2 when both assets repay. So when asset 1 repays and asset 2 defaults the demand for asset 1 is higher than when both assets repay. This implies that the probability that $p_1 = \frac{1}{R}$ and the repay of asset 1 is revealed is higher when asset 2 defaults and p_2 is unrevealing. Therefore, default or repay of asset 2 changes the probability that p_1 is revealing the repay of asset 1. Therefore, p_1 can not be independent of the repay or default of asset 2 and p_2 .

But since prices are interdependent by the demands of N_3^I managers, uninformed managers face an adverse selection problem. When prices are unrevealing, uninformed managers receive asset 1 with a lower probability when asset 2 defaults and with a higher probability when asset 2 repays or asset 1 is defaulting. In equilibrium, price of asset 1 must compensate uninformed managers for this adverse selection problem and must decrease following any shock to the ex-ante default probabilities of both asset 1 and asset 2. When q_2 increases, the default of asset 2 is more likely and price of asset 2 suffers. Also, the adverse selection problem in market 1 is more severe because the probability of receiving the repaying asset 1 decreases. Hence, price of asset 1 must also decrease to compensate uninformed managers for the risk of not receiving the repaying asset 1. When there is no global fund, there is no adverse selection problem and there is no co-movement.

As long as there are some global funds and the mass of N_3^I managers is not zero, their trades contain information regarding both assets and in equilibrium market clearing prices reveal this information to all uninformed managers. When there is no I_3 fund- and no N_3^I manager- the price of asset 1 only contains the information revealed by the demands of N_1^I managers. Since N_1^I managers never demand asset 2, p_1 has no information regarding the repay or default of asset 2. When the investment strategy is specialization in one market managers are evaluated only based on the returns of that particular market. But when the investment strategy is to seek investment opportunity in as many markets as possible managers returns are compared with the highest return among all the markets. Therefore, even a small mass of global funds is enough to induce the rest of managers in I_1 (I_2) funds to extract information about market 1 from the actions of N_3^I managers at market 2. Notice that the interdependence is amplified by the continuation payoff of being employed. To see this better, let $\mathbf{p}^e(\chi, \mathbf{y}) = (P_{11}^U, P_{22}^U)$, that is N_1^U and N_2^U managers are marginal traders in equilibrium. This means that

$$P_{jj}^U = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \quad (17)$$

and

$$\frac{1 - f_j}{P_{jj}^U} - R = (2f_j - 1)\frac{W_j^U}{\gamma} \quad (18)$$

This premium is similar to the reputational premium in Guerrieri and Kondor (2012) and disappears as soon as $W_j^U = 0$. However, even if $W_j^U = 0$, $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ are still interdependent. This is because at (P_{11}^U, P_{22}^U) , uninformed traders face the same signal extraction problem of uninformed managers. Hence, the posteriors of uninformed traders, f_j , are not independent of $p_i^e = P_{ii}^U$ and the repay or default of asset i . Thus, P_{jj}^U are functions of q_1 and q_2 and any change in q_1 and q_2 shifts both P_{11}^U and P_{22}^U . The following Corollary summarizes the discussion.

Corollary 1. *As long as there are some global funds in the market, prices are co-moving in any equilibrium following any shock to priors.*

3.3 Simple Equilibria

In this section we characterize some simple partially revealing equilibria. The following Lemma gives the posteriors of uninformed managers at any

simple equilibrium with unequal unrevealing prices. In all the following results assume $r_j = \frac{N_j^I}{\bar{y}-y}$.

Lemma 1. *Assume $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$. In any simple equilibrium at which with positive probability $(p_1^e, p_2^e) = (p_1, p_2)$ where $\bar{p} < p_2 < p_1 < \frac{1}{R}$, posteriors of uninformed managers about risky assets at (p_1, p_2) are given as*

$$1 - f_1 = \frac{(1 - q_1)(1 - r_1 - q_2 r_3)}{1 - r_1 - (1 - q_1)q_2 r_3} \quad (19)$$

$$1 - f_2 = \frac{(1 - q_2)(1 - r_1)}{1 - r_1 - (1 - q_1)q_2 r_3} \quad (20)$$

Next, we derive the posteriors of uninformed managers when unrevealing prices are equal. When unrevealing prices are equal, informed managers of I_3 funds are indifferent between risky assets when both repay. So we can assume that α fraction of them only asks asset 1 and $1 - \alpha$ fraction of them asks asset 2 where α is determined in equilibrium so that the posteriors of uninformed managers at $p^e = (p, p)$ are equal. The following Lemma is giving these equal posteriors in an equilibrium with equal prices.

Lemma 2. *Assume $\frac{r_3}{1-r_1} > \frac{q_2-q_1}{(1-q_1)q_2}$. In any simple equilibrium at which with positive probability $(p_1^e, p_2^e) = (p, p)$; $\bar{p} < p < \frac{1}{R}$ is realized the posterior beliefs of uninformed managers about risky assets are equal and given as follows*

$$1 - f_1 = \frac{(1 - q_1)(1 - r_2 - (1 - \alpha^*)r_3)(1 - r_1 - r_3(q_2 + (1 - q_2)\alpha^*))}{(1 - q_1)G_0(r_1, r_2, r_3, \alpha^*) + q_1 G_1(r_1, r_2, r_3, \alpha^*)} \quad (21)$$

$$1 - f_2 = \frac{(1 - q_2)(1 - r_1 - \alpha^* r_3)(1 - r_2 - r_3 + \alpha^*(1 - q_1)r_3)}{(1 - q_1)G_0(r_1, r_2, r_3, \alpha^*) + q_1 G_1(r_1, r_2, r_3, \alpha^*)} \quad (22)$$

where

$$G_0(r_1, r_2, r_3, \alpha^*) = (1 - r_2 - (1 - \alpha^*)r_3)(1 - r_1 - r_3(q_2 + (1 - q_2)\alpha^*)) \quad (23)$$

$$G_1(r_1, r_2, r_3, \alpha^*) = (1 - r_1 - \alpha^* r_3)(1 - r_2 - r_3(1 - q_2 + (1 - \alpha^*)q_2)) \quad (24)$$

and α^* is the solution to

$$\begin{aligned} & (1 - q_1)(1 - r_2 - (1 - \alpha)r_3)(1 - r_1 - r_3(1 - q_2)\alpha - r_3 q_2) \\ & - (1 - q_2)(1 - r_1 - \alpha r_3)(1 - r_2 - r_3 + \alpha(1 - q_1)r_3) = 0 \end{aligned} \quad (25)$$

Lemmas 1 and 2 show clearly that at any non-revealing price, the posteriors of uninformed managers are not the same as their priors. The difference between f_j and q_j is the information leaked to the market at any equilibrium. Note that Lemmas 1 and 2 put mutually exclusive conditions on $\frac{r_3}{1-r_1}$ so the posteriors of uninformed managers are always well defined for any value of N_1^U and N_3^U .

In Sami and Brusco (2014), we proved that as long as there are no specialized funds in the market, there is no equilibrium with unequal unrevealing prices. Next proposition shows the existence of such equilibrium at this model. In this equilibrium, the information that is revealed to the market is not enough to convince uninformed managers that the probability of the repay of asset 2 is as high as asset 1.

Proposition 4. Assume $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$ and

$$(i). I_3 < \underline{C} - M^I - \bar{y}$$

$$(ii). \min\{I_1, I_2\} > \bar{C} + M^I$$

where \underline{C} and \bar{C} are given in the Appendix. There exists an equilibrium at which $p^e(\chi, y)$ takes the following values;

- some revelation;

$$p_j^e = P_{jj}^U = \frac{\gamma(1-f_j)}{\gamma R + (2f_j-1)\delta\omega\beta W_j^U} \quad (26)$$

where $1-f_1$ and $1-f_2$ are given in Lemma 1.

- partial revelation at which either $p_j^e \leq \bar{p}_j$ or $p_j^e = \frac{1}{R}$.

$$p_i^e = \frac{\gamma(1-q_i)}{\gamma R + (2q_i-1)\delta\omega\beta W_i^U} \quad (27)$$

- full revelation at which for each $i = 1, 2$ we have either $p_i \leq \bar{p}_i$ or $p_i = \frac{1}{R}$.

Moreover, $\underline{p} < P_{22}^U < P_{11}^U < \frac{1}{R}$ and marginal traders at (P_{11}^U, P_{22}^U) are uninformed managers of specialized funds.

When there are no specialized funds, the demands for any asset is only coming from the managers of global funds. At $p_1 > p_2$, uninformed managers are marginal traders; and again face the same adverse selection problem; the

probability of receiving asset 1 when it defaults or asset 2 repays is higher than the probability of receiving it when it repays and asset 2 defaults. Therefore, an equilibrium with unequal non-revealing prices exists only if we have enough specialized funds in the market so that the demands of N_j^U managers can clear the market and make them marginal traders.

When the informed managers hired at I_3 funds are relatively less than the informed managers hired at I_1 funds (so that $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$), the investments of N_3^I managers are not enough to change the prior belief of uninformed managers about asset 1 being the highest repaying asset. Moreover, assumptions (i) and (ii) imply that $I_3 < \min\{I_1, I_2\}$. This means that the total investments made by global funds is less than the investments of specialized funds. When the total investments of global funds are low, markets are cleared only if there is positive demands from specialized funds and equilibrium prices are set by uninformed managers of specialized funds. Thus, prices do not contain as much information as they would if there were more informed managers hired at I_3 funds and the size of I_3 funds were.

When marginal traders are uninformed managers of specialized funds and $p_j^e = P_{jj}^U$, maximum prices that N_3^U managers pay are solved from equations (11)-(12). Note that N_3^U managers continuation payoff of being employed, W_3^U , is less than the continuation payoff of uninformed managers of specialized funds. This is because if both risky assets repay, N_3^U managers who buy the more expensive asset are fired. Nevertheless, $N_j^U, j = 1, 2$, managers buying the repaying risky asset are always retained. Therefore, when N_j^U managers are marginal traders at p_j^e their payoff of buying risk less bond is $\gamma R + \delta\omega f_j \beta W_j^U$ while the payoff to N_3^U managers of buying risk free bond is $\gamma R + \delta\omega f_1 f_2 W_3^U$. Clearly, at $p_j^e = P_{jj}^U$ the payoff to N_3^U managers of buying riskless bond is less than the payoff of buying asset j , therefore $P_{j3}^U > P_{jj}^U$ and it is not optimal for N_3^U managers to be indifferent between risky assets and risk less bond at $p_j^e = P_{jj}^U$.

The next result characterizes the equilibrium when N_3^I is large. When the size of global funds is large relative to specialized funds, the demands of N_3^U managers clear the market and the only possible equilibrium is the one with equal unrevealing prices. In this equilibrium marginal traders are uninformed managers of global funds. When $p_1^e = p_2^e$ both assets are paying the same expected return, so managers hired at I_3 funds are fired if they buy the defaulting risky asset or risk free bond when at least one of the assets

is repaying. Thus, P_{j3}^U now solves

$$(1 - f_j)\left(\frac{\gamma}{P_{j3}^U} + \delta\omega\beta W_3^U\right) = \gamma R + \delta\omega f_1 f_2 W_3^U \quad (28)$$

where $1 - f_1$ and $1 - f_2$ are given in Lemma 2. The following proposition summarizes this discussion.

Proposition 5. *Assume that $\frac{r_3}{1-r_1} > \frac{q_2-q_1}{(1-q_1)q_2}$ and*

$$(i). I_3 > \frac{b_1+b_2}{R} + M^I.$$

$$(ii). \min\{I_1, I_2\} - M^I > \frac{\max\{b_1, b_2\}}{R}.$$

Then, there exists an equilibrium at which $\mathbf{p}^e(\chi, \mathbf{y})$ takes the following values;

- *some revelation;*

$$p_j^e = P_{j3}^U = \frac{\gamma(1 - f_j)}{\gamma R + (f_1 f_2 - f_j - 1)\delta\omega\beta W_3^U} \quad (29)$$

where $P_{13}^U = P_{23}^U \in (\bar{p}, \frac{1}{R})$, and $1 - f_1$ and $1 - f_2$ are given in Lemma 2.

- *partial revelation at which either $p_j^e \leq \bar{p}_j$ or $p_j^e = \frac{1}{R}$ and,*

$$p_i^e = \frac{\gamma(1 - q_i)}{\gamma R + (2q_i - 1)\delta\omega\beta W_i^U} \quad (30)$$

- *full revelation at which for each $i = 1, 2$ we have either $p_i \leq \bar{p}_i$ or $p_i = \frac{1}{R}$.*

Moreover, marginal traders at P_{j3}^U are uninformed managers of global funds.

3.4 Non-Simple Equilibrium

Up to now we just focused on simple equilibria where p^e only gets a unique value in $(\bar{p}, \frac{1}{R})^2$. Suppose $(\chi_1, \chi_2) = (0, 0)$. Suppose also that I_1 and I_3 funds have hired very few informed managers. Then, the mass of informed managers in I_3 funds is not enough to reveal enough information to convince the uninformed managers to bid the same price for both assets in equilibrium. Moreover, for some values of liquidity trading equilibrium prices do not reveal any information to uninformed managers and the posteriors of

uninformed managers are the same as their priors, i.e. $1 - f_i = 1 - q_i$. As we discussed in Sami and Brusco (2014) because of adverse selection problem that arises for marginal N_3^U managers when prices are different, the equilibrium only exists if the marginal traders are uninformed managers of specialized funds.

Proposition 6. *Suppose*

$$\frac{(r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)]}{(r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3)} < \frac{(1 - q_1)q_2}{q_1(1 - q_2)}$$

Assume also,

- (i). $I_3 < \underline{C} - M^I - \bar{y}$.
- (ii). $\min\{I_1, I_2\} > \bar{C} + M^I$.

There exists a partially revealing, non-simple equilibrium at which $\mathbf{p}^e(\chi, \mathbf{y})$ takes the following values;

- no revelation;

$$p_j^e = P_{jj}^U(q_j) = \frac{\gamma(1 - q_j)}{\gamma R + (2q_j - 1)\delta\omega\beta W_j^U} \quad (31)$$

where $j = 1, 2$.

- some revelation;

$$p_j^e = P_{jj}^U(f_j) = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \quad (32)$$

where $j = 1, 2$, and f_j is given in the appendix.

- partial revelation at which either $p_j^e \leq \bar{p}_j$ or $p_j^e = \frac{1}{R}$ and,

$$p_i^e = \frac{\gamma(1 - q_i)}{\gamma R + (2q_i - 1)\delta\omega\beta W_i^U} \quad (33)$$

- full revelation at which for each $i = 1, 2$ we have either $p_i \leq \bar{p}_i$ or $p_i = \frac{1}{R}$.

Moreover, marginal traders at P_{jj}^U are uninformed managers of specialized funds.

$p_1^e((\chi, \mathbf{y}))$ and $p_2^e((\chi, \mathbf{y}))$ co-move because at $(P_{11}^U(f_1), P_{22}^U(f_2))$, the probability of $p_j^e = P_{jj}^U$ is not independent of χ_i and $p_i^e = P_{ii}^U$. However, when no information is revealed through prices at $(P_{11}^U(q_1), P_{22}^U(q_2))$, the probability that $p_1^e = P_{11}^U(q_1)$ is independent of the repay or the default of asset 2 and $p^e = P_{22}^U$.

Price co-movements only disappear when there is no I_3 fund in the market. But in that case, less information leaks to the market as well.

3.5 Optimal Retention Rule

The optimal behavior of funds in our model is identical to the one in Guerrieri and Kondor (2012). We have to prove that the firing rule of funds are optimal. Specialized funds fire their managers when they buy the defaulting asset or riskless bond when asset repays. Global funds fire the managers if they don't achieve the highest ex-post return. Given that the return signals to informed managers are perfect, any manager with wrong investment decision is immediately revealed uninformed with probability 1. If the percentage of the informed managers in unemployment pool is always non-zero, it is optimal to fire an uninformed manager. But recall that a fraction δ of informed managers is always separated from the funds. Separated or unemployed informed managers always search for a job because by free entry condition for uninformed managers, informed managers get a positive expected pay-off if they look for a job. Thus, unemployment pool is never empty of informed managers and it is optimal to fire a manager that is revealed uninformed. It remains to show that funds retain a manager who has made the right investment decision and is not revealed uninformed by exogenous signal. This is the case when the updated belief of funds about manager being informed is higher than the probability that a just hired manager is informed, i.e.,

$$\eta_{t+1} > \epsilon_t = \frac{\tilde{L}_t^I}{\tilde{L}_t^I + \tilde{L}_t^U} \quad (34)$$

But (34) holds given the assumption $\omega > \frac{1}{1+\delta}$, by exactly the same arguments in the proof of Proposition 1 of Guerrieri and Kondor (2012) and Proposition 7 of Sami and Brusco (2014) and assuming . This assumption ensures that when a manager is not revealed uninformed and has not made any mistake the beliefs of funds improves with a high enough speed that surpasses the probability of hiring an informed manager from the unemployment pool.

4 Conclusion

This paper discussed price co-movement between two financial markets in a risk neutral world with independent liquidity and return shocks. The investment decisions of funds are delegated to fund managers who are informed or uninformed on the return of the assets and face dismissal if they don't make the highest possible return. We showed that in any equilibrium of the model prices co-move with each other following a shock to the priors on any asset. In equilibrium, market clearing prices reflect all the information available in the market. As long as there are some global funds in the market, the demands of informed managers hired at these funds reveal information about both assets. When the total mass of informed managers is so low that market is not cleared, the equilibrium price must make uninformed managers marginal traders. But as long as there are some global funds in the market, with higher probability uninformed managers receive the defaulting asset or riskfree bond when both prices are unrevealing. In equilibrium, prices must compensate uninformed managers for this adverse selection problem and are functions of the ex-ante default probabilities of both assets. Hence, any shock to the ex-ante default probability of one asset changes the price of both assets.

The reputationally concerned managers always ask for a premium over the risk free rate that compensates them for the risk of being dismissed. This premium magnifies the co-movement between prices. However, the co-movement doesn't disappear if there is no reputational concern. This means that even if investors were directly investing their capital and weren't delegating the investment decision to fund managers, prices would still co-move with each other. Co-movement only disappears when there is no global fund in the market. But if there is no global fund, less information is revealed to the market. This suggests that there is a trade-off between market stability and information revelation. Global funds increase the price co-movement but reveal more information to the markets. Without global funds there is no price co-movement and more market stability, but less information revelation as well.

5 Appendix

In all the following results assume $q_2 > q_1$. Note that this leads to $P_{22}^U < P_{11}^U$. Besides, note that by (19), in the equilibrium with unequal prices $1 - f_2 > 1 - q_2$ and $1 - f_1 < 1 - q_1$, hence,

$$P_{11}^U < \bar{P} = \frac{\gamma(1 - q_1)}{\gamma R + \delta\omega\beta(2q_1 - 1)W_1^U} \quad (35)$$

$$P_{22}^U > \frac{\gamma(1 - q_2)}{\gamma R + \delta\omega\beta(2q_2 - 1)W_2^U} > \underline{P} = \frac{(1 - q_2)(1 - \frac{M^I}{\Delta y})^2}{\gamma R + \delta\omega\beta W_3^U} \quad (36)$$

Let $\bar{C} = \frac{\max\{b_1, b_2\}}{R}$ and $\underline{C} = \min\{b_1, b_2\}\underline{P}$. We rewrite the supply assumption as follows;

$$M^I + \bar{y} < \min\{b_1, b_2\}\underline{P} \quad (37)$$

Claim 1. Suppose $p^e(\chi, y)$ is an equilibrium price mapping. Define

$$\mathcal{I} = \left\{ (p_1, p_2) \in (\bar{p}, \frac{1}{R})^2 \mid p_1 > p_2, p^e(\chi, y) = (p_1, p_2) \text{ for some } (\chi, y) \right\}$$

Also, define

$$\Phi_{(\chi_1, \chi_2)} = \{y = (y_1, y_2) \mid p^e(\chi, y) = (p_1, p_2) \text{ for some } (p_1, p_2) \in \mathcal{I}\} \quad (38)$$

Then,

$$(A1). \quad \Phi_{00} = \{(y_1, y_2) \mid \underline{y} \leq y_1 \leq \bar{y} - N_1^I, \underline{y} \leq y_2 \leq \bar{y} - N_2^I - N_3^I\}.$$

$$(A2). \quad \Phi_{01} = \{(y_1, y_2) \mid \underline{y} \leq y_1 \leq \bar{y} - N_1^I - N_3^I, \underline{y} + N_2^I + N_3^I \leq y_2 \leq \bar{y}\}.$$

$$(A3). \quad \Phi_{10} = \{(y_1, y_2) \mid \underline{y} + N_1^I \leq y_1 \leq \bar{y}, \underline{y} \leq y_2 \leq \bar{y} - N_2^I - N_3^I\}.$$

$$(A4). \quad \Phi_{11} = \{(y_1, y_2) \mid \underline{y} + N_1^I \leq y_1 \leq \bar{y}, \underline{y} + N_2^I + N_3^I \leq y_2 \leq \bar{y}\}$$

Proof. (i). Let $y \in [\underline{y}, \bar{y} - N_1^I] \times [\underline{y}, \bar{y} - N_2^I - N_3^I]$, then $z_i((0, 0), (y_1, y_2)) = z_i((1, 1), (y_1 + N_1^I, y_2 + N_2^I + N_3^I))$. Thus, by belief consistency condition we must have $p_i^e((0, 0), (y_1, y_2)) = p_i^e((1, 1), (y_1 + N_1^I, y_2 + N_2^I + N_3^I))$. But this is possible only if equilibrium prices are unrevealing and $p_2^e \leq p_1^e$. Therefore, $[\underline{y}, \bar{y} - N_1^I] \times [\underline{y}, \bar{y} - N_2^I - N_3^I] \subset \Phi_{00}$.

Let $(y_1, y_2) \in \Phi_{00}$ but $(y_1, y_2) \notin [\underline{y}, \bar{y} - N_1^I] \times [\underline{y}, \bar{y} - N_2^I - N_3^I]$. This means that $z_2((0, 0), (y_1, y_2)) \geq \bar{y}$ which reveals repay for asset 2 and hence p_2^e must be $1/R$. Contradiction.

(ii). Let $y \in [\underline{y}, \bar{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \bar{y}]$, then $z_i((0, 1), (y_1, y_2)) = z_i((1, 1), (y_1 + N_1^I + N_3^I, y_2))$. Thus, by belief consistency condition we must have $p_i^e((0, 1), (y_1, y_2)) = p_i^e((1, 1), (y_1 + N_1^I + N_3^I, y_2))$ which is possible only if prices are unrevealing. Therefore, $[\underline{y}, \bar{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \bar{y}] \subset \Phi_{01}$.

If $(y_1, y_2) \in \Phi_{01}/[\underline{y}, \bar{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \bar{y}]$, then $y_1 > \bar{y} - N_1^I - N_3^I$ and $z_1((0, 1), (y_1, y_2)) > \bar{y}$ which implies $p_1 = 1/R$. Contradiction.

(iii). Similar to (2).

(iv). Let $y \in [\underline{y} + N_1^I, \bar{y}] \times [\underline{y} + N_2^I + N_3^I, \bar{y}]$, then $z_i((1, 1), (y_1, y_2)) = z_i((0, 0), (y_1 + N_1^I, y_2 + N_2^I + N_3^I))$ and by belief consistency $p_i(1, 1), (y_1, y_2) = p_i((0, 0), (y_1 + N_1^I, y_2 + N_2^I + N_3^I))$ which is possible only if prices are not revealing. So $[\underline{y} + N_1^I, \bar{y}] \times [\underline{y} + N_2^I + N_3^I, \bar{y}] \subset \Phi_{11}$.

If $y \in \Phi_{11}/[\underline{y} + N_1^I, \bar{y}] \times [\underline{y} + N_2^I + N_3^I, \bar{y}]$, then $y_1 < \underline{y} + N_1^I$ and $z_1((1, 0), (y_1, y_2)) < \underline{y} + N_1^I$ which implies that N_1^I managers haven't demanded asset 1. This only happens when asset is defaulting so $p_1 = \underline{p}$ which is a contradiction. \square

Claim 2. Suppose $p^e(\chi, y)$ is an equilibrium price mapping. Define

$$\mathcal{I} = \{(p, p) \in (\bar{p}, \frac{1}{R})^2 \mid p^e(\chi, y) = (p, p), \text{ for some } (\chi, y)\}$$

Also, define

$$\Phi'_{(\chi_1, \chi_2)} = \{y = (y_1, y_2) \mid p^e(\chi, y) = (p, p) \text{ for some } (p, p) \in \mathcal{I}\} \quad (39)$$

Then,

(i). $\Phi'_{00} = \{y = (y_1, y_2) \mid \underline{y} \leq y_1 \leq \bar{y} - (N_1^I + \alpha N_3^I), \underline{y} \leq y_2 \leq \bar{y} - N_2^I - (1 - \alpha)N_3^I\}$.

(ii). $\Phi'_{01} = \{(y_1, y_2) \mid \underline{y} \leq y_1 \leq \bar{y} - N_1^I - N_3^I, \underline{y} + N_2^I + (1 - \alpha)N_3^I \leq y_2 \leq \bar{y}\}$.

(iii). $\Phi'_{10} = \{(y_1, y_2) \mid \underline{y} + N_1^I + \alpha N_3^I \leq y_1 \leq \bar{y}, \underline{y} \leq y_2 \leq \bar{y} - N_2^I - N_3^I\}$.

(iv). $\Phi'_{11} = \{(y_1, y_2) \mid \underline{y} + N_1^I + \alpha N_3^I \leq y_1 \leq \bar{y}, \underline{y} + N_2^I + (1 - \alpha)N_3^I \leq y_2 \leq \bar{y}\}$.

where α is the fraction of N_3^I that buy asset 1 when $p^e(\chi, y) = (p, p)$.

Proof. Similar to the proof of Claim 1. \square

Lemma 1. Note that by Claim 1

$$\begin{aligned} Pr((p_1^e, p_2^e) \in (\bar{p}, \frac{1}{R})) &= (1 - q_1)(1 - q_2)Pr((y_1, y_2) \in \Phi_{00}) + (1 - q_1)q_2Pr((y_1, y_2) \in \Phi_{01}) \\ &\quad + q_1(1 - q_2)Pr((y_1, y_2) \in \Phi_{10}) + q_1q_2Pr((y_1, y_2) \in \Phi_{11}) \end{aligned} \quad (40)$$

Therefore,

$$1 - f_1 = \frac{(1 - q_1)(1 - r_1 - q_2r_3)}{1 - r_1 - (1 - q_1)q_2r_3} \quad (41)$$

$$1 - f_2 = \frac{(1 - q_2)(1 - r_1)}{1 - r_1 - (1 - q_1)q_2r_3} \quad (42)$$

It is clear that $0 < 1 - f_1 < 1$. Since, $\frac{1}{1 - q_1} > \frac{r_3}{1 - r_1}$, $0 < 1 - f_2 < 1$ as well. Also since $\frac{r_3}{1 - r_1} < \frac{q_2 - q_1}{(1 - q_1)q_2}$, $1 - f_1 > 1 - f_2$. \square

Proof of Lemma 2. Recall that if in equilibrium $\underline{p} < p_i^e < 1/R$, marginal traders are either uninformed managers hired at I_3 funds or uninformed managers at I_j funds. Then $p_1^e = p_2^e = p$ is only possible if posteriors of uninformed managers are also equal. If informed managers of global funds are indifferent between the risky assets when both repay and have the same price, the only strict demand for any asset is coming from the informed managers in specialized funds. But in this case, the posteriors are not equal. Thus, we need to have α^* fraction of N_3^I managers demanding asset 1 and $(1 - \alpha^*)$ of them demanding only asset 2 when both assets repay and have the same price where α^* is determined in equilibrium to equate the posteriors on risky assets.

Next, note that

$$1 - f_i = Pr(\chi_i = 0 | p_1^e = p, p_2^e = p) = \frac{Pr(p_1^e = p, p_2^e = p, \chi_i = 0)}{Pr(p_1^e = p, p_2^e = p)} \quad (43)$$

Using Claim 2,

$$\begin{aligned} Pr(p_1^e = p, p_2^e = p, \chi_1 = 0) &= (1 - q_1)(1 - q_2)Pr((y_1, y_2) \in \Phi_{00}) + (1 - q_1)q_2Pr((y_1, y_2) \in \Phi_{01}) \\ Pr(p_1^e = p, p_2^e = p) &= (1 - q_1)(1 - q_2)Pr((y_1, y_2) \in \Phi_{00}) + (1 - q_1)q_2Pr((y_1, y_2) \in \Phi_{01}) \\ &\quad + q_1(1 - q_2)Pr((y_1, y_2) \in \Phi_{10}) + q_1q_2Pr((y_1, y_2) \in \Phi_{11}) \end{aligned} \quad (44)$$

Therefore,

$$1 - f_1 = \frac{(1 - q_1)(1 - r_2 - (1 - \alpha^*)r_3)(1 - r_1 - r_3(1 - q_2)\alpha^* - r_3q_2)}{(1 - q_1)G_0(r_1, r_2, r_3, \alpha^*) + q_1G_1(r_1, r_2, r_3, \alpha^*)} \quad (45)$$

$$1 - f_2 = \frac{(1 - q_2)(1 - r_1 - \alpha^*r_3)(1 - r_2 - r_3 + \alpha^*(1 - q_1)r_3)}{(1 - q_1)G_0(r_1, r_2, r_3, \alpha^*) + q_1G_1(r_1, r_2, r_3, \alpha^*)} \quad (46)$$

where

$$G_0(r_1, r_2, r_3, \alpha^*) = (1 - r_2 - (1 - \alpha^*)r_3)(1 - r_1 - r_3 + (1 - q_2)(1 - \alpha^*)r_3) \quad (47)$$

$$G_1(r_1, r_2, r_3, \alpha^*) = (1 - r_1 - \alpha^*r_3)(1 - r_2 - r_3 + q_2\alpha^*r_3) \quad (48)$$

Since we must have equal posteriors, α^* is the solution to

$$H(\alpha) \equiv (1 - q_1)(1 - r_2 - (1 - \alpha)r_3)(1 - r_1 - r_3(1 - q_2)\alpha - r_3q_2) - (1 - q_2)(1 - r_1 - \alpha r_3)(1 - r_2 - r_3 + \alpha(1 - q_1)r_3) = 0 \quad (49)$$

Note that, $H(\alpha = 1) > 0$ by $q_2 > q_1$ and $H(\alpha = 0) < 0$ by assumption. Thus, there exists $0 < \alpha^* < 1$ at which $1 - f_1 = 1 - f_2$. \square

Proof of Proposition 3. At any simple equilibrium with unequal prices,

$$Pr(p_1^e = p_1, p_2^e = p_2 \mid (\chi_1, \chi_2) = (0, 0)) = Pr(\phi_{00}) = (1 - r_1)(1 - r_2 - r_3) \quad (50)$$

. But,

$$Pr(p_1^e = p_1 \mid \chi_1 = 0) = (1 - r_1)(1 - q_2) + (1 - r_1 - r_3)q_2 \quad (51)$$

$$Pr(p_2^e = p_2 \mid \chi_2 = 0) = (1 - r_2 - r_3)(1 - q_1) + (1 - r_2 - r_3)q_1 = (1 - r_2 - r_3) \quad (52)$$

The probabilities in (50)-(52) show that at any simple equilibrium with different unrevealing prices, p_1^e and p_2^e are interdependent. The same argument together with the use of Claim 2 shows that at any simple equilibrium with equal prices, p_1^e and p_2^e interdependent.

Suppose $\mathbf{p}^e(\chi, \mathbf{y})$ is a non-simple equilibrium price vector. Define $\mathcal{P} = \{U_{\chi_1\chi_2}^i\}_{i=1}^n$ as a partition of $\phi_{\chi_1\chi_2}$ and suppose $\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \in (\bar{p}, \frac{1}{R})^2$; $p_1^i \geq p_2^i$, for any $(y_1, y_2) \in U_{\chi_1\chi_2}^i$. Also assume that $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ are independent. Therefore, for any $i = 1, \dots, n$,

$$Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) = Pr(p_1^e(\chi, \mathbf{y}) = p_1^i \mid \chi_1 = 0)Pr(p_2^e(\chi, \mathbf{y}) = p_2^i \mid \chi_2 = 0) \quad (53)$$

and

$$Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) = Pr(p_1^e(\chi, \mathbf{y}) = p_1^i \mid \chi_1 = 0)Pr(p_2^e(\chi, \mathbf{y}) = p_2^i \mid \chi_2 = 1) \quad (54)$$

Assume $p_1^i > p_2^i$ or $p_1^i = p_2^i$ for any i . By Claims 1 and 2

$$\sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) > \sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) \quad (55)$$

By (53) and (54),

$$\begin{aligned} \sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) &= \sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) \times \\ &\quad \frac{Pr(p_2^i \mid \chi_2 = 1)}{Pr(p_2^i \mid \chi_2 = 0)} \end{aligned} \quad (56)$$

But by (55), we must have $\frac{Pr(p_2^i \mid \chi_2 = 1)}{Pr(p_2^i \mid \chi_2 = 0)} < 1$ for some i . Since prices are independent for any i ,

$$\begin{aligned} Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 1) &= Pr(p_1^i \mid \chi_1 = 1) \cdot Pr(p_2^i \mid \chi_2 = 1) \\ &< Pr(p_1^i \mid \chi_1 = 1) \cdot Pr(p_2^i \mid \chi_2 = 0) \\ &= Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 0) \end{aligned} \quad (57)$$

Therefore,

$$\sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 1) < \sum_{i=1}^n Pr(\mathbf{p}^e(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 0) \quad (58)$$

But by Claims 1 and 2, (58) is equivalent to $Pr((y_1, y_2) \in \Phi_{11}) < Pr((y_1, y_2) \in \Phi_{10})$. Also, $Pr((y_1, y_2) \in \Phi_{11}) = Pr((y_1, y_2) \in \Phi_{00})$. This means that $Pr((y_1, y_2) \in \Phi_{00}) < Pr((y_1, y_2) \in \Phi_{10})$ but this is a contradiction by Claims 1 and 2.

Now, let $p^e(\chi, y)$ be a non-simple equilibrium price mapping. Suppose also that for some (χ, y) , $p^e(\chi, y) = (p_1^i, p_2^i)$ and $p_1^i \geq p_2^i$, and for some (χ, y) , $p^e(\chi, y) = (p_1^j, p_2^j)$ and $p_1^j < p_2^j$. Note that symmetric with Φ_{χ_1, χ_2} defined in

Claim 1, we can define Φ''_{χ_1, χ_2} as the superset of all $y = (y_1, y_2)$ that don't reveal $\chi = (\chi_1, \chi_2)$ and $p_1^e(\chi, y) = p_1 < p_2 = p_2^e(\chi, y)$.

For any (χ_1, χ_2) , the super set of all (y_1, y_2) for which $p^e(\chi, y) \in (\bar{p}, \frac{1}{R})^2$ is

$$\Psi_{\chi_1, \chi_2} = \Phi_{\chi_1, \chi_2} \cup \Phi'_{\chi_1, \chi_2} \cup \Phi''_{\chi_1, \chi_2}$$

Again, let $\{U_{\chi_1, \chi_2}^i\}_i$ be a partition of Ψ_{χ_1, χ_2} where $p^e(\chi, y) = (p_1^i, p_2^i)$ for $y \in U_{\chi_1, \chi_2}^i$. By the same arguments, inequality (55) holds and if $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ are independent, then $Pr((y_1, y_2) \in \Psi_{11}) < Pr((y_1, y_2) \in \Psi_{10})$ which is a contradiction. \square

Proof of corollary 1. By assumption (9), demands of informed managers and liquidity traders never clear the markets. So in any equilibrium, there must be some amount of risky assets that is allocated to uninformed managers. Since supply is not enough to allocate to all uninformed managers, unrevealing prices must make uninformed managers marginal traders. This means that when marginal traders are N_j^U or N_3^U , unrevealing prices are given by (15) or (16). But by Proposition 3, there is at least a pair of realizations of $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ for which equilibrium prices are interdependent. This means that for some values of $p^e(\chi, y) = (p_1, p_2)$, the posteriors of uninformed managers on asset i are not independent of p_j and χ_j . But this implies that $1 - f_i$ must be a function of both q_1 and q_2 . But since either uninformed managers of specialized funds or uninformed managers of global funds are marginal traders at $p^e(\chi, y) = (p_1, p_2)$, any shock to q_i changes both p_i and p_j . It only remains to show that $1 - f_i$ is a decreasing function of q_j in any equilibrium. Let $U_{\chi_1 \chi_2} = \{(y_1, y_2) \mid p^e(\chi, y) = (p_1, p_2)\}$. Note that,

$$1 - f_1 = \frac{(1 - q_1)[(1 - q_2)Pr(y \in U_{00}) + q_2Pr(y \in U_{01})]}{Pr p^e(\chi, y) = (p_1, p_2)} \quad (59)$$

where

$$\begin{aligned} Pr p^e(\chi, y) = (p_1, p_2) &= (1 - q_1)[(1 - q_2)Pr(y \in U_{00}) + q_2Pr(y \in U_{01})] \\ &\quad + q_1[(1 - q_2)Pr(y \in U_{10}) + q_2Pr(y \in U_{11})] \end{aligned} \quad (60)$$

At any (y_1, y_2) , the mass of strict demands for asset 1 at $\chi = (1, 0)$ and $\chi = (1, 1)$ are the same. Also $Z_2(p, (1, 0), (y_1, y_2)) = Z_2(p, (1, 1), (y_1, y_2 + N_2^I + N_3^I))$ for $y_2 < \bar{y} - N_2^I - N_3^I$. By belief consistency $p^e((1, 0), (y_1, y_2)) = p^e((1, 1), (y_1, y_2 + N_2^I + N_3^I))$ which means $U_{11} = \{y' \mid y'_1 = y_1, y'_2 = y_2 + N_2^I +$

$N_3^I, (y_1, y_2) \in U_{10}\} \supseteq U_{10} + (0, N_2^I + N_3^I)$. But again by belief consistency, $U_{10} = \{y' \mid y'_1 = y_1, y'_2 = y_2 - N_2^I - N_3^I, (y_1, y_2) \in U_{11}\} \supseteq U_{11} - (0, N_2^I + N_3^I)$. Hence, U_{10} and U_{11} are simple shifts of each other and $Pr(y \in U_{11}) = Pr(y \in U_{10})$. Also by belief consistency, U_{00} and U_{10} are simple shifts of each other and occur with the same probability which also implies that $Pr(U_{11}) = Pr(U_{00})$. However, by the same arguments in the proof of Proposition 3, particularly by inequality (55),

$$Pr(p^e(\chi, y) = (p_1, p_2) \mid \chi_1 = 0, \chi_2 = 0) > Pr(p^e(\chi, y) = (p_1, p_2) \mid \chi_1 = 0, \chi_2 = 1)$$

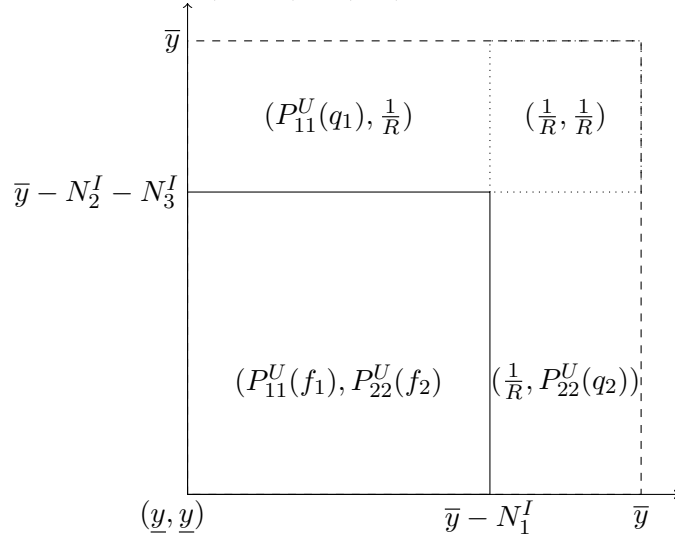
which is equivalent to $Pr(y \in U_{00}) > Pr(y \in U_{01})$. But then,

$$\begin{aligned} \frac{d(1 - f_1)}{dq_2} = & (1 - q_1)[Pr(U_{01} - Pr(U_{00}))]D - [(1 - q_1)(Pr(U_{01}) - Pr(U_{00})) \\ & + q_1(-Pr(U_{10} + Pr(U_{11})))N] D^{-2} < 0 \end{aligned} \quad (61)$$

where N is the nominator and D is the denominator of (59). Therefore, $1 - f_1$ is decreasing in q_2 . By the similar arguments, $1 - f_2$ is also decreasing in q_1 . □

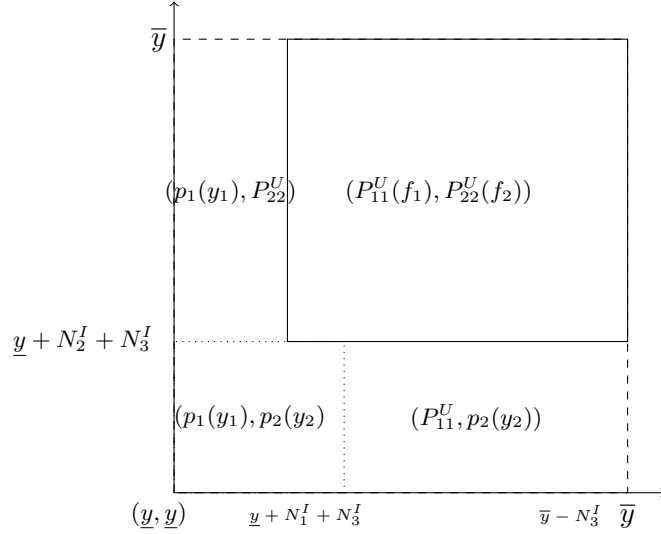
Proof of Proposition 4. Price mapping

Claim 1 specifies the regions where (χ_1, χ_2) are not revealed in any equilibrium in which unrevealing prices are different. Let us first describe the structure of the equilibrium and the regions where the equilibrium price function determines its revealing, non-revealing and partially revealing values. First consider $(\chi_1, \chi_2) = (0, 0)$.



When $(y_1, y_2) \in (\bar{y} - N_1^I, \bar{y}] \times (\bar{y} - N_2^I - N_3^I, \bar{y}]$, $Z_j(y_j, \chi) \geq \bar{y}$. Clearly, this reveals $(p_1, p_2) = (\frac{1}{R}, \frac{1}{R})$. When $(y_1, y_2) \in (\bar{y} - N_1^I, \bar{y}] \times [\underline{y}, \bar{y} - N_2^I - N_3^I)$, $Z_1 \geq \bar{y}$. Again, it's clear that asset 1 is repaying and it's price is $\frac{1}{R}$. However, $Z_2(y_2, (0, 0)) = Z_2(y_2 + N_2^I + N_3^I, (0, 1))$ so it's not revealed that asset 2 repays. Since we now have only one risky asset, the model is identical to the baseline model of Guerrieri and Kondor (2012) and for $(y_1, y_2) \in (\bar{y} - N_1^I, \bar{y}] \times [\underline{y}, \bar{y} - N_2^I - N_3^I)$ the posteriors on risky asset 2 are the same as priors, i.e., $1 - f_2 = 1 - q_2$. Thus, $p_2^e((0, 0), (y_1, y_2)) = P_{22}^U(q_2)$. By the symmetric argument, $p_1^e((0, 0), (y_1, y_2)) = P_{11}^U(q_1)$ for $(y_1, y_2) \in [\underline{y}, \bar{y} - N_1^I) \times (\bar{y} - N_2^I - N_3^I, \bar{y}]$.

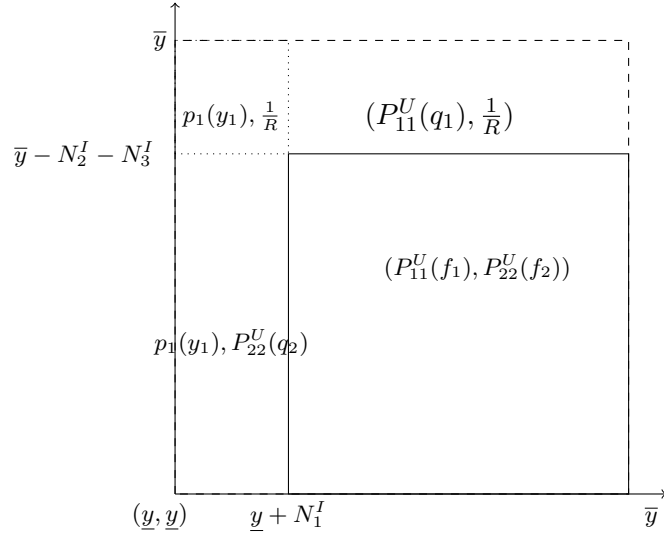
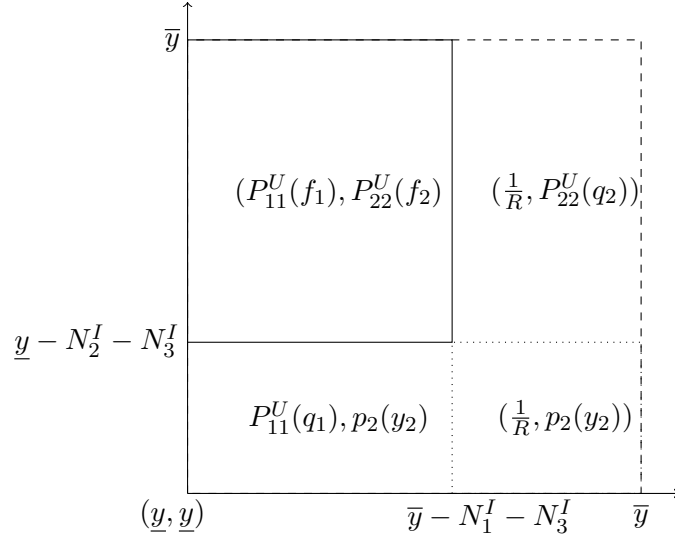
Consider now the case of $(\chi_1, \chi_2) = (1, 1)$.



Again, the solid rectangle shows Φ_{11} . Since $P_{22}^U < P_{11}^U$, N_3^I managers strictly demand asset 2 irrespective of the default or repay of asset 1 when $\chi_2 = 0$. So $\chi_2 = 1$ is immediately revealed whenever $Z_2 < \underline{y} + N_2^I + N_3^I$. Moreover, when $\chi_2 = 1$ all N_3^I managers buy asset 1 if it repays. So when the default of asset 2 is revealed, $\chi_1 = 1$ is revealed if the mass of strict demands of asset 1 is less than $\underline{y} - N_1^I - N_3^I$. Therefore, $(\chi_1, \chi_2) = (1, 1)$ is revealed when $(y_1, y_2) \in [\underline{y}, \underline{y} + N_1^I + N_3^I) \times [\underline{y}, \underline{y} + N_2^I + N_3^I)$ and $p^e((1, 1), (y_1, y_2)) = (p_1(y_1), p_2(y_2))$. Moreover, when asset 2 is revealed defaulting and $y_1 \in [\underline{y} + N_1^I + N_3^I, \bar{y}]$, $Z_1((1, 1), (y_1, y_2)) = Z_1((0, 1), (y_1 - N_1^I - N_3^I, y_2))$ so the default of asset 1 is not revealed. Again, since we are only left with one unrevealed risky asset we are back to Guerrieri and Kondor (2012) and $p_1^e = P_{11}^U(q_1)$. The symmetric argument applies to the region where $(y_1, y_2) \in [\underline{y}, \underline{y} + N_1^I) \times [\underline{y}, \underline{y} + N_2^I + N_3^I)$, hence, $p_2^e = P_{22}^U(q_2)$. Note that since $P_{11}^U > P_{22}^U$,

the default of asset 1 is only revealed when $Z_1 < \underline{y} + N_1^I$ if $\chi_2 = 1$ is not revealed. This is because, N_3^I managers always buy asset 2 when it repays, irrespective of the default or repay of asset 1. So when $y_2 \in [\underline{y}, \underline{y} + N_2^I + N_3^I)$ and it's not clear if N_3^I have bought asset 2 or not, the default of asset 1 is only revealed for $y_1 \in [\underline{y}, \underline{y} + N_1^I)$.

Consider $(\chi_1, \chi_2) = (0, 1)$ or $(1, 0)$. The following figures show the revelation, partial revelation and non-revelation regions for these cases.



Like the two other cases, when one asset is revealed we are back to Guerrieri and Kondor (2012) and $p_j^e = P_{jj}(q_j)$ when the price is j is not

revealing.

Demands

The demands of N_1^U and N_2^U managers are given as

$$d_1^U = \begin{cases} (1, 1, 0) & \text{if } p_1 \in \{\frac{1}{R}, P_{11}^U(f_1), P_{11}^U(q_1)\} \\ ((1, 0, 0) & \text{otherwise} \end{cases} \quad (62)$$

$$d_2^U = \begin{cases} (1, 0, 1) & \text{if } p_2 \in \{\frac{1}{R}, P_{22}^U(f_2), P_{22}^U(q_2)\} \\ ((1, 0, 0) & \text{otherwise} \end{cases} \quad (63)$$

where P_{jj}^U , $j = 1, 2$ are solved from (10). At $p_j^e = P_{jj}^U$, N_j^U managers are indifferent between risky asset j and bond. So their payoff of buying asset j is $\gamma R + \delta \omega f_j W_j^U$. When $p_1^e > p_2^e$, the maximum prices that N_3^U managers pay are solved from (11) and (12). But at P_{j3}^U the payoff to an N_3^U manager of receiving asset j is $\gamma R + \delta \omega f_1 f_2 W_3^U$. Recall that N_3^U managers are only retained when they buy the less expensive risky asset when both assets repay. This means that N_3^U managers are fired with a higher probability than N_j^U managers and therefore, $W_3^U < W_j^U$. But then, $\gamma R + \delta \omega f_j W_j^U > \gamma R + \delta \omega f_1 f_2 W_3^U$ and clearly $P_{jj}^U > P_{j3}^U$. Also, recall that $1 - f_1 > 1 - f_2$ so $P_{22}^U < P_{11}^U$. But this implies that for any (χ_1, χ_2) all the uninformed managers at global funds are strictly demanding asset 2. That is, at (P_{11}^U, P_{22}^U) for any (χ_1, χ_2) and $y_2 \in [\underline{y}, \bar{y}]$, $Z_2(\chi, y) \geq \underline{y} + N_3^U$. The demands of N_3^U contain no information and are constant at any (χ, \mathbf{y}) for which $\mathbf{p}^e(\chi, \mathbf{y}) = (P_{11}^U, P_{22}^U)$. But this is equivalent to assuming that at (P_{11}^U, P_{22}^U) - and only at this price pair- the amount of noise demands have increased by the mass of N_3^U managers. Hence, the nonrevelation regions when the nonrevealing prices are equal to (P_{11}^U, P_{22}^U) are identical to the regions specified in Claim 1. The demands of N_3^U managers are given as

$$d_3^U = \begin{cases} (1, 1, 1) & \text{if } (p_1, p_2) = (\frac{1}{R}, \frac{1}{R}) \\ (0, 0, 1) & \text{if } p_2 = P_{22}^U(f_2) \quad \text{or} \quad p_2 = P_{22}^U(q_2) \\ (0, 1, 0) & \text{if } (p_1, p_2) = (P_{11}^U(q_1), \frac{1}{R}) \quad \text{or} \quad (p_1, p_2) = (P_{11}^U(q_1), p_2(y_2)) \\ (1, 0, 0) & \text{otherwise} \end{cases} \quad (64)$$

Note that since $W_3^U \leq W_2^U$, N_3^U managers are willing to pay higher than $P_{jj}^U(q_j, W_j^U)$ for asset j and they strictly demand asset j .

Allocations

It only remains to show that the allocations are market clearing and feasible. Recall that $D_k(p)$ is the set of all the traders with strict demands for asset k at price p , $Z_k(\chi_k, y_k)$ is the mass of all d^q , $q \in D_k(p)$ at (χ, \mathbf{y}) . Let $A(p) = (A_1(p), A_2(p))$ be the aggregate demand vector at price p . Let $\bar{D}_k = \{q | q \notin D_k(p), d_k^q = 1, d_l^q = 0; l = k\}$. Then, at each price p the auctioneer allocates the asset $k = 1, 2$ according to the following rule.

- $x_k(d^q) = 1$ if $q \in D_k(p)$.
- $x_k(d^q) = \max\{\frac{b_k p_k - Z_k(p)}{A_k - Z_k}, 0\}$ if $q \in \bar{D}_k$
- $x_k(d^q) = 0$ if $d_k^q = 0$.

The probability of receiving risk-less bond is equal to

- $x_0(d^q) = \max\{1 - x_1(d^q) - x_2(d^q), 0\}$.

So at $(P_{11}^U(f_1), P_{22}^U(f_2))$ liquidity traders are assigned asset j with probability 1. If any of the risky assets repay, N_j^I managers have strictly demanded it and should be assigned as well. N_3^I managers always demand asset 2 when it repays and only demand asset 1 if it repays and asset 2 defaults. Note that N_3^U managers always demand asset 2 for any χ_2 so like noise traders they must be allocated asset 2 for sure. N_1^U and N_2^U managers are marginal traders at $(P_{11}^U(f_1), P_{22}^U(f_2))$, the allocation probabilities to N_1^U and N_2^U managers are

$$\begin{aligned} x_1(d_1^U) &= \frac{b_1 P_{11}^U - (1 - \chi_1)(N_1^I + \chi_2 N_3^I) - y_1}{N_1^U} \\ x_2(d_2^U) &= \frac{b_2 P_{22}^U - (1 - \chi_2)(N_2^I + N_3^I) - N_3^U - y_2}{N_2^U} \end{aligned} \quad (65)$$

Clearly by assumptions (i) and (ii) of Proposition 4 and (37),

$$x_1(d_1^U) > \frac{b_1 P_{11}^U - M^I - \bar{y}}{N_1^U} > \frac{\underline{C} - M^I - \bar{y}}{N_1^U} > 0 \quad (66)$$

$$x_1(d_1^U) < \frac{b_1 P_{11}^U}{N_1^U} < \frac{b_1 P_{11}^U}{(I_1 - M^I)} < \frac{\bar{C}}{I_1 - M^I} < 1 \quad (67)$$

and

$$x_2(d_2^U) > \frac{b_2 P_{22}^U - (M^I + I_3) - \bar{y}}{N_2^U} > \frac{\underline{C} - M^I - I_3 - \bar{y}}{N_2^U} > 0 \quad (68)$$

$$x_2(d_2^U) < \frac{b_2 P_{22}^U - N_3^U}{N_2^U} < \frac{\bar{C}}{I_2 - M^I} < 1 \quad (69)$$

So allocations are feasible at $(P_{11}^U(f_1), P_{22}^U(f_2))$.

At $(p_1^e = \frac{1}{R}, p_2^e = \frac{1}{R})$, everyone in I_k and I_3 funds has nonzero demand for asset k also noise traders are the only agents with strict demands for risky asset. Managers of global funds are indifferent between all assets while managers of specialized funds are indifferent between asset 1(2) and risk less bond.

Hence, $A_k = N_k^I + N_k^U + N_3^I + N_3^U + y_k$, $Z_k = y_k$. This means that the auctioneer must first allocate the risky assets to the managers of specialized funds and if anything remained to global funds. This means that,

$$x_k(d^q(\frac{1}{R}, \frac{1}{R})) = \frac{\frac{b_k}{R} - y_k}{N_k^I + N_k^U + N_3^I + N_3^U} \quad (70)$$

But by assumption $b_k < R \min\{I_1, I_2\} + y_k$ which implies that $x_k(d^q(\frac{1}{R}, \frac{1}{R})) < 1$

At $\mathbf{p} = (p_1(y_1), p_2(y_2))$, only liquidity traders demand risky assets and since $p_k(y_k) = \frac{y_k}{b_k}$ market is certainly cleared. At $(\frac{1}{R}, \bar{p}_2)$,

$$x_1(d^q(\frac{1}{R}, p_2(y_2))) = \frac{\frac{b_1}{R} - y_1}{N_1^I + N_1^U + N_3^I + N_3^U} \quad (71)$$

which is again feasible by supply assumptions.

It remains to show that allocations are feasible at $(P_{11}^U(q_1, W_1^U), \frac{1}{R})$ and $(\frac{1}{R}, P_{22}^U(q_2, W_2^U))$. At $p_j^e = P_{jj}^U(q_j, W_j^U)$, and $p_k^e = \frac{1}{R}$, N_j^I and N_3^I managers are strictly demanding asset j when it repays. Since $W_3^U \leq W_2^U$, N_3^U managers are willing to pay higher than $P_{jj}^U(q_j, W_j^U)$ for asset j and they strictly demand asset j . N_j^U managers are indifferent between asset j and bond. Thus,

$$x_j(d_j^U) = \frac{b_j P_{jj}^U(q_j) - (1 - \chi_j)(N_j^I + N_3^I) - N_3^U - y_j}{N_j^U} \quad (72)$$

Again by assumptions (i) and (ii), allocations are feasible.

At $p_j^e = P_{jj}^U$ and $p_i^e = p_i(y_i)$, no one demands asset i . N_j^U managers are indifferent between asset j and riskless bond but N_3^U strictly demand asset j . Thus, for $k = 3, j$

$$x_j(d_k^U) = \frac{b_j P_{jj}^U(q_j) - (1 - \chi_j)(N_j^I + N_3^I) - N_3^U - y_j}{N_j^U} \quad (73)$$

Again, this allocation is feasible by assumption. \square

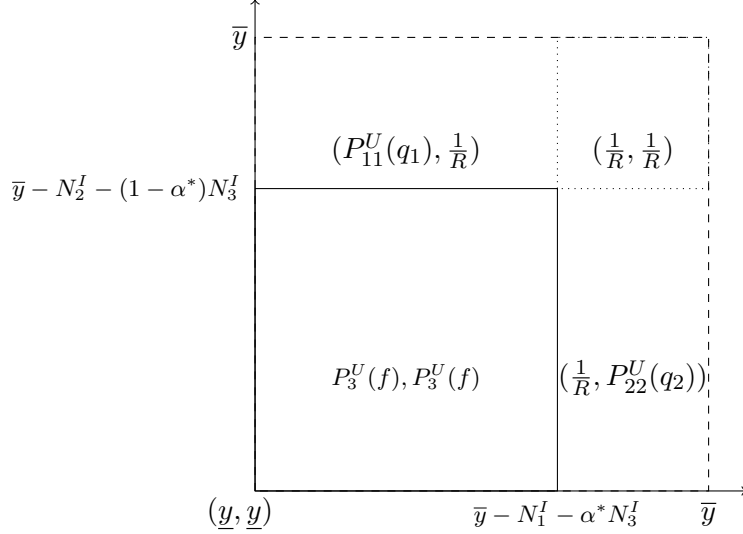


Figure 1: $(\chi_1, \chi_2) = (0, 0)$

Proof of Proposition 5. Price mapping

Like the proof of Proposition 4, Figures 1-4 show the values that price mapping takes at any (χ, \mathbf{y}) .

Recall that Claim 2 gives the regions of non-revelation for any (χ_1, χ_2) and the solid rectangle in any figure shows Φ_{χ_1, χ_2} . The main difference between the structure of this equilibrium and the equilibrium in Proposition 4 is the fixed fractions of N_3^I managers that demand each risky asset when both of them repay. It is worth noting that when $(\chi_1, \chi_2) = (1, 1)$ and $\chi_1 = 1$ is revealed, all N_3^I managers are buying asset 2 if it repays and $Z_2 = y_2 + N_2^I + N_3^I \geq \underline{y} + N_2^I + N_3^I$. So for $y_2 < \underline{y} + N_2^I + N_3^I$ asset 2 is also revealed defaulting. However, for $y_2 \geq \underline{y} + N_2^I + N_3^I$ the default is not revealed. But then we are left with only one risky asset and the model is identical to Guerrieri and Kondor (2012). Hence, the $1 - f_2 = 1 - q_2$ for $y_2 \geq \underline{y} + N_2^I + N_3^I$. The same argument applies to asset 1 when asset 2 is revealed defaulting.

Demands

We explained in the proof of Proposition 4 that given any pair of posteriors uninformed managers of specialized funds pay at most $P_{jj}^U(f_j)$ that is less than $P_{j3}^U(f_j)$, the maximum that N_3^U managers pay for asset j. So when $p_j^e = P_{j3}^U$ for $j = 1, 2$, N_j^U managers are out of risky asset markets and only demand risk free bond. Since $P_{13}^U = P_{23}^U$ and $1 - f_1 = 1 - f_2$, from this

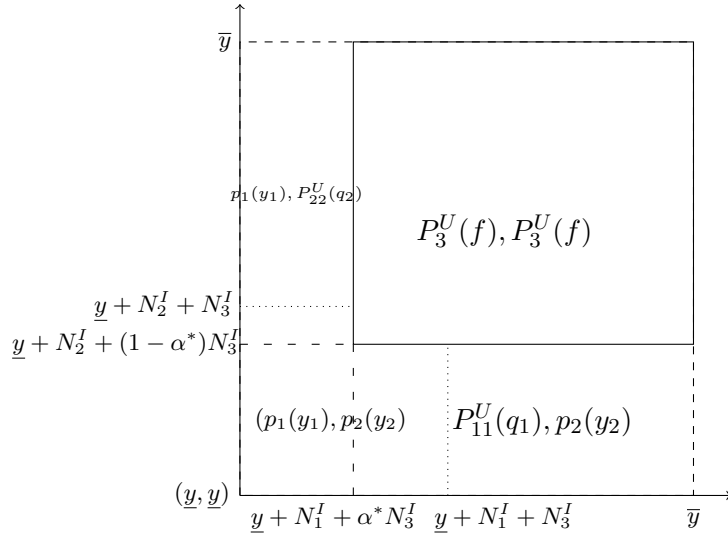


Figure 2: $(\chi_1, \chi_2) = (1, 1)$

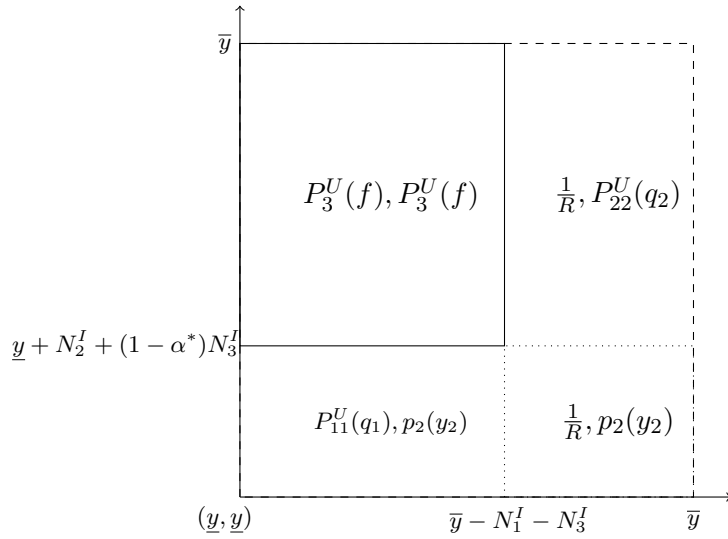


Figure 3: $(\chi_1, \chi_2) = (0, 1)$

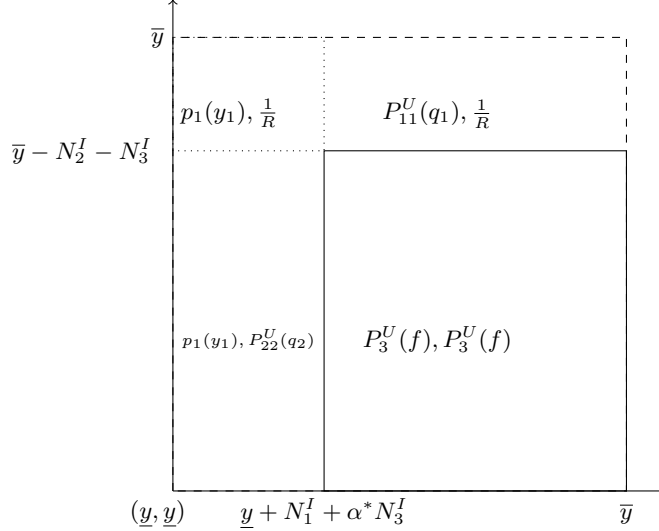


Figure 4: $(\chi_1, \chi_2) = (1, 0)$

point on we refer to both by P_3^U and $1 - f$. The demands of N_j^U managers are the same as (62) and (63). The demands of N_3^U managers are given as

$$d_3^U = \begin{cases} (1, 1, 1) & \text{if } (p_1, p_2) \in \{(\frac{1}{R}, \frac{1}{R}), (P_3^U(f), P_3^U(f))\} \\ (0, 1, 0) & \text{if } (p_1, p_2) \in \{(P_{11}^U(q_1), p_2(y_2)), (P_{11}^U(q_1), \frac{1}{R})\} \\ (0, 0, 1) & \text{if } (p_1, p_2) \in \{(p_1(y_1), P_{22}^U(q_2)), (\frac{1}{R}, P_{22}^U(q_2))\} \\ (1, 0, 0) & \text{otherwise} \end{cases} \quad (74)$$

Note that since $W_3^U \leq W_j^U$, N_3^U strictly demand asset j when asset i is revealed and $p_j = P_{jj}^U(q_j)$. The reason is at $p_j = P_{jj}^U(q_j)$, marginal traders are uninformed managers of I_j funds but then N_3^U managers can't be marginal and strictly demand asset j .

Allocations

To prove the existence, it only remains to show that the markets are cleared at $\mathbf{p}^e(\chi, \mathbf{y})$ for any (χ, \mathbf{y}) and the allocation probabilities are feasible. Recall that by allocation rule given in the proof of Proposition 4 at each $p = (p_1, p_2)$ the auctioneer assigns asset k to anyone with strict demand for asset k and then to anyone indifferent between assets and the bond. At $p^e = (P_3^U, P_3^U)$, noise traders strictly demand risky assets. In addition, informed managers of specialized funds and $\alpha^* - 1 - \alpha^*$ - fraction of informed managers of global funds strictly demand risky asset 1-2- when both assets

repay. However, all N_3^I managers strictly demand the repaying asset when either asset 1 or asset 2 defaults while N_j^I managers only demand riskless bond when risky asset j defaults. Consequently,

$$x_1(d_3^U) = \frac{b_1 P_3^U - y_1 - (1 - \chi_1) N_1^I + (1 - \chi_2) \alpha^* N_3^I + \chi_2 N_3^I}{N_3^U} \quad (75)$$

$$x_2(d_3^U) = \frac{b_2 P_3^U - y_2 - (1 - \chi_2) N_2^I + (1 - \chi_1)(1 - \alpha^*) N_3^I + \chi_1 N_3^I}{N_3^U} \quad (76)$$

It is clear that (75) and (76) are market clearing. We only need to show that they are feasible, that is $x_j(d_3^U) \in [0, 1]$. Notice that,

$$\frac{b_1 P_3^U - y_1 - N_1^I - N_3^I}{N_3^U} \leq x_1(d_3^U) \leq \frac{b_1 P_3^U - y_1}{N_3^U} \quad (77)$$

$$\frac{b_2 P_3^U - y_2 - N_2^I - N_3^I}{N_3^U} \leq x_2(d_3^U) \leq \frac{b_2 P_3^U - y_2}{N_3^U} \quad (78)$$

$$(79)$$

Besides,

$$1 - f_1 > (1 - q_1) \left(1 - \frac{M^I}{\Delta y}\right)^2 > (1 - q_2) \left(1 - \frac{M^I}{\Delta y}\right)^2. \quad (80)$$

This implies that

$$P_3^U = \frac{\gamma(1 - f_1)}{\gamma R - (1 - 2f_1)\delta\omega\beta W_3^U} > \frac{\gamma(1 - f_1)}{\gamma R + \delta\omega\beta W_3^U} > \frac{(1 - q_2) \left(1 - \frac{M^I}{\Delta y}\right)^2}{\gamma R + \delta\omega\beta W_3^U} \quad (81)$$

Using (81),

$$b_j P_3^U > b_j \frac{(1 - q_2) \left(1 - \frac{M^I}{\Delta y}\right)^2}{\gamma R + \delta\omega\beta W_3^U} > \underline{C} > \bar{y} + M^I \quad (82)$$

where the last inequality follows from the assumption on supply. But by (82),

$$x_j(d_3^U) > \frac{b_j P_3^U - \bar{y} - M^I}{N_j^U} > 0 \quad (83)$$

Additionally,

$$\frac{b_j P_3^U - y_j}{N_3^U} < \frac{b_j}{RN_3^U} < \frac{b_j}{R(I_3 - M^I)} < \frac{b_1 + b_2}{R(I_3 - M^I)} < 1$$

Hence, $x_1(d_3^U)$ and $x_2(d_3^U)$ are feasible. Note also that assumptions guarantee that $x_1(d_3^U) + x_2(d_3^U) \leq 1$.

When $\mathbf{p}^e(\chi, \mathbf{y})$ is partially revealing, the allocations are the same as the ones in the proof of Proposition 4 and are feasible by the assumption. \square

Claim 3. *Suppose (p_1, p_2) is a price pair such that $\mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2) \in (\bar{p}, \frac{1}{R})^2$ for some (χ, \mathbf{y}) . Also, suppose that $Pr(\chi_j = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = 1 - q_j$; $j = 1, 2$. Define $\tilde{\phi}_{\chi_1\chi_2} = \{y \in [\underline{y}, \bar{y}]^2 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)\}$. Then,*

$$(A1). \quad \tilde{\phi}_{00} = [\underline{y} + N_1^I + N_3^I, \bar{y} - N_1^I - N_3^I] \times [\underline{y} + N_2^I + N_3^I, \bar{y} - N_2^I - N_3^I]$$

$$(A2). \quad \tilde{\phi}_{01} = [\underline{y} + N_1^I, \bar{y} - N_1^I - 2N_3^I] \times [\underline{y} + 2(N_2^I + N_3^I), \bar{y}]$$

$$(A3). \quad \tilde{\phi}_{10} = [\underline{y} + 2N_1^I + N_3^I, \bar{y} - N_3^I] \times [\underline{y} + N_2^I + N_3^I, \bar{y} - N_2^I - N_3^I]$$

$$(A4). \quad \tilde{\phi}_{11} = [\underline{y} + 2N_1^I + N_3^I, \bar{y} - N_3^I] \times [\underline{y} + 2(N_2^I + N_3^I), \bar{y}]$$

Proof. First note that $\tilde{\phi}_{\chi_1\chi_2} \subset \Phi_{\chi_1\chi_2}$ for any (χ_1, χ_2) . Thus, (p_1, p_2) are not revealing (χ_1, χ_2) . Moreover, posteriors of managers given (p_1, p_2) are the same as priors. This is because the size of $\tilde{\phi}_{\chi_1\chi_2}$ is constant for any (χ_1, χ_2) so the probability of $\tilde{\phi}_{\chi_1\chi_2}$ is the same at any (χ_1, χ_2) . Hence, $Pr(\mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = Pr(y \in \tilde{\phi}_{\chi_1\chi_2})$. But then,

$$\begin{aligned} Pr(\chi_1 = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) &= \frac{(1 - q_1)[Pr(y \in \tilde{\phi}_{00})(1 - q_2) + Pr(y \in \tilde{\phi}_{01})q_2]}{Pr(y \in \tilde{\phi}_{\chi_1\chi_2})} \\ &= (1 - q_1) \end{aligned} \tag{84}$$

By the same argument $Pr(\chi_1 = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = (1 - q_2)$ \square

Proof of proposition 6. Price mapping

Claim 3 specifies the regions where prices are unrevealing. Figures 5-8 show these regions and the regions where prices are partially and fully revealing. Note that except for the no-revelation regions, the rest of the regions are the same as the ones in Proposition 4. Furthermore, the partial revelation regions where prices equal $P_{jj}^U(f_j)$ are not fully revealing (χ_1, χ_2) but are not entirely unrevealing either. As Figures 5-8 show, $\mathbf{p}^e(\chi, \mathbf{y}) =$

$(P_{11}^U(f_1), P_{22}^U(f_2))$ for $y \in \Phi_{\chi_1\chi_2} - \tilde{\Phi}_{\chi_1\chi_2}$. Therefore, the posteriors of uninformed managers at $(P_{11}^U(f_1), P_{22}^U(f_2))$ are given as

$$1 - f_1 = \frac{(1 - q_1) (1 - q_2)A + q_2B}{[(1 - q_2) + q_1q_2]A + (1 - q_1)q_2B} \quad (85)$$

$$1 - f_2 = \frac{(1 - q_2)A}{[(1 - q_2) + q_1q_2]A + (1 - q_1)q_2B} \quad (86)$$

where,

$$\begin{aligned} A &= (r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)] \\ B &= (r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3) \end{aligned}$$

Note that since

$$\frac{(r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)]}{(r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3)} < \frac{(1 - q_1)q_2}{q_1(1 - q_2)}$$

$1 - f_1 > 1 - f_2$ Demands are the same as the demands given in Proposition 4 and as we showed in the proof of Proposition 4 the allocations are feasible as well. □

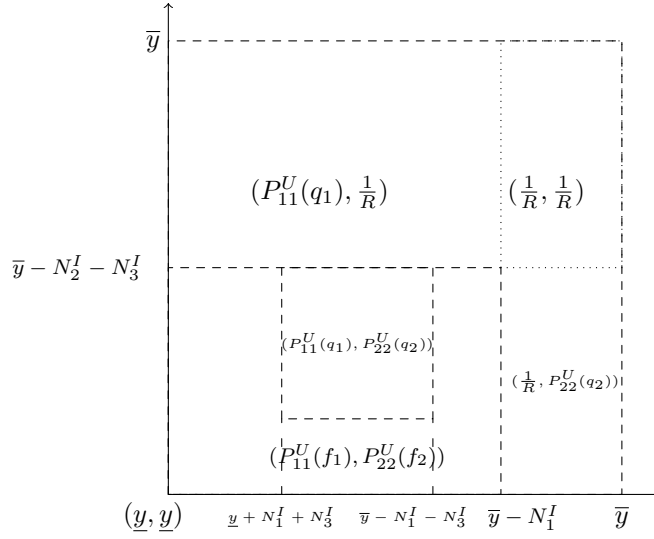


Figure 5: $(\chi_1, \chi_2) = (0, 0)$

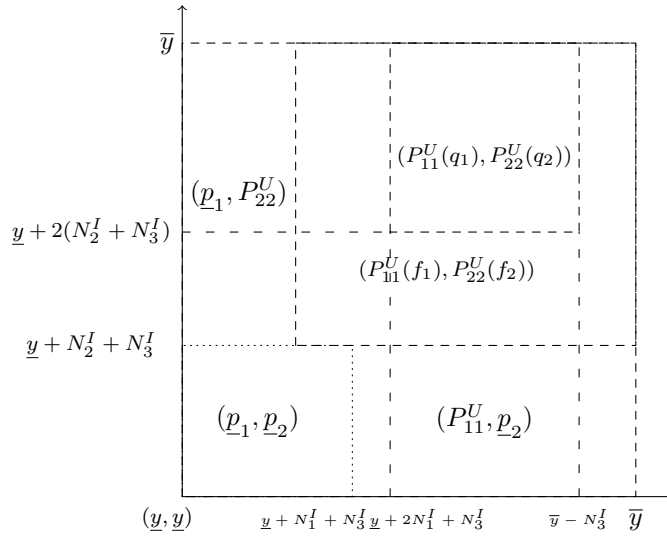


Figure 6: $(\chi_1, \chi_2) = (1, 1)$

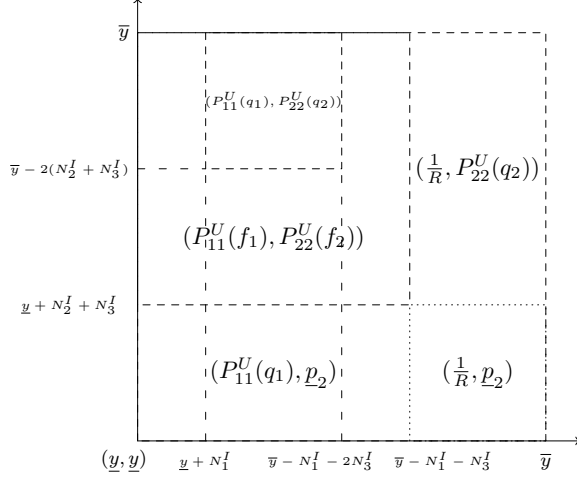


Figure 7: $(\chi_1, \chi_2) = (0, 1)$

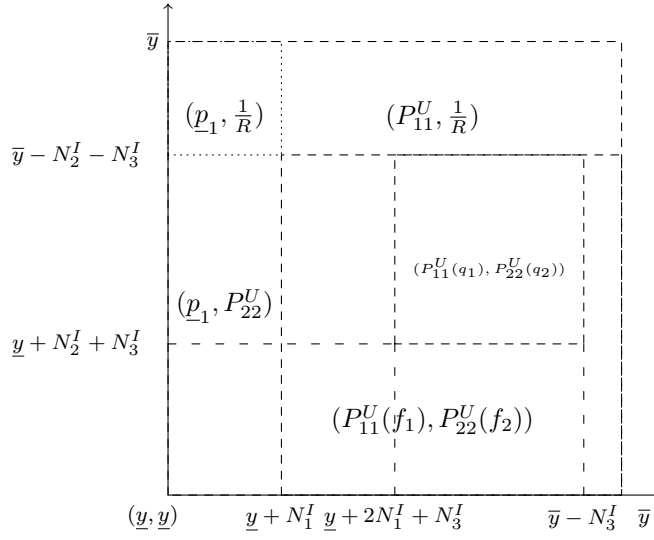


Figure 8: $(\chi_1, \chi_2) = (1, 0)$

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