Chapter 15: Factor Markets

Matthew MacLachlan

July 15, 2013

15.1 Competitive Factor Market

- Types of factor purchasers:
  - Price takers
  - Noncompetitive price setters, such as the monopsony firm.

- Types of factor sellers (same as any other industry):
  - Competitive
  - Monopolistically competitive,
  - Oligopolistic
  - Monopolistic

- Factor markets are considered competitive when there are many buyers and sellers.

Firm’s Short-Run Factor Demand

- In the short-run, the level of capital possessed by a firm, $K$, is considered fixed.
  - A firm may only change output by changing the amount of labor, $L$, it employs.

- The short-run production function may be expressed as follows:
  $$q = \hat{q}(L, K) = q(L)$$

- Cost is also only a function of the amount of labor:
  $$C(L) = wL + F$$, where $F$ is the fixed cost.

- The revenue function is expressed generally as $R(q(L))$.

- Putting it all together to express the firms profit maximization decision,
  $$\max_L \pi = R(q(L)) - wL - F$$

  - First order condition for profit maximization:
    $$\frac{d\pi}{dL} = \frac{dR}{dq} \frac{dq}{dL} - w = 0$$ or written another way
    $$\frac{dR}{dq} \frac{dq}{dL} = w$$

  - In words, this condition requires that the marginal contribution of labor to revenues, $\frac{dR}{dq} \frac{dq}{dL}$ (marginal revenue product of labor ($MRP_L$)), must equal the marginal cost of labor, $w$. For a competitive firm, the following must hold:
\[ MRP_L = \frac{MR \times MP_L}{p} = \frac{dq}{dL} = w \]

\[ \frac{dq}{dL} = \frac{w}{p} \]

Therefore, the function of labor demanded is a function of the wage-price ratio

\[ L = L \left( \frac{w}{p} \right) \]

**A Thread Mill’s Short-Run Labor Demand Function.**

\[ q = L^{0.6} K^{0.2} \]

\[ \bar{L} = 32 \]

\[ q = L^{0.6} 32^{0.2} = 2L^{0.6} \]

\[ MP_L = \frac{d(2L^{0.6})}{dL} = 1.2L^{-0.4} \]

\[ MRP_L = p \times 1.2L^{-0.4} = w \]

Generally, \( L = \left( \frac{5w}{6p} \right)^{-2.5} \)

\[ p = $50; \ w = $15 \]

\[ L = \left( \frac{5 \times 15}{6 \times 50} \right)^{-2.5} = 32 \]

**Change in the Wage.**
Short proof that labor decreases as an exogenous wage increases.

Begin with our optimality condition

\[ pMP_L = w \]

Differentiate with respect to \( w \)

\[ p \frac{MP_L}{L} \frac{dL}{dw} = 1 \]

Rearrange terms

\[ \frac{dL}{dw} = \frac{1}{p \frac{MP_L}{L}} \]

The law of diminishing marginal returns indicates that \( \frac{MP_L}{L} < 0 \)

Therefore, \( \frac{dL}{dw} < 0 \)

Firm’s Long-Run Factor Demands

When the firm plans in the long run, it is able to adjust both its level of labor and capital.

Choice of Inputs.

- Costs are no longer fixed. Instead, the producer may also choose the level of capital.

\[ C = wL + rK \]
• The firms production and revenue functions are also now functions of the level of capital.

\[ q = q(L, K); \quad R = R(q(L, K)) \]

• The firms production function may be expressed as follows for the long run:

\[ \max_{L,K} \pi = R(q(L, K)) - wL - rK \]

  – This problem has the following first order conditions:

\[ \frac{d\pi}{dL} = \frac{\partial R}{\partial q} \frac{\partial q}{\partial L} - w = 0 \]
\[ \frac{d\pi}{dK} = \frac{\partial R}{\partial q} \frac{\partial q}{\partial K} - r = 0 \]

  – These functions may be rearranged as they were in the perfectly competitive case to provide tractable conditions:

\[ MRP_L = pMP_L = p \frac{\partial q}{\partial L} = w \]
\[ MRP_K = pMP_K = p \frac{\partial q}{\partial K} = r \]

Cobb-Douglas Factor Demand Functions.

Suppose that we have the Cobb-Douglas production function: \( q = AL^{a-1}K^b \).

\[ paAL^{a-1}K^b = w \]
\[ pbAL^{a}K^{b-1} = r \]

These relationships may be used to find optimal levels of \( L \) and \( K \) in terms of the parameters only:

\[ L = \left( \frac{a}{w} \right)^{(1-b)/d} \left( \frac{b}{r} \right)^{b/d} (AP)^{1/d} \]
\[ K = \left( \frac{b}{w} \right)^{a/d} \left( \frac{b}{r} \right)^{(1-a)/d} (AP)^{1/d} \]
\[ d = 1 - a - b \]

Comparing Short-Run and Long-Run Labor Demand Curves.
1. $w = 10 \rightarrow 15$

2. $L = 32 \rightarrow 88; \ a \rightarrow b$

3. $K = 32 \rightarrow 108$

4. $L = 88 \rightarrow 162; \ b \rightarrow c$

• For both the long and short-run the labor demand curve is the marginal revenue product curve of labor.

• In the long run, the firm is able to vary capital, so the labor demand curve is much flatter.

  – Increasing the amount of capital makes each unit of labor more effective (if the two inputs are complements). Therefore, when capital is increased in response to a decrease in the wage, the increase in demand is amplified.

**Factor Market Demand**

• Factor demand is linked to output demand. Similarly, output prices are linked to input prices.
<table>
<thead>
<tr>
<th>Decrease in factor price</th>
<th>Increase in factor price</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>↑</td>
</tr>
<tr>
<td>( L )</td>
<td>↓</td>
</tr>
<tr>
<td>( q )</td>
<td>↑</td>
</tr>
<tr>
<td>( p )</td>
<td>↓</td>
</tr>
<tr>
<td>( q )</td>
<td>↓</td>
</tr>
<tr>
<td>( L )</td>
<td>↑</td>
</tr>
</tbody>
</table>

Competitive Factor Market Equilibrium

...Perloff makes a a section header, but leaves the explanation to Chapters 2, 8 and 9.

15.2 Noncompetitive Factor Market

- When the market structure is non competitive, price is not equal to \( pMP_L \). Instead,
  \[
  MR = p \left( 1 + \frac{1}{\varepsilon_i} \right), \quad \text{where } \varepsilon_i \text{ is the price elasticity of demand of output } i.
  \]
- This means that the marginal productivity of labor may be expressed as follows:
  \[
  MRP_L = p \left( 1 + \frac{1}{\varepsilon_i} \right) MP_L
  \]
- Consider the case in which the elasticity of demand is \( n\varepsilon \), where \( n \) is the number of firms. In this case, the labor demand (capture by \( MRP_L \)) is increasing in the number of firms.
  \[
  MRP_L = p \left( 1 + \frac{1}{n\varepsilon} \right) MP_L
  \]
  - The labor demanded in the presence of a duopoly will be greater than in the presence of a monopoly but less than in the presence of perfect competition.
– In a competitive market, the second term in the parentheses will go to zero.

15.3 **Monopsony**

*Monopsony:* a single buyer of a good.

– Picks a single price/quantity combination to maximize profits.
– Mirror image of a monopoly.

**Monopsony Profit Maximization**

* A monopsonist that faces an upward sloping labor supply curve recruits employees until the marginal expenditure of an additional employee equals its demand for an additional employee.

– **Marginal Expenditure:** The additional cost of hiring an additional employee.
  * Because a firm must increase its wage for all employees to attract addition labor, an additional employee costs more than the wages paid to that employee.

* A monopsony’s total expenditure on labor: \( E = w(L)L \)

\[
ME = w(L) + \left(\frac{dw}{dL}\right)_L L
\]

\(\text{C of new}\) \(\text{C of old}\)
Assuming the firm is a price taker in the output market and that its output is a function of labor only, we may set up the profit maximization problem:

\[
\max_L \pi = pQ(L) - w(L)L,
\]

where \(Q(L)\) is the production function. We may take the first order condition:

\[
p \frac{dQ}{dL} - w(L) - \frac{dw}{dL}L = 0
\]

or

\[
MRP_L = p \frac{dQ}{dL} = w(L) + \frac{dw}{dL} = ME
\]

In words, the monopsony hires labor until the marginal revenue product of labor equals the marginal expenditure on the last employee.

- **Monopsony power**: the ability of a single buyer to pay less than the competitive price profitably.
  
  - The ability of the buyer to pay less depends on the shape of the supply curve. The dependency can be captured by expressing the marginal expenditure in terms of the elasticity:

\[
ME = w(L) + \frac{dw}{dL}L = w(L) \left(1 + \frac{dw}{dL} \frac{L}{w}\right) = w(L) \left(1 + \frac{1}{\eta}\right)
\]

- From this, we may derive a relationship analogous to the Lerner Index:

\[
\frac{ME - w}{w} = \frac{1}{\eta}
\]

  - The ability to underpay labor is inversely proportional to the elasticity of supply for labor.
15.4 Capital Markets and Investing

- The dynamical nature of durable capital which may not be rented. Capital levels are described by stocks and flows.
  - **Stock**: The level of capital in a given period.
  - **Flow**: The level of change in a given period. May apply to
    * Money
    * Capital
    * Labor

- The presence of dynamical processes requires us to extend our toolkit to complete our analysis.

Interest Rates

- Money now is almost always preferred to money in the future. Therefore, it is necessary to pay interest when money is received now rather than in the future.
– **Interest rate:** The percentage more that must be repaid to borrow money for a fixed period of time.

– **Present value:** The value of money right now.

\[ FV = PV \times (1 + i)^t \], where \( i \) = interest rate, and \( t \) = number of periods

– **Future value:** The value of money received at a later date.

\[ PV = \frac{FV}{(1 + i)^t} \]

**Discount Rate**

– **Discount Rate:** A person’s discount rate is analogous to the social interest rate.

– Most people have different discount rates

– Determines borrowing and lending practices

**Stream of Payments**

Individuals or firms may choose to make purchases outright or to make periodic payments. How do we calculate the value of those monthly payments? We use the following formula:

\[ PV = f \left[ \frac{1}{(1 + i)^1} + \frac{1}{(1 + i)^2} + \ldots + \frac{1}{(1 + i)^t} \right] \]

What happens if the payments go on forever (a perpetuity):

\[ PV = \frac{f}{i} \]

We use the future value formula if we want to know the value of something in the future if we make contributions to it (e.g. a bank account):

\[ FV = f \left[ 1 + (1 + i)^1 + (1 + i)^2 + \ldots + (1 + i)^{t-1} \right] \]

**Investing**

– Ignore uncertainty and inflation

– **Rate of return on and investment:** the percent of the original investment which is paid to the investor each period.

**Net Present Value Approach.**

– A firms should only make an investment if the revenues generated from the investment, \( R \), exceed the costs, \( C \). That is, the net present value is positive.

\[ NPV = R - C > 0 \]
• When the revenues and costs are distributed over time, this may be expressed as follows:

\[ NPV = R - C \]

\[ = \left[ R_0 + \frac{R_1}{(1 + i)^1} + \frac{R_2}{(1 + i)^2} + \ldots + \frac{R_T}{(1 + i)^T} \right] \]

\[ - \left[ C_0 + \frac{C_1}{(1 + i)^1} + \frac{C_2}{(1 + i)^2} + \ldots + \frac{C_T}{(1 + i)^T} \right] \]

\[ = (R_0 - C_0) + \frac{R_1 - C_1}{(1 + i)^1} + \frac{R_2 - C_2}{(1 + i)^2} + \ldots + \frac{R_T - C_T}{(1 + i)^T} \]

\[ = \pi_0 + \frac{\pi_1}{(1 + i)^1} + \frac{\pi_2}{(1 + i)^2} + \ldots + \frac{\pi_T}{(1 + i)^T} \]

### Internal Rate of Return Approach.

• **Internal rate of return (irr):** The discount rate such that the net present value of investment is zero. To determine, solve:

\[ NPV = \pi_0 + \frac{\pi_1}{(1 + irr)^1} + \frac{\pi_2}{(1 + irr)^2} + \ldots + \frac{\pi_T}{(1 + irr)^T} \]

• ...or simply solve:

\[ irr = \frac{f}{PV} \]

• A firm will choose to make the investment if \( irr > i \)

### Durability

### Return to this

### Human Capital

• Investments in human capital may be made to enhance future earnings potential.

  – Example: Should one invest in college or not? That depends on whether the red area is bigger than the green area.
Time-Varying Discounting

Time Consistency.

- In the previous sections, the form of discounting is called *exponential discounting*. This form of discounting is *time consistent*: In each period, the agent discounts future income by the same amount.

- In reality, it may be the case that agents exhibit present biased discounting.
  
  - Example: If I offered you $100 in 10 years or $200 in 10 years and one month, you would likely take the $100. However, if I offered you $100 today or $200 in a month, you may take the $100 now.

Behavioral Economics.

- It may be the case that individuals indeed have time-inconsistent preferences.
  
  - People may have a bias for events that happen in the very near term.
  - Suggests structured planning may have large benefits for individual planners.

Falling Discount Rates and the Environment.
For environmental decisions, such as CO₂ emissions, for which the effects of a decision is felt long into the future, we may wish to have a social decision maker with a decreasing discount rate.

- Increases the social cost of emissions, which in turn reduces the optimal emissions level.
- May be plausible if we have little preference for individuals in, for example, the 10th or 11th generation.

15.5 Exhaustible Resources

- **Exhaustible resource**: nonrenewable natural asset that cannot be increased, only depleted.
  - Examples: gold, oil, uranium.
  - Extractors of the natural resource attempt to maximize the net present value (NPV) of the resource.

When to Sell and Exhaustible Resource:

- Example: coal mine.
  - Two periods: now and the next period.
    - Coal is sold at \( P_1 \) in period 1 and at \( P_2 \) in period 2.
    - Coal is extracted at a constant marginal cost of \( m \)
    - Future income is discounted by \( \frac{1}{1+i} \)
    - Therefore, we run into three different cases based on the profitability of coal in each period:
      - Sell all coal this year if \( P_1 - m > \frac{P_2-m}{1+i} \)
      - Sell all coal next year if \( P_1 - m < \frac{P_2-m}{1+i} \)
      - Sell coal in either year if \( P_1 - m = \frac{P_2-m}{1+i} \)

Price of a Scarce Exhaustible Resource

- We may extend the two period model to a multi-period model
- Coal is only sold if the price (after discounting) is the same in each period. Therefore, the sale price must increase over time.

\[
P_t - m = \frac{P_{t+1} - m}{1+i}
\]

\[
(P_t - m)(1+i) = P_{t+1} - m
\]

\[
P_{t+1} = P_t + iP_t - \mu - im + \mu
\]

\[
P_{t+1} = P_t + i(P_t - m)
\]

We may use this expression to find the price difference between periods:

\[
\Delta P = P_{t+1} - P_t = i(P_t - m)
\]
Price in a Two-Period Example.

- \( m = 0 \)
- \( Q_t = 200 - P_t \)
- \( Q_1 + Q_2 = (200 - P_1) + (200 - P_2) = Q \)

Now we can rearrange terms to get price in terms of quantity:

\[
\begin{align*}
(200 - P_1) + (200 - \frac{P_2}{P_1(1 + i)}) &= Q \\
400 - P_1 - P_1(1 + i) &= Q \\
400 - P_1(2 + i) &= Q \\
P_1 &= \frac{400 - Q}{2 + i} \\
P_2 &= P_1(1 + i) = \frac{(400 - Q)(1 + i)}{2 + i}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( Q = 169 )</th>
<th>( Q = 400 )</th>
<th>( i = 10% )</th>
<th>( i = 20% )</th>
<th>Any ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = \frac{400 - Q}{2 + i} )</td>
<td>$110</td>
<td>$105</td>
<td>$0</td>
<td></td>
</tr>
<tr>
<td>( P_2 = P_1(1 + i) )</td>
<td>$121</td>
<td>$126</td>
<td>$0</td>
<td></td>
</tr>
<tr>
<td>( \Delta P = P_1 - P_2 = iP_1 )</td>
<td>$11</td>
<td>$21</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( Q_1 = 200 - P_1 )</td>
<td>$90</td>
<td>$95</td>
<td>$0</td>
<td></td>
</tr>
<tr>
<td>( Q_2 = 20 - P_2 )</td>
<td>$79</td>
<td>$74</td>
<td>$0</td>
<td></td>
</tr>
<tr>
<td>Share sold in Year 2</td>
<td>47%</td>
<td>44%</td>
<td>50%</td>
<td></td>
</tr>
</tbody>
</table>
The figure below shows that relationship between the price of coal in the first period and the initial stock of coal.

Rents.

- Even within a competitive market when coal is scarce, it may be priced above its marginal cost, receiving *rent*.
  - *Rent*: a payment to the owner of an input beyond the minimum necessary for the input to be supplied.

Rising Prices.

- Prices of a resource rise if the resource
  1. is scarce
  2. can be extracted at a constant marginal cost
  3. is sold in a competitive market

- Example: redwoods
  - Grow so slowly that they act as a finite resource
  - Prices increased over time due to extraction and purchases by the government for preservation.

Why Price May Be Constant of Fall

Reasons why the price of a resource might fall over time:

**Abundance.**

The initial gap between the price and marginal cost of nonrenewable resource depends on the abundance of that resource. For minerals with very large *reserves* (amount still not extracted),
there may be virtually no difference between price and cost, indicating that rents are will be very close to zero.

**Technical Progress.**
Technical progress may reduce the marginal cost of extraction over time.

**Changing Market Power.**
Changes in market structure will also likely lead to changes in the price of a resource.

\[
\begin{align*}
\text{Perfect Competition} & \rightarrow \text{Oligopoly/Monopoly} \rightarrow \uparrow P \\
\text{Oligopoly} & \rightarrow \text{Monopoly} \rightarrow \uparrow P \\
\text{Oligopoly/Monopoly} & \rightarrow \text{Perfect Competition} \rightarrow \downarrow P \\
\text{Monopoly} & \rightarrow \text{Oligopoly} \rightarrow \downarrow P
\end{align*}
\]