GETTING AWAY WITH ROBBERY? PATENTING BEHAVIOR WITH THE THREAT OF INFRINGEMENT

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Abstract

The paper examines the relationship between the innovator's patenting and patent breadth decisions as well as how these two decisions affect, and are affected by, the innovator's ability to enforce her patent rights. An important feature of the model is that the entrant may be able, by his choice of location in product space, to affect the innovator's decision to defend her patent. An interesting finding of the paper is that the innovator might find it optimal to patent her innovation even when she chooses to not defend her patent by invoking a trial when patent infringement occurs. The paper also shows that, in most cases, the greater is the entrant's R&D effectiveness, the smaller is the innovator's incentive to patent her product. If patenting occurs, however, the greater is R&D effectiveness, the greater is the patent breadth that could be chosen without triggering infringement.

Keywords: patent breadth, entry deterrence, patent infringement, patent validity.

JEL Classification Codes: L20; L13; O34.

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1. Introduction

Patents have been used over the last 500 years as means of protecting intellectual property. The decision to patent an innovation implies that patenting is perceived as generating more rents than when no protection is in place. Under no patent protection the innovator cannot generally influence the market entry and location decisions of potential entrants; at best, the innovator may be able to hinder the generation of competing innovations by keeping her innovation a secret. Under patent protection, however, potential entrants are required to locate a certain distance away from the patentee's innovation to not infringe the patent. This distance is specified by the breadth of the patent, which is determined, to a large extent, by the patentee through the claims that she makes in the patent application. Thus, once the decision to patent has been made, the innovator needs to make another important decision, namely, how broad of a protection to claim.

The innovator's patent breadth choice is a strategic decision that has important implications both for the level of competition that she will face in the market and for her ability to enforce/defend her patent right. The greater is the breadth of patent protection, the harder it is for potential competitors to enter into the patentee's market with non-infringing innovations and thus the longer the patentee can maintain the limited monopoly that the patent grants. However, the broader is a patent, the greater is the likelihood of both infringement and patent validity challenges by competitors and/or third parties, which if successful will reduce the effective patent life and consequently the innovator's ability to capture innovation rents (Merges and Nelson 1990, Lerner 1994, Lanjouw and Schankerman 2001). This possibility is especially critical in light of the increase in patent litigation during the last decades, particularly in the field of biotechnology, and the increase in the number of patents that are invalidated after being challenged (Barton 2000, Choi 1998). In addition, empirical evidence suggests that courts tend to uphold narrow patents and revoke or limit the scope of broad ones (Waterson 1990). The possibility that broad patent protection may impede a firm's ability to safeguard and defend its technological territory raises the question as to what constitutes an optimal patenting behavior for the innovator.

The analysis of the innovator's patenting behavior in the existing patent literature has focused on either the decision to patent the innovation or to keep it a secret (Waterson 1990, Horstmann et al. 1985), or on the optimal patent breadth decision by the patentee under the case where invoking an infringement trial when the patent has been infringed is always optimal (Yiannaka and Fulton 2003). While the above decisions have been studied in isolation, they are related; in addition other possibilities exist, such as the case where the patentee finds that it is not desirable to defend her patent by invoking a trial when infringement occurs.

The purpose of this paper is to examine the relationship between the innovator's patenting and patent breadth decisions as well as how these two decisions affect, and are affected by, the innovator's ability to enforce her patent rights. On this last point, the paper also examines whether it is possible for the innovator to affect the entrant's location decision (and thus the rents that can be captured by the patent) even when defending the patent under infringement is not optimal. To address the above issues, the paper develops a game theoretic model that examines the optimal patenting behavior of an incumbent innovator who has generated a patentable product innovation and who is faced with potential entry by another firm. The incumbent/innovator has to decide whether she should patent her innovation, and if so, what patent breadth should be claimed. If her patent is infringed, the incumbent also has to decide whether she should invoke a trial to defend the patent. An important feature of the model is that the entrant may be able, by his choice of location in product space, to affect the incumbent's decision to defend her patent.

An interesting finding of the paper is that the innovator might find it optimal to patent her innovation even when she chooses not to defend her patent by invoking a trial when patent infringement occurs. In other words, patenting may be a profitable strategy even if the patent will not subsequently be defended. This result occurs because, by choosing to patent her innovation, the incumbent can induce the entrant to choose a location in the product space that, even though it infringes the patent, is still advantageous for the incumbent (i.e., it is further away from the incumbent's location than the location chosen under no patent protection). Under this case, the entrant, knowing that his location decision affects the incumbent's decision to invoke a trial, strategically chooses a location that will not be challenged by the incumbent. This result is more likely to occur when reverse engineering is costless to the potential entrant and the entrant's R&D effectiveness is low – his R&D costs are high. The paper also shows that, in most cases, the greater is the entrant's R&D effectiveness, the smaller is the innovator's incentive to patent her product. If patenting occurs, however, the greater is R&D effectiveness, the greater is the patent breadth that could be chosen without triggering infringement. This result occurs because the greater is the entrant's R&D effectiveness, the further away from the incumbent the entrant can locate in the product space. The outcome is increased product differentiation, less competition and thus higher profits for both players. Other factors affecting the innovator's optimal patenting behavior are the probability that the patent will be found valid at trial and the monopoly profits earned when market entry can be deterred.

The rest of the paper is organized as follows. Section two describes the theoretical development of the strategic patent breadth model; it describes the market conditions, defines patent breadth and models the choice of patent breadth as a sequential game of complete information. Section three provides the analytical solution of the model. Finally, section four concludes the paper.

2. The Patent Breadth Model

2.1 Model Assumptions

The model builds upon the model developed by Yiannaka and Fulton (2003) to study the optimal patent breadth decision when under infringement a trial always takes place. In our model the optimal patent breadth strategy is determined in a sequential game of complete information. The agents in the game are an incumbent innovator who has invented a patentable drastic product innovation and decides whether to seek patent protection, how broad of a protection to claim and whether to defend her patent when infringement occurs and a potential entrant who decides on whether to enter the incumbent's market and, if entry occurs, where to locate in a vertically differentiated product space. Both the incumbent and the entrant are risk neutral and maximize profits. It is assumed that the regulator (e.g., Patent Office) always grants the patent as claimed; thus, the regulator is not explicitly modeled.

The incumbent's investment decision that led to the development of a new product is not examined – this decision is treated as exogenous to the game. In addition, it is assumed that the incumbent and the entrant each produce at most one product and that the entrant does not patent his product since further entry is not anticipated. The production process for the entrant is assumed to be deterministic, so that once the entrant chooses a location she can produce the chosen product with certainty. It is also assumed that there is no time lag between making and realizing a decision.

The incumbent and the entrant, if he enters, operate in a vertically differentiated product market. To keep the analysis tractable, it is assumed that no substitute exists for the products produced by the incumbent and the entrant. Consumers differ according to some attribute λ, uniformly distributed with unit density $f(\lambda)=1$ in the interval $\lambda \in [0,1]$, each buying one unit of either the incumbent's or the entrant's product but not both. The incumbent is assumed to have developed a product that provides consumers with utility $U_p = V + \lambda q_p - p_p$, where V is a base level of utility, q_p is the quality of the incumbent's product p_p is the price of the product produced by the incumbent. The entrant's product has quality $q_e > q_p$, $q_e \in (0,1]$, that provides consumers with utility $U_e = V + \lambda q_e - p_e$, where p_e is the price of the entrant's product. Without affecting the qualitative nature of the model, the quality of the incumbent's product q_p is set equal to zero (i.e., $q_p = 0$). As a result, the entrant's quality q_e is interpreted as the difference in quality between his product and that of the incumbent, or more generally as the distance the entrant has located away from the incumbent.[1]

Product i ($i = p,e$) is consumed as long as $U_i \geq 0$ and $U_i > U_j$. It is assumed that V is large enough so that $V \geq p_i \ \forall i = p,e$ and the market is always served by at least one product. The consumer who is indifferent between the two products has a λ denoted by λ^*, where λ^* is determined as follows: $V \geq p_i \ \forall i = p,e$

$$U_p = U_e \Rightarrow \lambda^* = \frac{p_e - p_p}{q_e} \qquad (1)$$

Since each consumer consumes one unit of the product of her choice, the demand for the products produced by the incumbent and the entrant are given by $y_p = \lambda^*$ and $y_e = 1 - \lambda^*$, respectively.

The incumbent's decision to patent the innovation implies patenting costs denoted by z, z>0, that are assumed to be independent of patent breadth. This assumption is in line with our assumption that the Patent Office always grants the patent as claimed (for a discussion on the Patent Office's role in the patent granting process for drastic innovations see Yiannaka and Fulton (2003)).

[1] With $q_p \neq 0$, equation (1) becomes $\lambda^* = \dfrac{(p_e - p_p)}{q_e - q_p}$. Since the quality difference, $q_e - q_p$, in the denominator is the relevant parameter of interest in the subsequent analysis, the assumption that $q_p = 0$ can be made to ease the notation without affecting the qualitative nature of the model.

The incumbent has already incurred the development costs associated with the product quality that she wants to patent. Thus, the R&D costs for the incumbent are sunk. For the entrant, however, market entry can only occur if he develops a higher quality product. To do so, he incurs R&D costs $F_e(q_e)$, where $F_e = \beta \frac{q_e^2}{2}$ and $\beta \geq \frac{4}{9}$. The restriction on the parameter β ensures that the quality chosen by the entrant, q_e, is bounded between zero and one. Note that with this formulation, $F_e'(q_e) > 0$ and $F_e''(q_e) > 0$, thus, it is increasingly costly for the entrant to locate away from the incumbent in the one-dimensional product space (i.e., to produce the better quality product). In addition, since q_e represents the quality difference between the incumbent's and the entrant's product the filing of a patent by the incumbent provides the entrant with knowledge of how to produce the incumbent's product (i.e., $F_e(q_p) = 0$ – the assumption of perfect information disclosure by the patent is made). An important assumption of the model is that in the absence of the patent reserves engineering of the product innovation is possible and costless. The R&D costs are assumed sunk once they have been incurred and neither the incumbent nor the entrant find it optimal to relocate once they have chosen their respective qualities. Once the R&D costs are incurred, production of the products by both the incumbent and the entrant occur at zero marginal cost.

The patent breadth claimed and granted to the incumbent's product is denoted by b and it defines the area in the one-dimensional product space that the patent protects, thus, $b \in (0,1]$. Patent breadth values close to zero indicate protection of the patented innovation only against duplication. When the entrant locates at a distance $q_e < b$ away from q_p the patent is infringed and the incumbent must decide whether to invoke an infringement trial or not. It is assumed that the filing of an infringement lawsuit by the incumbent is always met with a counterclaim by the accused infringer that the patent is invalid.[2] The costs incurred during the infringement trial/validity attack by the incumbent and the entrant are denoted by C_p^T and C_e^T, respectively. These costs are assumed to be independent of the breadth of protection and of the entrant's location. The trial costs will only be incurred if $q_e < b$ and they are assumed to be sunk – once made they cannot be recovered by either party.[3]

The patent system being modeled is assumed to be that of the fencepost type, in which patent claims define an exact border of protection. Under the fencepost system, infringement will always be found when an entrant locates within the incumbent's claims, unless the entrant proves that the patent is invalid (Cornish 1989).[4] In the fencepost system the probability that infringement is found does not depend on how close the entrant has located to the incumbent. The implication of assuming a fencepost patent system is that the probability that infringement will be found (given that the entrant has located at $q_e < b$ distance away from q_p) is equal to the probability that the validity of the patent will be upheld. Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.[5]

Patent validity is directly linked to patent breadth. In general, the broader is the patent protection, the harder it is to show novelty, nonobviousness and enablement (Miller and Davis 1990). Thus, the broader is patent protection, the harder it is to establish validity. In addition, evidence from the literature shows that courts tend to uphold narrow patents and invalidate broad

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[2] This is a standard defence adopted by accused infringers (Cornish 1989, Merges and Nelson 1990).
[3] With this assumption we exclude the possibility of the court awarding lawyers' fees to either party.
[4] In contrast, a signpost patent system implies that claims provide an indication of protection and the claims are interpreted using the doctrines of equivalents and reverse equivalents. Under a signpost system the closer the entrant locates to the incumbent the easier it is to prove infringement using the doctrine of equivalents. In addition, infringement may be found even when the entrant locates outside the incumbent's claims using the doctrine of reverse equivalents.
[5] Note that, our analysis and results are not affected by whether only certain claims are invalidated during the infringement/validity trial or the entire patent; that is, when patent breadth is narrowed rather than the entire patent revoked. This occurs because further entry is not anticipated in our model.

ones (Waterson 1990, Cornish 1989, Merges and Nelson 1990). To capture these observations, the probability $\mu(b)$ that the patent will be found to be valid, or equivalently that infringement will be found, is assumed to be inversely related to patent breadth – i.e., $\mu'(b) < 0$. Specifically, $\mu(b) = 1 - \alpha b$.[6] Thus, $1 - \mu(b) = \alpha b$ is the probability that the patent will be found to be invalid. The validity parameter α, $\alpha \in (0,1)$, reflects the degree that patent breadth affects patent validity. For any given patent breadth, the greater is the validity parameter α, the greater is the probability that the patent will be found invalid.

2.2 The Game

The patent breadth game consists of five stages. In the first stage of the game, the incumbent decides whether to seek patent protection or not. If the incumbent decides not to patent her innovation then, given the assumption of possible and costless reverse engineering, the entrant enters at his most preferred location and he and the incumbent compete in prices at the last stage of the game and earn duopoly profits Π_e^{NP} and Π_p^{NP}, respectively. If the incumbent decides to patent her innovation then at the second stage of the game she decides on the patent breadth, b, claimed. In the third stage of the game, a potential entrant observes the incumbent's product and the breadth of protection granted to it and chooses whether or not to enter the market. If the entrant does not enter he earns zero profits while the incumbent operates as a monopolist in the last stage of the game and earns monopoly profits Π_m. If the entrant enters, he does so by choosing the quality q_e of his product relative to that of the incumbent. This decision determines whether the entrant

[6] Patent breadth is not the only factor affecting the validity of the patent. A patent may also be invalidated because of unallowable amendments during patent examination and because the innovation is not regarded as an invention under the patent law (Cornish 1989). By assuming that the innovator has generated a patentable innovation, the latter case is excluded. To keep the analysis simple, it is also assumed that the probability of patent invalidation due to unallowable amendments is negligible.

infringes the patent or not, as well as whether the incumbent will invoke a trial in the case the patent is infringed.

If the entrant chooses a quality greater than or equal to the patent breadth claimed by the incumbent (i.e., $q_e \geq b$), then no infringement occurs, and he and the incumbent compete in prices in the last stage of the game and earn duopoly profits Π_e^{NI} and Π_p^{NI}, respectively. If the entrant locates inside the patent breadth claimed by the incumbent (i.e., $q_e < b$), the patent is infringed and the incumbent needs to decide whether to invoke a trial or not. This decision is made in the fourth stage of the game. The payoffs for the incumbent and the entrant when the entrant chooses $q_e < b$ and the incumbent chooses not to invoke a trial are $(\Pi_e^I)^{NT}$ and $(\mathit{\Pi}_p^I)^{NT}$, respectively. If the incumbent invokes a trial then the validity of the patent is examined. With probability $\mu(b)$, the patent is found to be valid (i.e., infringement is found), the entrant is not allowed to market his product and the incumbent operates as a monopolist in the last stage of the game. With probability $1-\mu(b)$, the patent is found to be invalid, and the entrant and the incumbent compete in prices. The payoffs for the incumbent and the entrant when the entrant chooses $q_e < b$ and the incumbent invokes a trial are $E(\mathit{\Pi}_e^I)^T$ and $E(\mathit{\Pi}_p^I)^T$, respectively. Figure 1 illustrates the extensive form of the game outlined above.

The solution to this game is found by backward induction. The fifth stage of the game in which the incumbent and the entrant – when applicable – compete in prices is examined first, followed by the fourth stage in which the incumbent makes her trial decision, the third stage in which the entrant makes his entry decision, the second stage in which the incumbent makes her decision regarding patent breadth and finally the first stage in which the incumbent decides whether to patent her innovation or not.

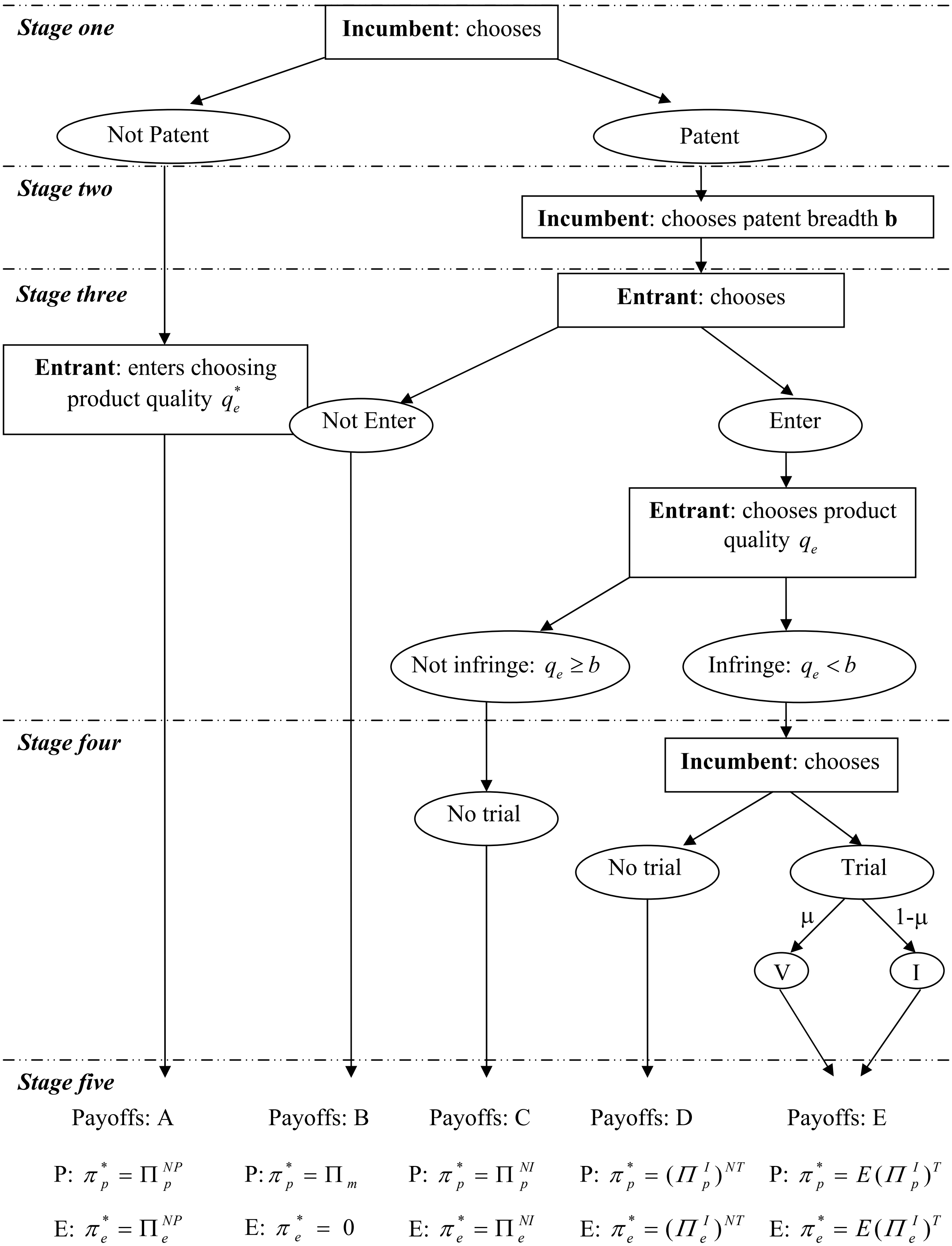

Figure 1. The Patenting Game

3. Analytical Solution of the Game

3.1 Stage 5 – The Pricing Decisions

In the fifth stage of the game, two cases must be considered – the case where the entrant has entered and the case where the entrant has not entered. Considering the last case first, in the absence of entry by the entrant, the incumbent will charge $p_p = V$ and earn monopoly profits $\Pi_m = V - F_p$.

If entry occurs, the problem facing duopolist *i* is to choose price p_i to maximize profit

$\pi_i = p_i y_i - F_i$ ($i = p, e$), where $y_p = \dfrac{p_e - p_p}{q_e}$ and $y_e = \dfrac{q_e + p_p - p_e}{q_e}$. Recall that the R&D costs, F_p and F_e for the incumbent and the entrant, respectively, are assumed to be sunk at this stage in the game. The Nash equilibrium in prices, as well as the resulting outputs and profits, are given by:

(2) Incumbent: $$p_p^* = \frac{q_e}{3},\ y_p^* = \frac{1}{3},\ \pi_p^* = \frac{q_e}{9}$$

(3) Entrant: $$p_e^* = \frac{2q_e}{3},\ y_e^* = \frac{2}{3},\ \pi_e^* = \frac{4q_e}{9}$$

Since the entrant has the higher quality product, he charges the higher price. Profits are increasing in the distance q_e between the incumbent's and the entrant's location. The greater is the difference in quality between the two products, the less intense is competition at the final stage of the game and the greater are the profits for both the incumbent and the entrant.[7]

3.2 Stage 4 – The Incumbent's Trial Decision

As illustrated in Figure 1, under patenting, the entrant's location decision (his quality choice q_e) will determine whether the patent will be infringed and whether in the case of infringement a trial

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[7] This is a well-established result in the product differentiation literature in simultaneous games. When competitors first simultaneously choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Motta 1993, Shaked and Sutton 1982).

will take place. When the entrant infringes the patent, the incumbent/patentee needs to decide whether to invoke an infringement trial or not. Given the quality chosen by the entrant, the patentee will invoke a trial when the patent is infringed as long as her expected profits when a trial takes place, $E(\Pi_p^I)^T$, are greater than her profits when a trial does not take place, $(\Pi_p^I)^{NT}$, i.e.,

$E(\Pi_p^I)^T > (\Pi_p^I)^{NT}$.

When the patentee invokes a trial her expected profits are given by:

$$(4) \qquad E(\Pi_p^I)^T = \mu\Pi_m + (1-\mu)\pi_p - C_p^T = (1-ab)\Pi_m + ab\frac{q_e}{9} - C_p^T$$

Equation (4) demonstrates that with probability μ infringement will be found (or equivalently that the validity of the patent will be upheld) at trial and the entrant will not be allowed in the market while the patentee will have a monopoly position and with probability $1-\mu$ infringement will not be found, the entrant will be allowed to market his product and the patentee and the entrant will operate as duopolists.

When the patentee does not invoke a trial her profits are given by:

$$(5) \qquad (\Pi_p^I)^{NT} = \pi_p^* = \frac{q_e}{9}$$

Equation (5) shows that when the patentee does not invoke a trial when infringement occurs she shares the market with the entrant realizing duopoly profits which depend on the entrant's choice of location in the quality product space.

Given the above the patentee will invoke a trial when her patent is infringed if:

$$(6) \qquad E(\Pi_p^I)^T > (\Pi_p^I)^{NT} \Rightarrow q_e < 9(\Pi_m - \frac{C_p^T}{1-ab})$$

Equation (6) shows that the patentee's decision on whether to invoke a trial when her patent is infringed may be affected by the entrant's location decision. We denote the quality that makes the patentee indifferent between invoking and not invoking a trial by $\overline{q}_e$, i.e., $\overline{q}_e = 9(\Pi_m - \frac{C_p^T}{1-ab})$, $\overline{q}_e \in (0,1)$ and assume that when the patentee is indifferent she will choose to not invoke a trial. The quality $\overline{q}_e$ depends among other things on the patent breadth chosen by the incumbent and is decreasing in patent breadth at an increasing rate, i.e., $\frac{\partial \overline{q}_e}{\partial b} < 0, \frac{\partial^2 \overline{q}_e}{\partial b^2} < 0$. Thus, the greater is the patent breadth chosen, the smaller is the quality chosen by the entrant that will infringe the patent without invoking a trial. Figure 2 below illustrates the relationship between the quality chosen by the entrant, q_e, and the patentee's decision to invoke a trial or not for any patent breadth choice, b.

As depicted in Figure 2, as long as the entrant chooses a product quality $q_e \geq b$ the patent is not infringed. When the entrant chooses a product quality q_e such that $\overline{q}_e < q_e < b$ (i.e., a quality to the right of locus $\overline{q}_e$ and below the locus $b = q_e$) the patent will be infringed but the patentee will not invoke a trial. This outcome is depicted by the dotted area in Figure 2. When the entrant chooses a product quality q_e such that $q_e < b$ and $q_e < \overline{q}_e$ (i.e., a quality to the left of locus $\overline{q}_e$ and below the locus $b = q_e$) the patent will be infringed and the patentee will invoke a trial. This outcome is depicted by the horizontally hatched area in Figure 2. As the monopoly profits that can be earned by the incumbent under no market entry or when the patent is infringed and its validity is upheld during trial increase, the locus $\overline{q}_e$ shifts upward and the more likely it becomes that a trial will take place under infringement (the infringement and trial area becomes larger). The greater are the incumbent's trial costs the less likely it is that she will find it optimal to invoke an infringement trial under infringement (as trial costs increase the locus $\overline{q}_e$ shifts downward and the infringement and trial area becomes smaller).

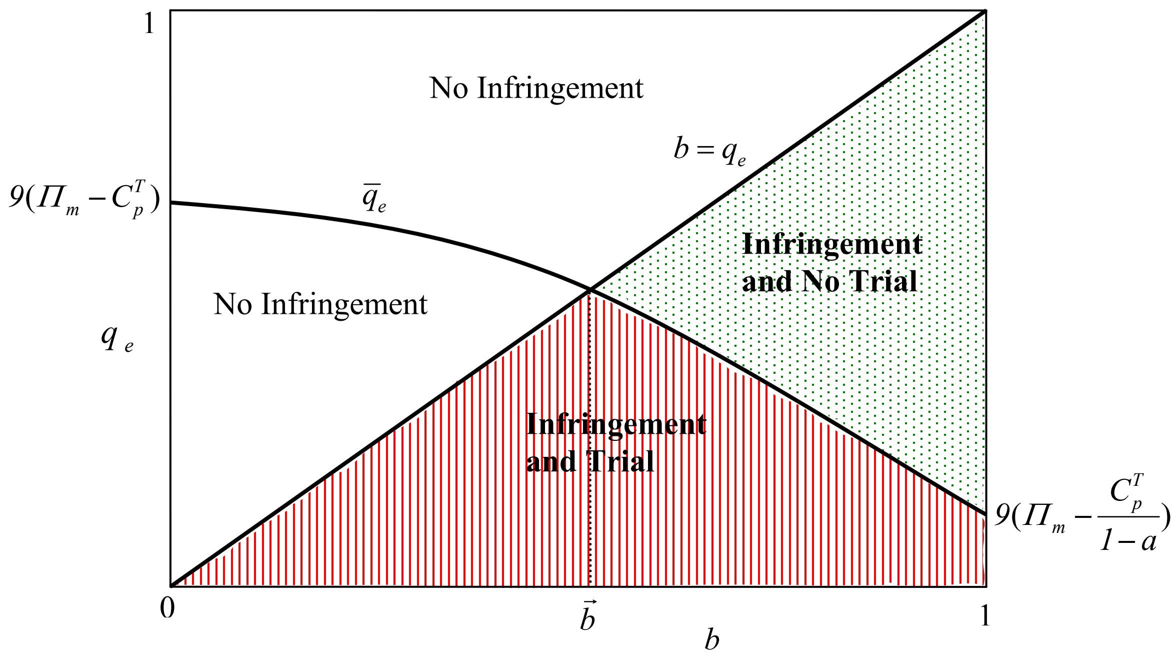

Figure 2. The Entrant's Quality Choice, q_e, and the Patentee's Trial Decision

Note that, when $q_e \leq \bar{q}_e \; \forall \, q_e \in (0,1]$, invoking a trial when the patent is infringed ($q_e < b$) is always an optimal strategy for the patentee, regardless of the quality chosen by the entrant. This case occurs when $9(\Pi_m - \frac{C_p^T}{1-a}) \geq 1 \Rightarrow \Pi_m \geq \frac{1}{9} + \frac{C_p^T}{1-\alpha} \; \forall \, \Pi_m \geq 0, C_p^T \geq 0$ and $\alpha \in (0,1)$ (i.e., the locus $\bar{q}_e$ is above the locus $q_e = b$ in Figure 2 $\forall q_e \in (0,1)$ and $b \in (0,1]$. The case where under infringement a trial always occurs regardless of the entrant's product quality choice, q_e, has been examined by Yiannaka and Fulton (2003) and will not be considered here.

Also note that, when $q_e > \bar{q}_e \; \forall \, q_e \in (0,1]$, invoking a trial when the patent is infringed ($q_e < b$) is never an optimal strategy for the patentee, regardless of the quality chosen by the entrant. This case occurs when $9(\Pi_m - C_p^T) \leq 0 \Rightarrow \Pi_m \leq C_p^T \; \forall \, \Pi_m \geq 0, C_p^T \geq 0$ (i.e., the locus $\bar{q}_e$ is below the locus $q_e = b$ in Figure 2 $\forall q_e \in (0,1)$ and $b \in (0,1]$); the monopoly profits earned by the patentee when the patent is infringed and its validity is upheld or when the entrant does not enter the market are smaller than the trial costs that she incurs when she invokes an infringement trial. In this case, however, it is straightforward to show that, as long as the patenting costs, z, are positive, the incumbent will not have an incentive to take a patent. This is so because, given our model assumptions of complete information and costless and possible reverse engineering, if the entrant knows that irrespective of his quality choice a trial will never take place, he will always find it optimal to locate at his most preferred location, q_e^*, (where he also locates under no patent protection) regardless of the patent breadth chosen.[8]

As mentioned above, we are interested in examining the case under which the entrant by his choice of location in the quality product space, q_e, affects the incumbent's trial and patent breadth decisions, i.e., the case where $\overline{q}_e \in (0,1)$. This case occurs when the following conditions are satisfied. First, the monopoly profits realized by the incumbent when the entrant does not enter or when the validity of the patent is upheld during an infringement trial, Π_m, must satisfy the condition $C_p^T < \Pi_m < \frac{1}{9} + \frac{C_p^T}{1-\alpha}$ (i.e., the locuses $\overline{q}_e$ and $b = q_e$ in Figure 2 cross for $q_e \in (0,1)$ and $b \in (0,1]$). Thus, our analysis focuses on the case where a patent breadth $\vec{b} \in (0,1)$ that satisfies the condition $b = \overline{q}_e$ exists. It can be easily shown (see Appendix for a proof) that for $\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$ the patent breadth $\vec{b} = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ satisfies the $b = \overline{q}_e$ condition.

Second, if we denote the entrant's most preferred location under no patent protection or under patent protection when patent breadth is not binding by q_e^* then the condition $q_e^* < \vec{b}$ must

[8] Note that this is not necessarily true when reverse engineering is possible and costly because the entrant's optimal location choice q_e^* will be different under patenting where the information about the incumbent's product is public knowledge and under no patenting where the entrant has to incur a cost to obtain the information.

also be satisfied. Note that, when $q_e^* \geq \vec{b}$ the entrant will always choose q_e^* and the patentee will not invoke a trial when the patent is infringed. However, knowing that when $q_e^* \geq \vec{b}$, regardless of the patent breadth chosen she won't be able to enforce/defend her patent rights, the incumbent will not seek patent protection. Thus, for positive patenting costs when $q_e^* \geq \vec{b}$ a patent will not be sought by the incumbent. The condition $q_e^* < \vec{b}$ is satisfied for R&D cost values, β, such that $\beta > \beta_0 = \dfrac{8\alpha}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})}$. The greater is the entrant's R&D effectiveness (i.e., the smaller is β), the more likely it is that patenting will not be profitable for the incumbent. In addition, the smaller is the validity parameter, α, and the incumbent's trial costs, C_p^T, and the greater are the monopoly profits Π_m, the smaller should be the entrant's R&D costs to satisfy the condition $q_e^* < \vec{b}$ (for a proof see the Appendix).

3.3 Stage 3 – The Entrant's Location Decision

As illustrated in Figure 1, two cases must be considered regarding the entrant's location decision depending on whether the incumbent has patented her innovation or not.

3.3.1 No Patent Protection

Under no patent protection the entrant can freely locate at any point in the quality product space. Note that given our assumption of possible and costless reverse engineering the entrant cannot be deterred from entering the market under no patent protection; at the very least, the entrant can locate at $q_e = q_p$, share the market with the incumbent and realize zero profits. Let q_e^* be the optimal quality the entrant chooses under no patent protection, where q_e^* solves the following problem:

$$\max_{q_e} \Pi_e = \pi_e - F_e = \frac{4q_e}{9} - \beta\frac{q_e^2}{2} \tag{7}$$

Optimization of equation (7) yields the optimal quality q_e^*:

$$(8) \qquad q_e^* = \frac{4}{9\beta}$$

Equation (8) gives the entrant's most preferred location and indicates that the less costly it is to produce the better quality product (i.e., the smaller is β), the further away from the incumbent the entrant locates.

3.3.2 Patent Protection

Under patent protection and anticipating the incumbent's behavior concerning trial given q_e, the entrant must choose one of four options – Not Enter, Enter and Not Infringe the Patent, Enter, Infringe the Patent and Induce a Trial or Enter, Infringe the Patent and Not Induce a Trial. For any given patent breadth, b, the entrant will choose the option that generates the greatest profit.

The outcome of the Not Enter option is straightforward – the entrant earns zero profits. The outcomes of the other three options depend on a number of factors, including patent breadth, R&D costs and trial costs. The benefits and costs associated with the Enter and Not Infringe option are examined below, followed by an examination of the benefits and costs associated with the Enter and Infringe option. The examination of the Enter and Infringe option consists of the examination of the Enter, Infringe and Not Induce a Trial and the Enter Infringe and Induce a Trial options. Once the net benefits of each option are formulated, the most desirable option for the entrant is determined for any given patent breadth.

3.3.2.1 Entry with No Infringement ($q_e \geq b$)

For the entrant to enter without infringing the patent, the entrant must choose a quality location that is greater than or equal to the patent breadth – i.e., $q_e \geq b$. Given that q_e^* is the entrant's most preferred location, as long as $q_e^* \geq b$, the patent breadth does not affect the location chosen by the entrant (patent breadth is not binding), since the entrant can choose his optimal quality without fear of infringement. Thus, patent breadth will only be binding if $q_e^* < b$. Note that, when the entrant's R&D cost parameter takes its minimum value ($\beta = \frac{4}{9}$) the entrant never infringes the patent as $q_e^* = 1$. Since an increase in quality beyond q_e^* results in a reduction in profits, the entrant's profit is decreasing in q_e for all $q_e > q_e^*$. As a result, the entrant, when faced with a binding patent breadth, will always choose a quality equal to the patent breadth chosen by the incumbent (i.e., $q_e = b$).

Thus, a profit-maximizing entrant that wishes to not infringe the patent will choose his entry location q_e^{NI} as follows:

$$(9) \qquad q_e^{NI} = \begin{cases} \dfrac{4}{9\beta} & \text{if } b < \dfrac{4}{9\beta} \\ \text{b} & \text{if } b \geq \dfrac{4}{9\beta} \end{cases}$$

while the profits earned by the entrant are:

$$(10) \qquad \Pi_e^{NI} = \begin{cases} \dfrac{8}{81\beta} & \text{if } b < \dfrac{4}{9\beta} \\ \dfrac{4}{9}b - \dfrac{\beta}{2}b^2 & \text{if } b \geq \dfrac{4}{9\beta} \end{cases}$$

3.3.2.2 Entry with Infringement ($q_e < b$)

As discussed earlier the analysis focuses on the case where the entrant by his choice of location in the quality product space can affect the incumbent's trial decision, i.e., $\overline{q}_e \in (0,1) \wedge q_e^* < \vec{b}$. Thus, when the entrant decides to infringe the patent he must determine whether to induce the incumbent to invoke a trial or not. The entrant's profits under infringement and trial are determined below followed by his profits when he infringes and does not induce the patentee to invoke a trial.

<u>*The Entrant's Profits under Infringement and Trial*</u>

Recall that during an infringement trial there is a probability $\mu = 1-\alpha b$ that the validity of the patent will be upheld (or equivalently that infringement will be found) and a probability $1-\mu = \alpha b$ that the patent will be revoked. If the patent is found to be valid during trial, the entrant cannot enter and the patentee has a monopoly position in the market. If the patent is found to be invalid, the entrant is allowed to market his product and the patentee and the entrant operate as duopolists. With this background, the quality chosen by the entrant is determined by solving:

$$(11) \qquad \max_{q_e} E\left(\Pi_e^I\right)^T = (1-\mu)\cdot \pi_e - F_e - C_e^T = \alpha b \frac{4q_e}{9} - \beta \frac{q_e^2}{2} - C_e^T$$

The optimal quality chosen is given by:

$$(12) \qquad (q_e^I)^T = \frac{4\alpha b}{9\beta}$$

From Equation (12) it follows that the optimal quality under infringement satisfies the condition $(q_e^I)^T < b \Rightarrow \alpha < \frac{9}{4}\beta \ \forall \alpha \in (0,1)$ and $\beta > \beta_0 > \frac{4}{9}$. Equation (12) shows that when the entrant infringes the patent he finds it optimal to locate at a distance proportional to the breadth of the patent. Because there is uncertainty with respect to whether the entrant will be able to continue in the market, he 'underlocates'; to reduce the R&D costs, which are incurred with certainty, the entrant locates closer to the patentee than he would have done had infringement not been a possibility.

The expected profits for the entrant are given by equation (13):

$$(13) \qquad E\left(\Pi_e^I\right)^T = \frac{8\alpha^2 b^2}{81\beta} - C_e^T$$

When patent breadth is negligible (i.e., b approaches zero), the expected profits from infringement approach $-C_e$, since the probability of the patent being found valid approaches one. As patent breadth increases, expected profits from infringement also increase, a reflection of the rising probability that the patent will be found invalid.

The Entrant's Profits under Infringement and No Trial

When the choice of the entrant's most preferred quality q_e^* results in infringement and trial and the entrant wishes to infringe but not induce a trial, he maximizes his profits by choosing the lowest q_e associated with ensuring that the patentee does not invoke a trial. Thus, to maximize his profits under the infringement and no trial outcome the entrant will choose the quality $(q_e^I)^{NT} = \overline{q}_e$ (recall that when the patentee is indifferent between invoking and not invoking a trial she will choose to not invoke a trial). Given the above, the entrant's profits under infringement and no trial are given by equation (14):

$$(14) \qquad (\Pi_e^I)^{NT} = \pi_e - F_e = \frac{4\overline{q}_e}{9} - \beta\frac{\overline{q}_e^2}{2} = 4(\Pi_m - \frac{C_p^T}{1-\alpha b}) - \frac{81\beta}{2}(\Pi_m - \frac{C_p^T}{1-\alpha b})^2$$

The entrant's profits under infringement and no trial $(\Pi_e^I)^{NT}$ are non decreasing in b, $\forall b \in [\vec{b}, \overline{b})$, $\beta > \beta_0$ and $\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha b})$ (for a proof see Appendix). Thus, the greater is patent breadth, b, the smaller is $\overline{q}_e$ and thus the closer to q_e^* the entrant can locate without inducing the patentee to invoke a trial.

Figure 3 illustrates the entrant's quality choices and the patentee's trial decision for $q_e^* < \vec{b}$ as well as the entrant's profits under no infringement, infringement and trial and infringement and no trial. As illustrated in Figure 3, for patent breadth values $b \in (0, b_0] \vee b \in [\overline{b}, 1]$ where $b_o = q_e^* = \frac{4}{9\beta}$, $b_0 \in (0, \vec{b})$ and $\overline{b} : q_e^* = \overline{q}_e \Rightarrow \overline{b} = \frac{81\beta\Pi_m - 81C_p^T\beta - 4}{\alpha(81\beta\Pi_m - 4)}$, $\overline{b} \in (\vec{b}, 1]$, the entrant will always find it optimal to enter the market and locate at his most preferred location, q_e^*, without invoking a trial; patent breadth is not binding for $b \in (0, b_0]$ and for $b \in [\bar{b}, 1]$ the incumbent always finds it optimal to not invoke a trial under infringement. When the patent breadth chosen is such that $b \in (b_0, \vec{b})$, the entrant cannot locate at his most preferred location, q_e^* without infringing the patent while the patentee will always find it profitable to invoke a trial when the patent is infringed (i.e., $q_e < b$). In this case, the entrant will have to decide whether to enter and if entry occurs whether to infringe or not the patent knowing that if he infringes a trial will always take place. Finally, when the patent breadth chosen is such that $b \in [\vec{b}, \bar{b})$, the entrant cannot locate at his most preferred location, q_e^* without infringing the patent but he can, by his choice of location on the quality product space, q_e, affect whether the patentee will invoke a trial or not when the patent is infringed.

For the profit curves depicted in Figure 3, if the patentee chooses patent breadth b_1 the entrant will find it optimal to choose the product quality, $(q_e^I)^T$, that infringes the patent and induces the patentee to invoke a trial while if the patentee chooses patent breadth b_2 the entrant will find it optimal to choose the product quality $(q_e^I)^{NT}$, that infringes the patent and induces the patentee to not invoke a trial. Thus, under this scenario the entrant has to decide whether to enter and if entry occurs whether to induce the patentee to invoke a trial or not. Note that when $b \in [\vec{b}, \bar{b})$ the entrant will never choose to not infringe the patent since the non infringement strategy is always dominated by the infringement and no trial strategy (i.e., the entrant's profits under no infringement and under infringement and no trial are equal at $\vec{b}$, $\Pi_e^{NI}(\vec{b}) = E(\Pi_e^I)^{NT}(\vec{b})$ and Π_e^{NI} and $E(\Pi_e^I)^{NT}$ are decreasing and increasing in b, respectively, for any $b \in (\vec{b}, 1]$).

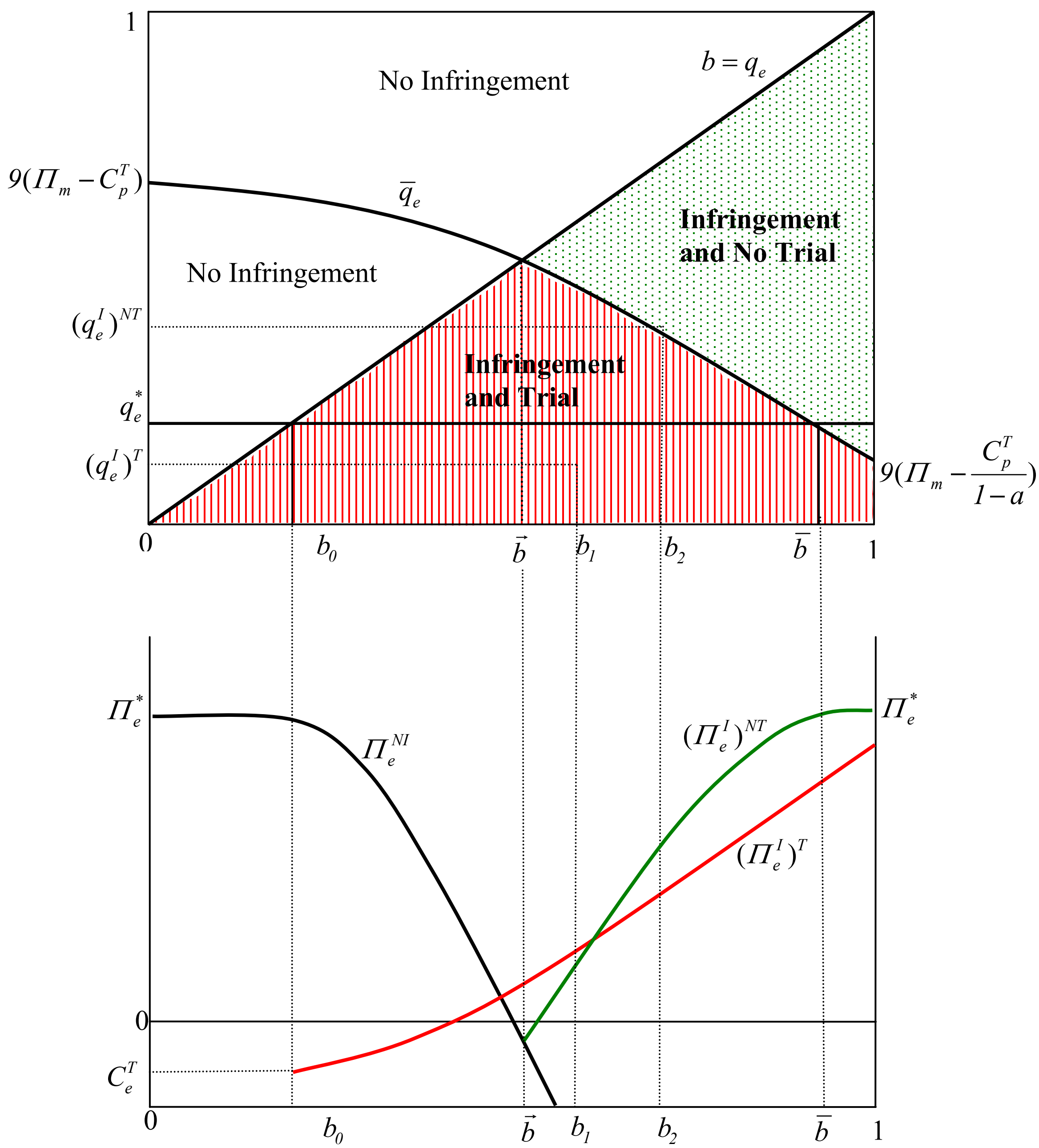

Figure 3. The Entrant's Quality Choice and the Patentee's Trial Decision for $q_e^* < \vec{b}$ and the Entrant's profits under No Infringement, Infringement and Trial and Infringement and No Trial.

The Entry/Infringement Decision

To determine the entrant's optimal strategy we must first determine whether there exists a patent breadth $\hat{b} \in (b_o, \vec{b})$ that can deter the entrant from entering in the market, i.e.,

$\Pi_e^{NI}(\hat{b}) \le 0 \wedge E(\Pi_e^I)^T(\hat{b}) \le 0 \wedge E(\Pi_e^I)^{NT}(\hat{b}) \le 0$. Define b_I^T as the patent breadth that makes the entrant indifferent between entering the market, infringing the patent and inducing a trial and not entering the market. Thus, b_I^T solves: $E(\Pi_e^I)^T(b_I^T) = 0$ where $b_I^T \in (b_0, 1]$.[9] Also, define b_I^{NT} as the patent breadth that makes the entrant indifferent between entering the market, infringing the patent and not inducing a trial and not entering the market. Thus, b_I^{NT} solves: $E(\Pi_e^I)^{NT}(b_I^{NT}) = 0$ where $b_I^{NT} \in (b_0, 1]$. Finally, define b_{NI} as the patent breadth that makes the entrant indifferent between entering the market without infringing the patent and not entering the market – i.e., b_{NI} satisfies $\Pi_e^{NI}(b_{NI}) = 0$, where $b_{NI} \in (b_0, 1]$. It is straightforward to show that $b_I^T = \sqrt{\dfrac{81\beta C_e^T}{8\alpha^2}}$, $b_I^{NT} = \dfrac{81\beta(\Pi_m - C_e^T) - 8}{81\beta\alpha\Pi_m - 8\alpha}$ and $b_{NI} = \dfrac{8}{9\beta}$; since $b_{NI} \in (b_0, 1]$, b_{NI} exists only for β values such that $\beta \ge \dfrac{8}{9}$ or $\beta \ge \beta_0$ whichever is greater. Given the above, any $b \in (b_0, 1]$ such that $b \le b_I^T$ makes entry under infringement and trial unprofitable for the entrant, any $b \in (b_0, 1]$ such that $b \le b_I^{NT}$ makes entry under infringement and no trial unprofitable for the entrant, while any $b \in (b_0, 1]$ such that $b \ge b_{NI}$ makes entry under no infringement unprofitable for the entrant.

Scenario A: Entry Deterrence

The entrant will not find it profitable to enter the market if there exists a $\hat{b} \in (b_0, 1]$ such that $b_{NI} \le \hat{b} \le b_I^T$ for $b_I^T < b_I^{NT}$ or $b_{NI} \le \hat{b} \le b_I^{NT}$ for $b_I^T > b_I^{NT}$ and $\vec{b} \ge b_{NI}$. The entry deterrence outcome is illustrated in Figure 4. The larger is the R&D cost parameter β, the easier it is to deter entry, *ceteris paribus,* because as β increases b_{NI} becomes smaller, b_I^T becomes larger while $\vec{b}$ is

[9] The assumption is made that the entrant will not enter when he is indifferent between entering and infringing the patent and not entering.

unaffected, making it more likely that the entry deterrence condition $b_{NI} \leq \hat{b} \leq b_I^T$ and $\vec{b} \geq b_{NI}$ will be satisfied.

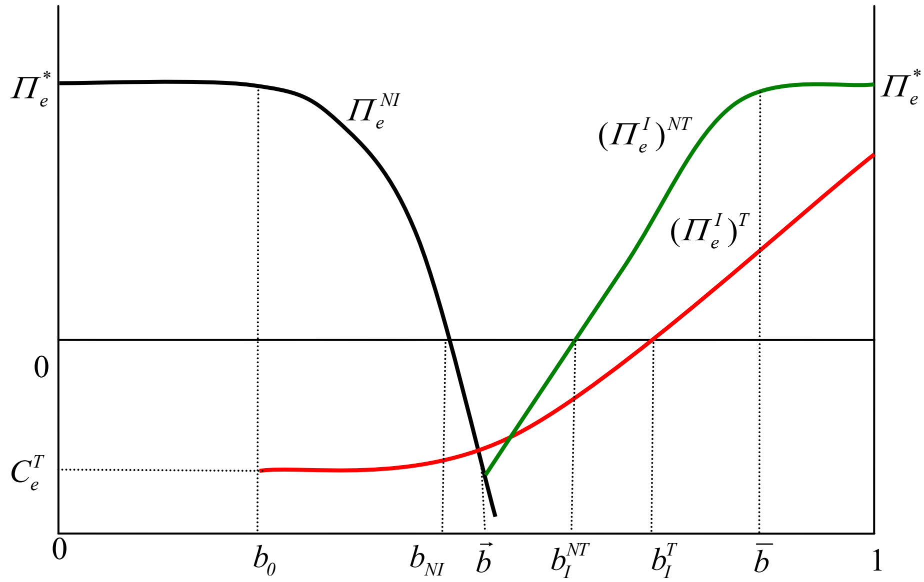

Figure 4. The Entrant's Profits under No Infringement, Infringement and Trial and Infringement and No Trial when Entry can be Deterred.

Scenario B: Entry Cannot be Deterred

There are a number of different cases where entry cannot be deterred leading to different optimal strategies for both the incumbent and the entrant. Generally, entry cannot be deterred when either a patent breadth b_{NI} that makes the entrant indifferent between entering without infringing and not entering does not exist, when it exists and $\vec{b} < b_{NI}$ and/or a patent breadth, $\tilde{b}$, exists that makes the entrant indifferent between infringing the patent and inducing a trial and not infringing the patent, while still generating positive profits for the entrant – i.e., $\tilde{b}$ solves $\Pi_e^{NI}(\tilde{b}) = E((\Pi_e^I)^T(\tilde{b})) > 0$ where $\tilde{b} \in (b_0, 1]$. The expression for $\tilde{b}$ is derived in the Appendix. To examine the different cases that can emerge when entry cannot be deterred, let $\tilde{\tilde{b}}$ be the patent breadth that makes the entrant indifferent between infringing the patent and inducing a trial and infringing the patent and not inducing a trial, while still generating positive profits for the entrant – i.e., $\tilde{\tilde{b}}$ solves $(\Pi_e^I)^T(\tilde{\tilde{b}}) = E((\Pi_e^I)^{NT}(\tilde{\tilde{b}})) > 0$ where $\tilde{\tilde{b}} \in (b_0, 1]$. The expression for $\tilde{\tilde{b}}$ is derived in the Appendix. As will become evident in the cases below, the optimal strategy for the entrant when entry cannot be deterred (scenario B) depends on the relationship between $\vec{b}$, b_{NI} and $\tilde{b}$ as well as on the existence of $\bar{b}$. Cases I and II examine the entrant's profits when $\bar{b}$ exists which implies that at $b = 1$ the entrant's profits under infringement and no trial will always be greater than his profits under infringement and trial, $(\Pi_e^I)^{NT}(b = 1) > (\Pi_e^I)^T(b = 1)$. Cases III and IV examine the entrant's profits when $\bar{b}$ does not exist, which implies that at $b = 1$ the entrant's profits under infringement and no trial can be greater, equal to or smaller than his profits under infringement and trial. Cases III and IV consider the situation where $(\Pi_e^I)^{NT}(b = 1) < (\Pi_e^I)^T(b = 1)$. When $\bar{b}$ does not exist and $(\Pi_e^I)^{NT}(b = 1) \geq (\Pi_e^I)^T(b = 1)$ cases III and IV are equivalent to cases I and II, respectively.

- Case I: $\exists \bar{b} \wedge \tilde{b} \ \wedge \vec{b} \leq \tilde{b}$

Under this case, the entrant's optimal strategy is to either not infringe the patent for relatively low patent breadth values ($b \in (b_0, \vec{b})$) or to infringe the patent and not induce a trial for relatively high patent breadth values ($b \in [\vec{b}, 1)$); the infringement and trial strategy is a dominated strategy for all patent breadth values. This case is most likely to emerge when the entrant's trial costs, C_e^T, are relatively high, making infringement and trial less attractive to the entrant, and the incumbent's monopoly profits, Π_m, and trial costs, C_p^T are relatively, low and high, respectively (making $\bar{q}_e$ smaller for any patent breadth and thus the infringe and no trial strategy attractive to the entrant). Case I is depicted in panel (i) in Figure 5.

- Case II: $\exists \bar{b} \wedge \tilde{b} \ \wedge \vec{b} > \tilde{b}$

Under this case, the entrant's optimal strategy is to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \tilde{b})$), infringe the patent and induce a trial for intermediate patent breadth values ($b \in [\tilde{b}, \tilde{\tilde{b}})$) and infringe the patent and not induce a trial for relatively large patent breadth values ($b \in [\tilde{\tilde{b}}, \bar{b})$). This case is most likely to emerge when the entrant's trial costs, C_e^T, and the validity parameter, α, are relatively low, and the R&D cost parameter, β, is relatively high, (making infringement with trial attractive to the entrant) and when the incumbent's monopoly profits, Π_m, and trial costs, C_p^T, are relatively high and low, respectively (making infringement and no trial attractive only for large values of patent breadth). Case II is depicted in panel (ii) in Figure 5.

- Case III: $\exists \tilde{b}, \nexists \bar{b} \wedge \vec{b} \leq \tilde{b}$

Under this case, the entrant will find it optimal to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \vec{b}]$), he will infringe the patent and not induce a trial for intermediate patent breadth values, $b \in (\vec{b}, \tilde{\tilde{b}}]$ and he will infringe the patent and induce a trial for relatively high patent breadth values $b \in (\tilde{\tilde{b}}, 1]$. This case is most likely to emerge when the entrant's trial costs, C_e^T, R&D cost parameter, β, and the validity parameter, α, are relatively high (making infringement and trial attractive only for high patent breadth values) and the incumbent's monopoly profits Π_m, and trial costs, C_p^T, are low and high, respectively (making infringement and no trial attractive even for relatively low patent breadth values). Case III is depicted in panel (iii) in Figure 5.

- Case IV: $\exists \tilde{b}, \nexists \bar{b} \wedge \vec{b} > \tilde{b}$

Under this case, the entrant will find it optimal to not infringe the patent for relatively low patent breadth values ($b \in (b_0, \tilde{b}]$) and infringe the patent inducing the incumbent to invoke a trial for relatively large patent breadth values ($b \in (\tilde{b}, 1]$); the infringe-and-not-induce-a-trial strategy is dominated by the other two strategies for all patent breadth values. This case is most likely to occur when the entrant's trial costs, C_e^T, and R&D cost parameter, β, are relatively low and high, respectively, the validity parameter, α, is high (making infringement and trial attractive) and the incumbent's monopoly profits Π_m, and trial costs, C_p^T, are relatively high and low, respectively (making infringement and no trial less attractive). Case IV is depicted in panel (iv) in Figure 5.

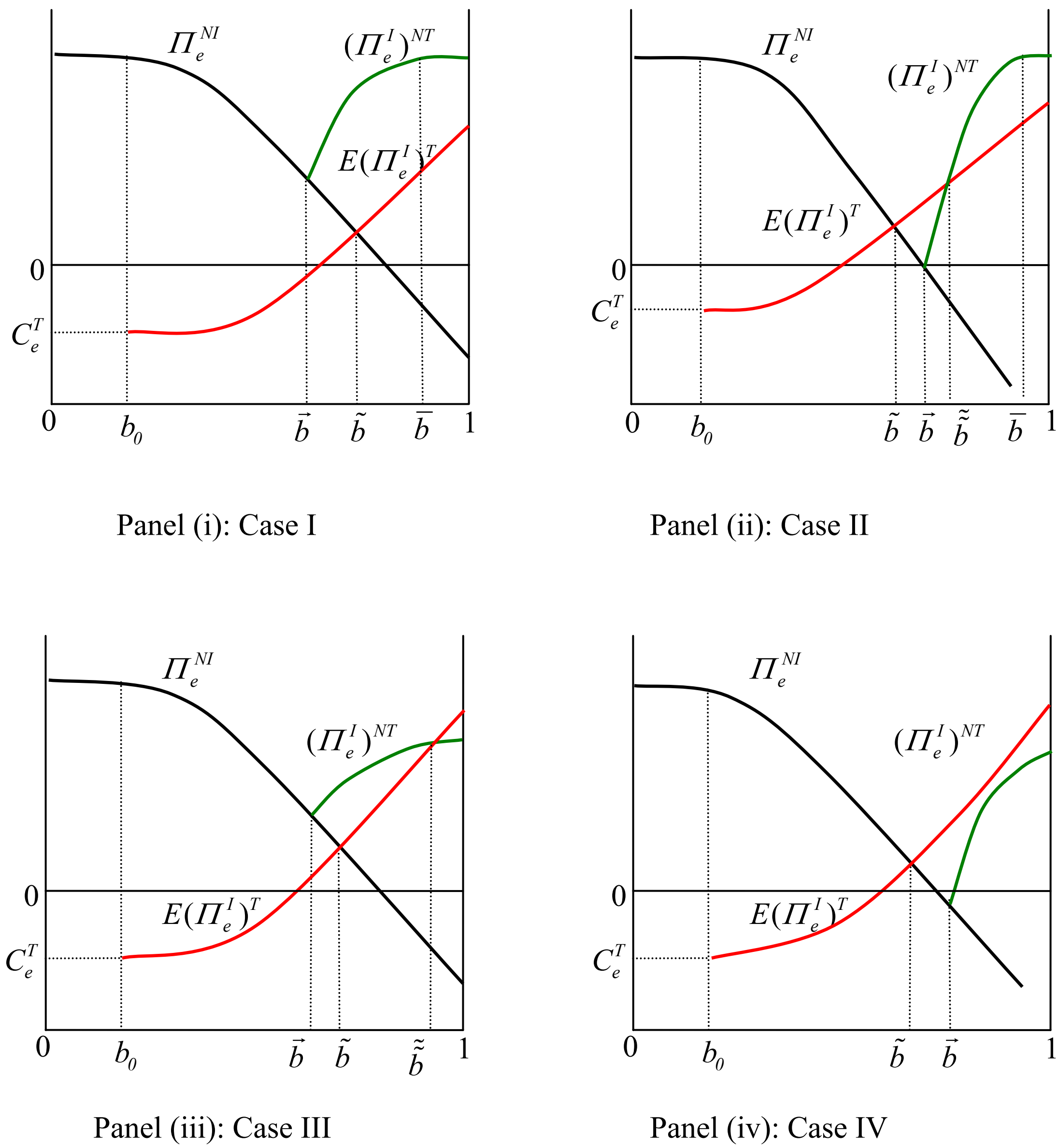

Figure 5. The Entrant's Profits under No Infringement, under Infringement and Trial and under Infringement and No Trial when Entry cannot be Deterred and $\exists\bar{b}$ – panels (a) and (b) and when $\nexists\bar{b}$ – panels (c) and (d).

3.3 Stage 2 – The Patent Breadth Decision

In stage 2 of the game, the incumbent chooses the patent breadth b that maximizes profits, given her knowledge of the entrant's behavior in the third stage of the game. Since the entrant's behavior depends on the values of Π_m, C_p^T, C_e^T, α and β, the patent breadth chosen by the incumbent also depends on these parameters. Specifically, the following situations are possible, each one corresponding to one of the scenarios and cases outlined above.

Scenario A: Choose Patent Breadth to Deter Entry

If there exists a patent breadth $\hat{b} \in (b_0, 1]$ such that $b_{NI} \leq \hat{b} \leq b_I^T$ for $b_I^T < b_I^{NT}$ or $b_{NI} \leq \hat{b} \leq b_I^{NT}$ for $b_I^T > b_I^{NT}$ and $\vec{b} \geq b_{NI}$ then the incumbent will choose this patent breadth and deter entry. By deterring entry, the incumbent earns monopoly profits, Π_m. Since these profits are higher than what can be earned under a duopoly, the incumbent always finds it optimal to deter entry.

Scenario B: Entry Cannot be Deterred

When entry cannot be deterred the incumbent will never find it optimal to choose patent breadth values such that $b \in (0, b_0]$ and $b \in [\bar{b}, 1]$ as for these patent breadth values the patenting strategy is always dominated by the no patenting strategy. As illustrated in Figure 3, in this case the entrant will find it optimal to locate at his most preferred location, q_e^* (the location chosen under no patent protection) as when $b \in (0, b_0]$ patent breadth is not binding while when $b \in [\bar{b}, 1]$ the incumbent's optimal strategy when the patent is infringed is to not invoke a trial. Given the above, as long as patenting costs are positive ($z > 0$), the incumbent maximizes her profits when she does not patent the innovation. Thus, the relevant patent breadth values that can be chosen by the incumbent when entry cannot be deterred are patent breadth values such that $b \in (b_0, \bar{b})$.

- Case I: $\exists \bar{b} \wedge \tilde{b} \;\wedge\; \vec{b} \leq \tilde{b}$

Under this case, it is never optimal for the entrant to infringe the patent and induce the incumbent to invoke a trial. The incumbent has to decide whether to choose a patent breadth $b \in (b_0, \vec{b})$ that will induce the entrant not to infringe the patent or to choose a patent breadth $b \in [\vec{b}, \bar{b})$ that will induce the entrant to infringe the patent without inducing a trial. Note that the entrant is indifferent between not infringing the patent and infringing the patent without inducing a trial when $b=\vec{b}$, i.e., $\Pi_e^{NI}(\vec{b})=(\Pi_e^I)^{NT}(\vec{b})$. In both cases, the incumbent's profits are increasing in the entrant's quality choice q_e, i.e., $\Pi_p^{NI}=(\Pi_p^I)^{NT}=\pi_p^*=\frac{q_e}{9}$, thus, the incumbent maximizes her profits by forcing the entrant to locate the furthest away in the quality product space. Since under no infringement the entrant will choose $q_e^{NI}=b$ while under infringement and no trial he will choose $(q_e^I)^{NT}=\overline{q}_e$ and $\overline{q}_e=b$ for $b=\vec{b}$ while $\overline{q}_e<b\,\forall\, b\in(\vec{b},1]$ the incumbent's profit maximizing strategy under Case I is to choose the patent breadth $b=\vec{b}$.

- Case II: $\exists\,\overline{b}\wedge\tilde{b}\;\wedge\;\vec{b}>\tilde{b}$

Under this case, the incumbent has to decide whether to choose a patent breadth $b\in(b_0,\tilde{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b\in(\tilde{b},\tilde{\tilde{b}})$ and induce the entrant to infringe the patent and induce a trial or choose a patent breadth $b\in[\tilde{\tilde{b}},\overline{b})$ and induce the entrant to infringe the patent and not induce a trial. The optimal strategy for the incumbent depends on her profits under no infringement, Π_p^{NI}, infringement and trial, $(\Pi_p^I)^T$ and infringement and no trial, $(\Pi_p^I)^{NT}$. It is straightforward to show that the infringement and no trial strategy is always dominated by the non infringement strategy. The reasoning is as follows. If the incumbent were to choose to induce non infringement the optimal strategy would be to choose the patent breadth $\tilde{b}$ since this is the patent breadth that forces the entrant to locate the furthest away possible in the quality space without infringing the patent. If the incumbent were to choose to induce infringement and no trial then the optimal strategy would be to choose patent breadth $\tilde{\tilde{b}}$ since this is the patent breadth that induces the entrant to locate the furthest away possible under infringement and no trial (for any $b>\tilde{\tilde{b}}$ the entrant locates closer to the incumbent – note that in panel (ii) in Figure 5 the entrant's profits under infringement and no trial are increasing in patent breadth). Moreover, as it can be seen in panel (ii) in Figure 5, the entrant's profits at $\tilde{\tilde{b}}$ are greater than his profits at $\tilde{b}$ which implies that the quality chosen by the entrant when $\tilde{b}$ is chosen by the incumbent, $q_e = \tilde{b}$ is greater than the quality chosen by the entrant when $\tilde{\tilde{b}}$ is chosen by the incumbent, $\overline{q}_e = 9(\Pi_m - \frac{C_p^T}{1-\alpha\tilde{\tilde{b}}})$. Since the incumbent's profits under non infringement and under infringement and no trial are both increasing in the quality chosen by the entrant, q_e, the incumbent is better off choosing $\tilde{b}$ rather than $\tilde{\tilde{b}}$, i.e., $\Pi_p^{NI}(\tilde{b}) > (\Pi_p^I)^{NT}(\tilde{\tilde{b}})$. Given the above, in this case the incumbent's choice is between inducing non infringement by claiming $\tilde{b}$ and inducing infringement and trial by claiming either $\tilde{b}+e$ or $\tilde{\tilde{b}}-e$ where $e \to 0$.

The choice that maximizes the incumbent's profits depends in a complex way on the relative values of the parameters, Π_m, C_p^T, C_e^T, α and β. In general, the greater are the incumbent's monopoly profits, the greater is the incumbent's incentive to induce infringement since the only opportunity the incumbent has to realize monopoly profits (when entry cannot be deterred) is when her patent is infringed and its validity is upheld during the infringement trial. The larger are the incumbent's monopoly profits and the validity parameter and the smaller are the entrant's R&D costs the more likely it is that the incumbent will find it optimal to induce non infringement. It is important to note that, in this case, the incumbent never finds it optimal to claim the maximum breadth of patent protection or choose a patent breadth that allows the entrant to infringe the patent without facing an infringement trial.

- Case III: $\exists \tilde{b}$, $\nexists \overline{b} \wedge \vec{b} \leq \tilde{b}$

In this case, the incumbent has to decide whether to choose a patent breadth $b \in (b_0, \vec{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b \in (\vec{b}, \tilde{\tilde{b}}]$ and induce the entrant to infringe the patent and not induce a trial or choose a patent breadth $b \in (\tilde{\tilde{b}}, 1]$ and induce the entrant to infringe the patent and induce a trial. When deciding between inducing non infringement and infringement and no trial the incumbent's optimal strategy is to choose the patent breadth $\vec{b}$ as demonstrated in Case I. Thus, under Case III the incumbent will either choose the patent breadth $\vec{b}$ which makes her indifferent between inducing non infringement and inducing infringement and no trial or she will choose to induce infringement by choosing either $\tilde{\tilde{b}} + e$ or $b = 1$. The choice that maximizes the incumbent's profits depends in a complex way on the relative values of the parameters, Π_m, C_p^T, C_e^T, α and β; their effect on the incumbent's optimal decision is as described in Case II.

- Case IV: $\exists \tilde{b}, \nexists \bar{b} \wedge \vec{b} > \tilde{b}$

Under this case, it is never optimal for the entrant to infringe the patent without inducing a trial. The incumbent has to decide whether to choose a patent breadth $b \in (b_0, \tilde{b}]$ and induce the entrant to not infringe the patent or to choose a patent breadth $b \in (\tilde{b}, 1]$ and induce the entrant to infringe the patent and induce a trial. This case has been examined by Yiannaka and Fulton (2003) who find that the incumbent will induce non infringement by claiming $b = \tilde{b}$ or induce infringement and trial by claiming either $b = \tilde{b} + e$ where $e \to 0$ or $b = 1$. The optimal strategy for the incumbent depends in a complex way on the values of the parameters Π_m, C_p^T, C_e^T, α and β.

3.4 Stage 1 – The Patenting Decision

In stage 1 of the game the incumbent decides whether to patent her innovation or not given her knowledge of the entrant's response to her patent breadth and trial decisions. The incumbent will choose to patent her innovation when the profits earned under patenting are greater than the profits earned under no patent protection. As described in the preceding sections under no patent protection entry cannot be deterred and the entrant will enter the market choosing his most preferred quality q_e^*. Thus, the incumbent's profits under no patent protection are given by $\Pi_p^{NP} = \frac{q_e^*}{9} = \frac{4}{81\beta}$ and, as expected, are increasing in the entrant's R&D effectiveness; the lower are the entrant's R&D costs, the further away he locates from the incumbent and the less intense is price competition at the last stage of the game. The profits realized by the incumbent under patent protection depend on whether she can deter entry (Scenario A) or not (Scenario B) and when entry cannot be deterred whether the entrant will infringe the patent or not and induce her to invoke a trial or not (Cases I, II, III and IV).

Note that, as discussed in section 3.2, when $q_e^* \geq \vec{b}$, the patenting strategy is always dominated by the no patenting strategy. In this case, under patenting the entrant always chooses q_e^* and the patentee does not invoke a trial. As long as patenting costs are positive the incumbent maximizes her profits by not seeking patent protection, i.e., $\Pi_p^{NP} > \Pi_p^{P}(= \frac{q_e^*}{9} - z)\ \forall z > 0$. Thus, when under patenting the incumbent can never enforce her patent rights when the patent is infringed she always chooses to not patent. When, however, the entrant's location choice affects whether the incumbent will find it optimal to invoke a trial under infringement (i.e., when $q_e^* < \vec{b}$) the optimal strategy for the incumbent depends on the values of the parameters Π_m, C_p^T, C_e^T, α and β.

Under patent protection when entry can be deterred (Scenario A), the incumbent's profits are given by $\Pi_p^P = \Pi_m - z$. The decision to patent or not in this case depends on the monopoly profits that can be captured by the incumbent, the magnitude of the patenting costs and the entrant's R&D effectiveness. As long as patenting costs are such that $z \leq \Pi_m - \frac{4}{81\beta} : \Pi_p^P \geq \Pi_p^{NP}$, patenting is more profitable than no patenting for the incumbent. The greater are the monopoly profits and the entrant's R&D costs (the greater is β), the more likely it is that patenting will result in greater profits than no patenting for the incumbent.

When entry cannot be deterred under patent protection (Scenario B) and under Case I, the incumbent's optimal patent breadth strategy is to choose patent breadth $b = \vec{b}$. The incumbent is indifferent in this case between inducing the entrant to not infringe the patent and allowing infringement without invoking a trial as $\Pi_p^{NI}(\vec{b}) = (\Pi_p^I)^{NT}(\vec{b})$. Thus, the incumbent's profits under patenting under Case I are given by

$\Pi_p^P = \frac{q_e(=\vec{b})}{9} - z = \frac{1 + 9a\Pi_m - \sqrt{1 + 36aC_p^T - 18a\Pi_m + 81a^2\Pi_m^2}}{18a} - z$. As long as patenting costs are such that $z \leq \frac{1 + 9a\Pi_m - \sqrt{1 + 36aC_p^T - 18a\Pi_m + 81a^2\Pi_m^2}}{18a} - \frac{4}{81\beta}$ patenting will be more profitable than no patenting for the incumbent. The greater are the monopoly profits and the entrant's R&D costs and the smaller is the validity parameter and the incumbent's trial costs the more likely it is that patenting will be more profitable than no patenting for the incumbent under Case I. Thus, under certain parameter values, the incumbent may find it optimal to patent her innovation even when she does not find it optimal to defend her patent under patent infringement.

When entry cannot be deterred under patent protection and under Cases II and IV it is never optimal for the incumbent to allow infringement without invoking a trial. Under these cases the incumbent will either induce the entrant to not infringe the patent by claiming patent breadth $b=\tilde{b}$ or to infringe the patent by claiming $b=\tilde{b}+e$ or $b=\tilde{\tilde{b}}-e$ (Case II), $b=1$ (Case IV). When the incumbent's optimal patent breadth strategy is to choose $b=\tilde{b}$ and induce non infringement her profits under patent protection are given by

$\Pi_p^P=\frac{q_e(=\tilde{b})}{9}-z=\frac{9(4\beta+\sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2+8\beta+81C_e^T\beta^2})}{16\alpha^2+81\beta^2}-z$. In this case, as long as patenting costs are such that $z\le\frac{729\beta(4\beta+\sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2+8\beta+81C_e^T\beta^2})-(64a^2+324\beta^2)}{1296\alpha^2\beta+6561\beta^3}$ patenting will be more profitable than no patenting for the incumbent. Under this case, the smaller is the validity parameter, α, and the entrant's R&D costs, β, and the greater are the entrant's trial costs the more likely it is that patenting will be more profitable than no patenting.

When the incumbent's optimal strategy under patenting is to induce infringement and trial by claiming $b=\tilde{b}+e$ then her profits under patent protection are given by

$\Pi_p^P=E(\Pi_p^I)^T-z=(1-\alpha(\tilde{b}+e))\Pi_m+\frac{4\alpha^2(\tilde{b}+e)^2}{81\beta}-C_p^T-z$. As long as patenting costs are such that

$$z\le\frac{4\alpha^2(4\sqrt{\beta}+\sqrt{32\alpha^2C_e^T+2\beta(8+81C_e^T\beta)})^2}{(16\alpha^2+81\beta^2)^2}+\Pi_m-\frac{9\alpha(4\beta+\sqrt{2}\sqrt{\beta}+\sqrt{16\alpha^2C_e^T+\beta(8+81C_e^T\beta)})}{16\alpha^2+81\beta^2}\Pi_m-\frac{4}{81\beta}-C_p^T$$

patenting will be more profitable than no patenting for the incumbent. When the optimal strategy for the incumbent is to induce infringement and trial by claiming $b=1$ then her profits under patent protection are given by $\Pi_p^P=E(\Pi_p^I)^T-z=(1-\alpha)\Pi_m+\frac{4\alpha^2}{81\beta}-C_p^T-z$ and as long as patenting costs are such that $z\le(1-\alpha)\Pi_m+\frac{4(\alpha^2-1)}{81\beta}-C_p^T$ patenting will be more profitable than no patenting for the incumbent. Under this case, the greater are the monopoly profits and the smaller are the incumbent's trial costs the more likely it is that patenting will be more profitable than no patenting for the incumbent.

Finally, when entry cannot be deterred under patent protection and under Case III the incumbent's optimal strategy is to either claim patent breadth $b = \vec{b}$ and induce non infringement or infringement and no trial or to claim $b = 1$ and induce infringement and trial. The patenting costs that would make patenting more profitable than no patenting for the incumbent under these cases have been examined in the Cases I, II and IV.

4. Concluding Remarks

A simple game theoretic model was developed to examine how an innovator's decision to seek patent protection and her optimal patent breadth decision affect and are affected by her ability to enforce her patent rights. The innovator in our model seeks patent protection for a product innovation under potential entry by a firm producing a better quality product. The innovator must decide whether to patent her innovation or not and under patenting how broad should be the patent protection claimed. The entrant observes the patent breadth granted to the innovator's product under patenting and decides whether to enter and if entry occurs where to locate in the quality product space. Finally, the innovator observes the entrant's quality choice and in the case of infringement decides on whether to invoke a trial or not. A key feature of the model is that the entrant can, by his choice of product quality, affect the innovator's trial decision when the patent is infringed.

Analytical results show when the innovator can use patent breadth to deter entry she will find it profitable to patent her innovation when her monopoly profits (earned under no entry or when the validity of the patent is upheld during an infringement trial) are large, patenting costs are small and the entrant's trial and R&D costs are large. Entry deterrence is achieved by claiming a patent breadth that is less than the maximum breadth possible while the greater are the entrant's

R&D and trial costs, the larger are the incumbent's monopoly profits and the smaller is the validity parameter and the incumbent's trial costs, the greater is the likelihood that entry can be deterred.

When entry cannot be deterred, the incumbent may find it optimal to patent her innovation even when she chooses to not defend her patent under infringement. Under this case the incumbent is able through patent protection to force the entrant to locate further away in the quality product space than he would have located under no patent protection. This case arises when patenting costs are low, the entrant's R&D costs are relatively large, the validity parameter is small and monopoly profits are relatively large.

The optimal patent breadth when entry cannot be deterred is in most cases smaller than the maximum breadth possible. This is so because as patent breadth increases, the closer the entrant can locate to his most preferred location without inducing a trial and the smaller are the profits earned by the incumbent. In general, the entrant's ability to affect the innovator's trial decision by his choice of product quality results in a smaller patent breadth claimed by the incumbent.

The above results hold under our model assumptions of complete and perfect information, single entry, a deterministic R&D process and possible and costless reverse engineering of the innovator's product. Relaxing the above assumptions is the focus of future research.

References

Cornish, W.R. *Intellectual Property*: *Patents, Copyright, Trademarks and Allied Rights*. New York: Matthew Bender & Company, 1989.

Horstmann, I., G. M. MacDonald and A. Slivinski. "Patents as Information Transfer Mechanisms: To Patent or (Maybe) Not to Patent." Journal of Political Economy 93(5, October 1985): 837-858.

Lanjouw, J.O. and M. Schankerman. "Characteristics of Patent Litigation: A window on Competition." *RAND Journal of Economics* 32(1, Spring 2001): 129-151.

Lerner, J. "The Importance of Patent Scope: An Empirical Analysis." *RAND Journal of Economics* 25(2, Summer 1994): 319-333.

Merges, R.P. and R. R. Nelson. "On the Complex Economics of Patent Scope." Columbia Law Review 90(4, May 1990): 839-916.

Waterson, M. "The Economics of Product Patents." The American Economic Review 80(4, September 1990): 860-869.

Yiannaka, A. and M. Fulton. "Strategic Patent Breadth and Entry Deterrence with Drastic Product Innovations." Paper presented at the First Biennial Conference of the Food System Research Group, Madison-Wisconsin, June 2003.

Appendix

- **The existence of $\vec{b} \in (0,1]$**

The solution of the condition $b = \overline{q}_e$ in terms of b yields the following two roots:

$$b_1 = \frac{1 + 9\alpha\Pi_m + \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha} \text{ and } b_2 = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}.$$

The root b_1 is rejected as a possible solution since $b_1 > 1 \; \forall \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0,1)$.

The root b_2 is accepted as a possible solution as $b_2 \in (0,1)$ for

$\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0,1)$. Given the above $b_2 = \vec{b}$.

- **The conditions for $q_e^* < \vec{b}$**

Given that $q_e^* = \frac{4}{9\beta}$ (see equation 8) and $\vec{b} = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ the condition

$q_e^* < \vec{b}$ can be written as $\beta > \frac{8\alpha}{9(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0$.

- The Effect of α, C_p^T and Π_m on β_0.

$$\frac{\partial\beta_0}{\partial\alpha} = \frac{-8\alpha(9\Pi_m - \frac{162\alpha\Pi_m^2 - 18\Pi_m + 36C_p^T}{2\sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}) + 8(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})}{9(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$.

$$\frac{\partial\beta_0}{\partial C_p^T} = \frac{16\alpha^2}{(\sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})(-1 - 9\alpha\Pi_m + \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$.

$$\frac{\partial\beta_0}{\partial\Pi_m}=\frac{8\alpha\left(\frac{162\alpha^2\Pi_m-18\alpha}{2\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}}-9\alpha\right)}{9\left(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m\right)^2}\leq 0\ \forall\alpha\in(0,1),C_p^T\geq 0\ \wedge\ \Pi_m\in\left(C_p^T,\frac{1}{9}+\frac{C_p^T}{1-\alpha}\right).$$

$$\frac{\partial^2\beta_0}{\partial\Pi_m{}^2}=-\frac{16\alpha\left(\frac{162\alpha^2\Pi_m-18\alpha}{2\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}}-9\alpha\right)^2}{9\left(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m-\right)^3}+$$

$$\frac{8\alpha\left(\frac{(162\alpha^2\Pi_m-18\alpha)^2}{4(1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2)^{3/2}}+\frac{18\alpha^2}{\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}}\right)}{9\left(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m\right)^2}\geq 0$$

$$\forall\alpha\in(0,1),C_p^T\geq 0\ \wedge\ \Pi_m\in\left(C_p^T,\frac{1}{9}+\frac{C_p^T}{1-\alpha}\right).$$

Figure A.1 depicts the combinations of β and Π_m values for given C_p^T and α values for which the condition $q_e^* < \vec{b}$ is satisfied. The shaded area in Figure A.1 includes all combinations of β and Π_m values for which the $q_e^* < \vec{b}$ condition is satisfied.

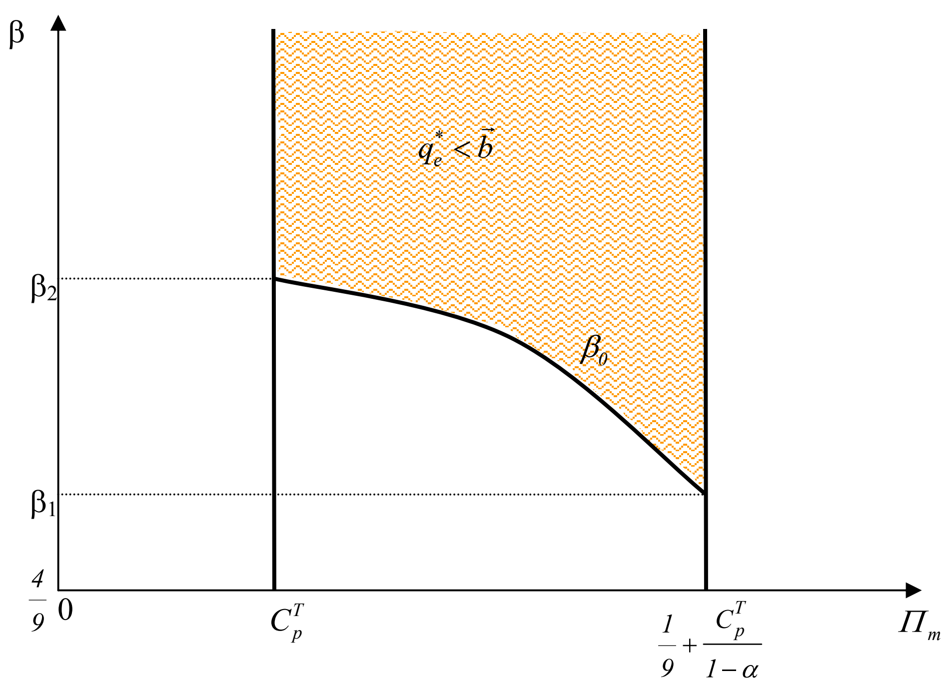

Figure A.1. Combinations of β and Π_m for which $q_e^* < \vec{b}$

- **The Entrant's Profits under Infringement and No Trial**

Since the entrant's profits are maximized at q_e^* and the analysis focuses on the case where $q_e^* < \vec{b}$ then the entrant maximizes his profits under infringement and no trial by choosing $\overline{q}_e$ (i.e., the quality that is closest to q_e^* that does not lead to an infringement trial). Since $\overline{q}_e$ is decreasing in b and at $\vec{b}$ $\overline{q}_e = \vec{b}$ as $b \in [\vec{b}, \overline{b})$ increases the entrant's profits under infringement and no trial also increase.

- **The Existence of $\tilde{b} \in (b_0, 1)$**

If a patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent, while still generating positive profits for the entrant, exists it should satisfy the conditions $\tilde{b} \in (b_0, 1]$ and $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) > 0$. The solution of $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) \Rightarrow (\frac{8\alpha^2}{81\beta} + \frac{\beta}{2})\tilde{b}^2 - \frac{4}{9}\tilde{b} - C_e = 0$ in terms of $\tilde{b}$ yields the following two roots: $\tilde{b}_{1,2} = \frac{9(4\beta \pm \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$.The root $\tilde{b}_1 = \frac{9(4\beta - \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \le 0 \;\; \forall \;\; \beta > \frac{4}{9}$, $\alpha \in (0,1) \wedge C_e^T \ge 0$ and it is thus rejected since $b_0 < \tilde{b} \le 1$. The root $\tilde{b}_1 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \ge 0 \;\; \forall \;\; \beta > \frac{4}{9}$, $\alpha \in (0,1) \wedge$ $C_e^T \ge 0$ and it is accepted as a possible solution. If $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$ exists it should also satisfy the conditions $b_0 < \tilde{b} \le 1$, $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^I)^T(\tilde{b})) > 0$. The condition $\tilde{b} > b_0$ is satisfied since $\tilde{b} - b_0 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} - \frac{4}{9\beta} > 0 \;\; \forall \;\; \beta > \frac{4}{9}$, $\alpha \in (0,1) \wedge C_e^T \ge 0$.

The condition $\tilde{b} \le 1$ is satisfied for certain combinations of β, α and C_e^T. To determine the combinations of β, α and C_e^T values which satisfy the condition $\tilde{b} \le 1$, the pairs of β, α and C_e^T values that satisfy the above constraint as an equality ($\tilde{b} = 1$) are determined first. The solution of $\tilde{b} = 1$ with respect to C_e^T yields

$C_e^T = \dfrac{16\alpha^2 - 72\beta + 81\beta^2}{162\beta}$. The area to the right of the locus $\tilde{b} = 1$ represents all combinations of β and C_e^T values, for a given α value, for which $\tilde{b} < 1$. If $\tilde{b}$ exists it must also satisfy the conditions $\Pi_e^{NI}(\tilde{b}) > 0$ and $E(\Pi_e^1)^T(\tilde{b})) > 0$. Thus, $\tilde{b}$ must violate the entry deterrence condition – $\tilde{b}$ must take values in the interval $b_I^T < \tilde{b} < b_{NI}$ when $\beta \geq \frac{8}{9}$ (i.e., when b_{NI} exists) or in the interval $b_I^T < \tilde{b}$ when $\frac{4}{9} < \beta < \frac{8}{9}$ (i.e., when b_{NI} does not exist). To determine the combination of β, α and C_e^T values for which $b_I^T < \tilde{b} < b_{NI}$ the locus $b_I^T = b_{NI}$ must first be determined. The locus $b_I^T = b_{NI}$ refers to the pairs of β, α and C_e values for which $\frac{8}{9\beta} = \sqrt{\frac{81 C_e^T \beta}{8\alpha^2}}$ holds true. Solution of the above condition with respect to C_e^T yields: $C_e^T = \dfrac{512\alpha^2}{6561\beta^3}$. All combinations of β and C_e^T values, for a given α value, below the locus $b_I^T = b_{NI}$ and to the right of locus $\beta = \frac{8}{9}$ are such that $b_I^T < \tilde{b} < b_{NI}$ while all combinations of β and C_e^T values, for a given α value, below the locus $\tilde{b} = 1$ and to the left of locus $\beta = \frac{8}{9}$ are such that $b_I^T < \tilde{b}$.