Managing Multiple Renewable Resources: Two Aquifers and a "Backstop"

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Abstract

Optimal sequencing of resource extraction is typically studied for non-renewable resources. We provide conditions for optimal use of multiple sources of a renewable resource and characterize the resulting extraction sequence, resource scarcity values, and (single) efficiency price path for two groundwater aquifers and an abundant alternative resource. Even with one demand, the optimal sequence depends on the differential opportunity costs of the two renewables. A numerical simulation for the South Oʻahu aquifer system, which also allows for different distribution costs, illustrates the case of using the "leakier" aquifer first and then switching to simultaneous use of both resources. The welfare gains relative to independent management are $4.7 billion.

Keywords: Renewable resources, dynamic optimization, multiple groundwater sources

JEL codes: Q25, Q28, C61

Introduction

Models of optimal renewable resource management typically involve an identifiable demand for a specific fishery, forest, or other confined resource. Efficient groundwater management, for example, typically considers a single aquifer serving a well-defined group of consumers. In many cases, however, the water manager must decide how to manage multiple aquifers simultaneously. In Florida, for example, five water management districts overlie a large system of interconnected aquifers (Florida DEP 2009), while in California, over 500 distinct groundwater systems underlie approximately 40% of the state's surface area (California DWR 2003). The O‘ahu case is similar, albeit with not nearly as many aquifer systems to potentially draw from. Previous studies of the Southern O‘ahu aquifer system have estimated the optimal intertemporal allocation of groundwater from the Pearl Harbor aquifer (Krulce, Roumasset, and Wilson 1997) and the optimal spatial and intertemporal allocation of groundwater from the Honolulu aquifer system (Pitafi and Roumasset 2009), under the assumption that each source serves separate water districts. In reality however, groundwater pumped from the Pearl Harbor aquifer is currently being transferred[1] to the Honolulu consumption region, suggesting that joint management is more appropriate.

The problem can be set up as one of a single demand supplied by multiple sources of a renewable resource, each with their own growth and extraction/harvest cost functions and can be extended to a set of linked demands. As such, the theory has potential

[1] Primary wells in both Honolulu and Pearl Harbor are connected directly to a common pipeline. Pressure within the pipe is kept high enough that water flows at no additional pumping cost between regions toward wherever groundwater is drawn out of the system (Glenn Oyama, personal communication via Geri Mason, June 2006).

applications beyond groundwater, e.g. to multiple fisheries or sources of energy. In the theory of nonrenewable resources, Herfindahl's rule states that when facing a single demand, a resource manager should extract deposits of a resource in the order of unit extraction costs (Herfindahl 1967). When multiple demands exist, the least-unit-extraction-cost-first rule is replaced by a least-price-first rule (Chakravorty and Krulce 1994; Chakravorty, et al. 2005), according to which the optimal shadow price is given by the sum of extraction cost, conversion cost, and the endogenous marginal user cost.[2] If there is only one demand, the extraction profile over multiple non-renewable resources is again determined exogenously, according to the sum of extraction and conversion costs for each resource (Chakravorty, Krulce, and Roumasset 2005; Im, Chakravorty, and Roumasset 2006).

In what follows, we show that Herfindahl's least-cost-first principle, valid in the case of a single demand, does not extend to renewables. A few recent studies consider similar problems involving multiple renewable resources. Zeitouni and Dinar (1997) construct a model with two adjacent sources of groundwater, but recharge is not allowed to vary with the quantity of groundwater in stock, and they do not derive a generalized rule to determine the optimal order of extraction from alternative sources. Horan and Shortle (1999) develop a theoretical framework for the optimal management of multiple Mink-Whale stocks but do not solve for the transitional dynamics.

[2] Gaudet, Moreaux, and Salant (2001) generalize the result to spatially differentiated resource sites and users. They find that in the presence of setup costs, however, the least-price rule need not hold. See also Chakravorty, Moreaux and Tidball (2008) for a generalization to include differential pollution across energy sources.

Costello and Polasky (2008) construct a more general multiple fisheries model, which incorporates stochasticity in addition to space and time. They find that harvest closure is optimal whenever the stock of a particular spatial patch falls below the patch-specific escapement target for breeding. When that occurs, the expected biological returns from escapement exceed the returns from current harvest. However, their results are based on the assumption of state independent control, i.e. they assume that marginal returns to harvest are independent of the amount harvested (constant price). Although this may be appropriate for many fisheries, downward sloping demand curves and stock-dependent extraction costs may be more common in other natural resource contexts. In the current study, an analytical optimization model is designed to jointly manage two coastal aquifers, where the natural net recharge of each aquifer varies with its respective head level. Whereas most of the above-cited studies are analytical, we also provide a numerical illustration of how to apply the model.[3]

The rest of the article is organized as follows. The next section extends the conventional single-aquifer economic-hydrologic optimization model (e.g. Brown and Deacon 1972; Gisser and Sanchez 1980; Feinerman and Knapp 1983) to allow for multiple groundwater resources. We derive a least-price rule for renewable resources analogous to Chakravorty and Krulce's (1994) rule for non-renewables. We then contrast it with Herfindahl's least-cost rule for the optimal extraction of non-renewable resources. The subsequent section utilizes data from the Honolulu and Pearl Harbor aquifers on the

[3] Zeitouni and Dinar (1997) numerically estimate a simplified version of their two-aquifer model. However, they do not provide a description of the algorithm used for the estimation, and the aquifer net recharge functions are state-independent.

island of O‘ahu (Hawai‘i) to provide a numerical illustration of a two-aquifer problem. The final section summarizes major analytical and empirical results and discusses general conclusions and potential research extensions.

The Model

Coastal groundwater aquifers are often characterized by a "Ghyben-Herzberg" lens (Mink 1980) of freshwater sitting on an underlying layer of seawater. The upper surface of the freshwater lens is buoyed up above sea level due to the difference in density between the freshwater and the displaced saltwater (figure 1). The volume of freshwater stored in an aquifer is dependent on the aquifer boundaries, porosity, and the head level (h), or the distance between the top of the lens and mean sea level. As the stock declines, i.e. the lens contracts, the head level falls, and water extraction becomes more costly for several reasons. Freshwater must be pumped longer distances, and that requires more energy. In addition, when the lens contracts to the point where the lower surface reaches the bottom of the deepest well, the remaining wells must take on larger shares of the pumping until physical limitations on the rate of pumping or capacity restrictions necessitate the construction of costly new wells. Thus, the unit extraction cost is a non-negative, decreasing, convex function of head: $c(h) \geq 0$, $c'(h) < 0$, and $c''(h) \geq 0$.

Leakage from a coastal aquifer is also a function of the head level. Low permeability caprock bounds the freshwater lens along the coast,[4] but pressure from the

[4] Caprock are coastal plain deposits (e.g. marine and terrestrial sediments, limestone, and reef deposits) that impede discharge of groundwater to the sea. Although caprock borders the coastline of Southern O‘ahu and is relevant to this study, it is not a general characteristic of coastal aquifers.

lens causes some freshwater to leak or discharge into the ocean as springflow and diffuse seepage through the caprock. As the head level declines, leakage decreases both because of the smaller surface area along the ocean boundary and because of the decrease in pressure due to the shrinking of the lens. Thus leakage is a positive, increasing, convex function of head: $l(h)\geq 0$, $l'(h)>0$, and $l''(h)\geq 0$. Assuming that natural inflow from precipitation and adjacent water bodies is fixed at some rate R, net recharge is defined as $f(h)\equiv R-l(h)$. Since coastal groundwater is a renewable resource whose pre-extraction growth rate depends on the stock of the resource, the following model may prove useful for other renewables.

The cost of distributing water from the primary wells to consumers at spatially heterogeneous locations varies, and the unit cost of transporting water to users in category j is denoted c_d^j. Distribution costs are higher for users at higher elevations because energy is required to boost the pumped water uphill. Consumption is also differentiated across spatial categories, so q_t^{ij} is the quantity extracted from aquifer i for consumption in category j at time t, and $\sum_j q_t^{ij}$ is the total quantity extracted for category j in period t. Although a model with a single aggregate rate of groundwater extraction does not fully characterize localized restrictions related to pump engineering, pipe or well pressure and flow, and three-dimensional effects such as cones of depression, such abstractions do not hinder the primary objective of this analysis, which is to characterize the long run drawdown and also the sequencing of extraction in the case of multiple aquifers. For a groundwater economics framework that specifically incorporates spatial dynamic groundwater flow equations corresponding to a multi-cell aquifer model, see Brozovic, Sunding, and Zilberman (2010).

Because coastal aquifers are located near the ocean, access to seawater is essentially unlimited. Desalination technology, although currently expensive, can produce freshwater as an alternative to groundwater extraction. The unit cost of this backstop resource (c_b) is constant, and the quantity of desalinated water produced for consumption in category *j* at time *t* is b_t^j.

The resource manager faces a non-autonomous optimal control problem with bounded controls and state-space constraints. The problem is non-autonomous because demand is growing over time. The control variables are restricted to be non-negative, and the state-space is constrained by minimum allowable head levels for each aquifer $h_{\min}^i$. Flow at the interface between the freshwater lens and underlying saltwater creates a thick transition zone comprised of brackish water that varies in salinity. As the aquifer is depleted, the transition zone eventually rises to the point where deeper wells begin to face saltwater intrusion. The US Environmental Protection Agency's standard for potable water is 2% of seawater salinity, so the model ensures maintenance of acceptable water quality through a constraint on the head level. The challenge of preventing saltwater intrusion is analogous to other renewable resource problems where a stock threshold exists, below which negative and potentially irreversible outcomes can occur. In some fisheries, for example, the biological growth curve exhibits critical depensation. Whenever the population falls below a critical level, an irreversible extinction process begins (Clark 2005).

Given a discount rate, $r>0$, the planner chooses the rates of extraction and desalination over time to maximize the present value of net social benefits:

(1) $$\underset{q_t^{ij},b_t^j}{Max}\sum_{t=0}^{\infty}\rho^t\left\{\sum_j\left(\int_0^{\sum_i q_t^{ij}+b_t^j} D_j^{-1}(x,t)dx-\sum_i\left(q_t^{ij}[c_i(h_t^i)+c_d^j]\right)-b_t^j[c_b+c_d^j]\right)\right\}$$

$$\text{subject to} \qquad \gamma_i[h_{t+1}^i-h_t^i]=f_i(h_t^i)-\sum_j q_t^{ij}$$

$$q_t^{ij}\geq 0, b_t^j\geq 0, h_t^i\geq h_{\min}^i \;\forall i,j$$

where $\rho \equiv (1+r)^{-1}$ is the discount factor, $D_j^{-1}(x,t)$ is the inverse demand function for consumption category *j*, and γ_i is a head-to-volume conversion factor for aquifer *i*. Pitafi and Roumasset (2009) use a similar setup to solve a single aquifer optimization problem with spatially differentiated consumers.

Optimality requires that the marginal benefit and marginal opportunity cost (MOC)[5] of groundwater are equated in every period. We define the efficiency price for consumption category *j* as the marginal benefit of groundwater extraction along the optimal trajectory, i.e. $p_t^j \equiv D_j^{-1}(\sum_i q_t^{ij}+b_t^j,t)$, and the MOC of a unit of water extracted from aquifer *i* is $\pi_t^{ij} \equiv c_i(h_t^i)+c_d^j+\rho\gamma_i^{-1}\lambda_{t+1}^i$. The following can be derived from the necessary conditions for the maximization problem (1):[6]

(2) $$\begin{aligned} &p_t^j \leq \pi_t^{ij}, && \text{if} < \text{then } q_t^{ij}=0 \\ &p_t^j \leq c_b+c_d^j, && \text{if} < \text{then } b_t^j=0 \end{aligned}$$

In other words, if the efficiency price for category *j* is less than the MOC of a unit of groundwater extracted from aquifer *i* for consumption in *j*, then no water is extracted for that purpose. Groundwater is only extracted from source *i* if its marginal benefit is at least

[5] We use this terminology in honor of David Pearce (see e.g. Pearce, et al. 1989).

[6] The discrete-time Hamiltonian and its necessary conditions are in appendix A.

equal to its MOC. Since it is never the case that zero water is consumed, it must be that

(3) $$p_t^j = \min\left(\pi_t^{1j}, \pi_t^{2j}, \ldots, \pi_t^{Ij}, c_b + c_d^j\right).$$

That the efficiency price is determined by the minimum MOC can be verified by a straightforward proof by contradiction. Without loss of generality, suppose that $\pi_t^{mj} < \pi_t^{nj}$ for $m < n$. Then let the efficiency price be determined by π_t^{kj}, where $k > 1$, i.e. $p_t^j = \pi_t^{kj} > \pi_t^{ij}$ for $i = 1, \ldots, k-1$, but that violates one of the necessary conditions, which states that $p_t^j \le \pi_t^{ij}$ for all *i*. □

The least-MOC-first rule (3) implies that $p_t^j = \pi_t^{mj}$ when aquifer *m* is being used $(q_t^{mj} > 0)$ and $p_t^j = \pi_t^{mj} = \pi^{nj}$, $\Delta p_t^j = \Delta\pi_t^{mj} = \Delta\pi_t^{nj}$ $(q_t^{mj} > 0, q_t^{nj} > 0)$ when aquifers *m* and *n* are being used simultaneously. This pricing and extraction rule accords with the least-price rule for nonrenewables. Herfindahl's least-cost rule for nonrenewables does not extend to renewables, however, even in the case of a single demand. Moreover, the result is robust to the case of constant extraction costs because the natural growth functions of the aquifers ensure that marginal user cost is still endogenous, and therefore ordering the resources by marginal opportunity cost is not necessarily equivalent to ordering by extraction cost.

Although the optimal order of resource extraction appears to be governed by a simple rule (3), solving for the MOC of each resource is not trivial. In the case of nonrenewable resources with constant extraction costs, the entire path of a given resource's shadow price is determined once the initial value is specified. When multiple renewable resources with stock-dependent extraction costs and growth are considered, however, each feasible MOC path must be solved for in conjunction with the associated feasible path of the aquifer head level. Forward shooting algorithms encounter difficulties of iterating on multiple starting values of MOC. Backward shooting algorithms encounter the same problem, and in addition, do not provide sufficient guidance in verifying candidate solution paths when switching is possible. Atoli and Buffie (2009) provide an automated search procedure for multi-state shooting problems in continuous time when no switching occurs. In our problem, however, the endogenous switch points create difficulties in solving the system of differential equations directly. In the presence of multiple non-linearities, gradient ascent algorithms take many iterations to converge or may not converge to the global maximum. Genetic or neural algorithms may do better than gradient ascent algorithms but implementation often proves challenging. Because of these difficulties, it may be useful to condition the search algorithm on different possible orders of extraction.

The aquifer head levels change over time due to extraction and/or natural recharge. Accordingly, the MOCs change inasmuch as the extraction costs and shadow prices are dependent on the head levels. Thus it may be optimal to extract exclusively from a single aquifer for a finite period and then switch to pumping from more than one source in the periods that follows. A resource manager with many aquifers at his disposal will face multiple endogenously determined switching points as described by the optimal pricing condition (2). In what follows, we explore the two-aquifer scenario, which is also the subject of our subsequent numerical illustration. In the case of even two aquifers, many scenarios are conceivable, but we focus on three of interest. Without loss of generality, we assume it is optimal to extract from aquifer A first in each scenario. We later establish a sufficiency condition that determines which aquifer is optimally used first. Our application illustrates the case wherein the condition is satisfied and indicates the larger aquifer for early withdrawal.

In the discussion that follows, a single "wholesale efficiency price" (*p*) is referred to in describing the optimal drawdown of the resources, rather than an efficiency price for each consumption category *j*. As long as consumption of groundwater is positive in every elevation category, the demands can be shifted down by their respective distribution costs and aggregated horizontally to determine the wholesale price. Adding the distribution cost back to the wholesale price at the original demand curves then determines the optimal quantity demanded for each elevation category.

In the initial stage of scenario 1, aquifer A is used exclusively, while aquifer B is built up.[7] As the head level of aquifer B rises, the marginal extraction cost declines but the shadow price rises, so the MOC may increase or decrease. Recall that $\pi_t^{ij} = c_i(h_t^i) + \rho\lambda_{t+1}^i$. Since $c_i'(h_t^i) < 0$ by assumption, the first term on the right hand side is decreasing as the head is building. Condition (A.4) in appendix A reduces to $\rho\lambda_{t+1}^i - \lambda_t^i = -\rho\gamma_i^{-1}\lambda_{t+1}^i f_i'(h_t^i)$ when extraction is zero. Rearranging yields $[\lambda_{t+1}^i - \lambda_t^i]/\lambda_t^i = r - \gamma_i^{-1}\lambda_{t+1}^i f_i'(h_t^i)/\lambda_t^i > 0$, i.e. the marginal user cost of aquifer *i* is always increasing when extraction from aquifer *i* is zero. Hence, MOC may be increasing or decreasing. At the same time, the MOC of aquifer A increases until the first switch-point occurs, whereupon aquifer A reaches its minimum allowable head constraint.

In stage 2, extraction from aquifer A is limited to net natural recharge, and the remaining optimal quantity demanded is supplied by aquifer B. The efficiency price, now

[7] Only by sheer coincidence would the MOCs of both aquifers be equal from the outset.

determined by the MOC of aquifer B, continues to increase, and the shadow price of groundwater for aquifer A rises apace. Although the extraction cost for aquifer A remains constant once MSY extraction is imposed, the MUC still rises as the multiplier on the head constraint becomes positive and rises (Eq. A.4).

The final stage of extraction is characterized by MSY extraction from both aquifers. A steady state is maintained, in which extraction from each aquifer is limited to net recharge, and the price remains at the backstop cost. The stages of extraction for scenario 1 are depicted graphically in figure 2. MOC_B "kinks" upward at T_1 because extraction switches from zero to positive for aquifer B. MOC_A also kinks at T_1 because in the stage of simultaneous extraction that follows, the optimal quantity demanded is split between the two aquifers (see appendix B for a proof of these results). In the usual case of a single demand and a single resource, the efficiency price of the resource is characterized entirely by the Hotelling condition, which is analogous to equation (A.2). For multiple aquifers, the efficiency price determined by equation (3) does not conform to a single Hotelling condition because it "kinks." For multiple resources in general, the price function associated with the optimal extraction path need only be piecewise continuous and is likely to be characterized by multiple "kinks" at various switch-points over time.

In scenario 2, stage 1 remains the same; aquifer A is used exclusively while aquifer B builds. At the first switch-point, however, the MOCs of the two aquifers become equal. Since neither aquifer is at its head level constraint, stage 2 is characterized by simultaneous (non-MSY) extraction of both aquifers, the rates of which are optimally chosen to keep the MOCs equal. Eventually, aquifer A reaches its head constraint, and extraction is limited to natural recharge. In the final stage, aquifer B is drawn down to its minimum allowable head level, and the system reaches a steady state, in which both aquifers are pumped at MSY. Scenario 3 is identical to scenario 2 except that aquifer B reaches its head constraint before aquifer A does.

The model can also be applied to the case where both aquifer head levels lie initially below their respective first best levels. In that scenario, optimal management entails using the "backstop" resource exclusively at the outset, thus allowing the aquifers to build toward their optimum steady state levels. Desalinated water is used in every period, and groundwater is only extracted in the steady state.

Generally, for autonomous, single renewable resource problems, optimality calls for monotonic state paths (Kamien and Schwartz 1991). If the stock starts below its steady state level, then it optimally builds monotonically, whereas if the stock starts above its steady state level, then it is optimally drawn down monotonically. With multiple resources, whether the problem is autonomous or not, optimality is not always characterized by monotonic state paths. The result can be attributed to the resource-specific extraction cost and growth functions, i.e. inter-resource comparisons must be made.

It is not necessarily the case that the exclusively used aquifer is drawn all the way down to its MSY level at the end of the first stage. We can verify that such an extraction path is optimal, however, by checking that the following sufficient condition holds (see appendix C):

$$(4)\qquad \begin{aligned} &c_1(h^1_{\min}) + \frac{[p_t - c_1(h^1_{\min})]f_1'(h^1_{\min})\gamma_1^{-1}}{r} - \frac{\gamma_1^{-1}c_1'(h^1_{\min})f_1(h^1_{\min})}{r} < \\ &c_2(h^2_{\max}) + \frac{[p_t - c_2(h^2_{\max})]f_2'(h^2_{\max})\gamma_2^{-1}}{r} - \frac{\gamma_2^{-1}c_2'(h^2_{\max})f_2(h^2_{\max})}{r} \end{aligned}$$

for all $p_t \in [\min\{c_1(h^1_{\min}), c_2(h^2_{\max})\}, c_b]$. In other words, if for all feasible head combinations and prices, the marginal opportunity cost of aquifer 1 is less than that of aquifer 2, then it is always optimal to draw aquifer 1 down to its MSY head level in the initial stage. In the illustration that follows, the sufficiency condition is satisfied, thus ensuring that Pearl Harbor aquifer (PHA), the larger of the two, is optimally drawn down first, and no water is extracted from the Honolulu aquifer (HNA) until PHA reaches its minimum head level, where maximum sustainable yield is achieved.

Application: Honolulu and Pearl Harbor Aquifers

As previously discussed, the volume of water stored in a coastal aquifer is a function of head, but it also depends on various hydrologic parameters such as the aquifer boundaries, lens geometry, and rock porosity (Mink 1980). Although the surfaces of the lens are technically parabolic, the hydraulic gradient in Southern O‘ahu is small enough that a triangular shape is a reasonable approximation. Consequently, the relationship between groundwater storage and head is modeled as linear. Following Krulce, Roumasset, and Wilson (1997), we assume that 78.149 billion gallons of freshwater are stored per foot of head in PHA. The net recharge function for PHA is constructed using Mink’s recharge estimate of 220 million gallons per day (mgd) in combination with the leakage function econometrically estimated by Krulce, Roumasset, and Wilson (1997): $l(h_t) = 0.24972h_t^2 + 0.022023h_t$. In a similar manner, we use Pitafi and Roumasset’s (2009) volume-head conversion factor of 61 billion gallons of water per foot of head for HNA and construct a net recharge function using Liu’s (2006) 64 mgd estimate of natural inflow. Taking into account average well depth below mean sea level, upconing, and the thickness of the brackish transition zone, Liu (2006, 2007) estimates the minimum allowable head levels required to avoid seawater intrusion for Pearl Harbor and Honolulu aquifer as 15.125 feet and 21 feet respectively.

The extraction cost is specified as a convex (linear) function of lift: $c_i(h_t^i) = \xi_i(e_i - h_t^i)$, where lift is defined as the difference between the average ground surface elevation of the wells, e_i, and the head level. To simplify the discussion, we will focus primarily on functional forms in the text and refer the reader to table 2 for a complete list of aquifer-specific parameter values. The cost parameter (ξ_i) is calculated using the initial unit extraction cost $c_i(h_0^i)$, which is a volume-weighted average of unit extraction costs[8] for all primary wells in the initial period, and the initial head level, h_0^i. Distribution costs (c_d^j) are calculated for each elevation category *j* from booster station pumping data (table 3). The unit cost of desalinating water (c_b) is estimated at $7.43/tg (appendix D).

The demand for water is modeled as a constant elasticity function: $D_j(x_t, t) = \alpha_j e^{gt}(x_t^j)^{-\eta}$. The coefficient ($\alpha_j$) for each elevation category is calculated using actual pumping data and the retail price for the year 2006 (table 3). Following Pitafi and Roumasset (2009), the exogenous rate of population growth *g* is assumed to be 1% in the baseline scenario. Demand elasticities vary considerably among studies, but recent estimates for increasing block price structures (Olmstead, Hanemann, and Stavins 2007) tend to be high relative to the elasticities used in previous studies of the South O‘ahu

[8] Expenditure on extraction and boosting include energy, labor, and equipment costs. 2006 costs are extrapolated from 2002 (unpublished) data provided by Honolulu Board of Water Supply.

aquifer system. In the current study, the baseline value is taken as η=0.3. Finally, the interest rate r=3%.[9]

Computational Strategy

The problem is solved using a forward shooting algorithm (see e.g. Judd 1998) conditioned on the expected extraction structure. The initial and terminal conditions for the head level are known. Growing demand ensures the implementation of the backstop at some point since the size of the aquifer system is finite, and steady state calculations reveal that the head level constraints are binding. Thus, the terminal price is the backstop cost, and the optimal path will be determined once we know the correct initial shadow prices. Along the optimal trajectory, the price rises to the backstop cost, and the head constraint becomes binding at precisely the same time (see appendix E).

The order of extraction will depend on which of the resources is "cheaper." In our study, the aquifers satisfy the sufficiency condition such that Pearl Harbor aquifer is used first and is drawn down exclusively to its MSY head level, while Honolulu aquifer is allowed to build.[10] Once Pearl Harbor aquifer reaches its MSY head level, extraction is limited to recharge and Honolulu aquifer is drawn down to its MSY head level as the resources approach a steady state. Given the order and stages of extraction, we proceed to solve for the endogenously determined switch-points and the time at which the aquifers reach a steady state.

[9] That the discount rate is greater than the population growth rate ensures convergence of the objective functional.

[10] If the sufficiency condition is not satisfied, then the algorithm described should be adjusted, inasmuch as there will likely be a stage of simultaneous extraction before either aquifer reaches its MSY level.

Trial values are assumed for the initial shadow prices, and condition (A.5) allows one to solve for the shadow prices in the following period. Once the period 2 shadow prices are determined, the efficiency price (A.2) and therefore the rates of extraction can be ascertained for the current period. The rates of extraction reveal the head levels in the next period via the equation of motion (A.6), and the whole process can be repeated, using the period 2 head levels and shadow prices as the new starting point. Eventually, one of the terminal conditions is reached. If at least one of the other terminal conditions is inconsistent, then the initial guesses for the shadow prices are revealed as incorrect. The guesses must be adjusted and the process repeated until all of the initial and terminal conditions are satisfied for the head of each aquifer and the price, so that the PV functional is maximized.

Results

We determine the optimal paths of price (figure 3), extraction (figure 4), consumption (figure 5), and head level (figure 6) for each of the aquifers. The efficiency price for consumption category 1 starts at $2.15 and rises relatively slowly until year 40, at which point PHA reaches its head level constraint. After year 40, the price path is determined by the MOC of HNA, and the price begins to rise more rapidly as HNA is depleted. During the second stage, any quantity demanded at the optimal price in excess of maximum sustainable yield for PHA is supplied by extraction from HNA. Eventually, the price rises to the cost of the backstop as HNA approaches its minimum allowable head level. The steady state is reached after 100 years, and optimal consumption in excess of MSY for both aquifers from that point forward is met by desalination.

Since the extraction cost functions are linear and relatively flat in this application, the order of extraction is determined primarily by the difference in net recharge functions. Intuitively, PHA is drawn down first exclusively because the resulting decrease in leakage (increase in net recharge) is greater than the increase in leakage (decrease in net recharge) that occurs as HNA builds. It should be noted, however, that the PHA is building for an initial period even when it is being used exclusively (figure 6). Because the problem is non-autonomous, non-monotonicity can result from the need to build the stock in anticipation of future scarcity.

To characterize the welfare effects of optimal joint management, the independent maximization problems for the Honolulu and Pearl Harbor aquifers are numerically solved following the methodology outlined in Pitafi and Roumasset (2009). Quantity and cost data are updated to 2006 (table 2). In addition, some of the hydrologic aspects of the basic model are adjusted to account for the brackish transition zone, exactly as was done for the joint optimization problem. We assume that for the independent scenarios, demand is partitioned by currently established management districts and water is not transported between the two consumption areas. The net present value gain of switching from independent optimization to joint optimization is $4.7 billion.

Sensitivity Analysis

In this section, parameter values are varied to test the sensitivity of the model's results, as well as to consider the possible impacts of climate change and watershed degradation, caused by, for example, invasive species. The growth rate of demand, the elasticity of demand for water, and the natural rate of infiltration to the aquifer are three parameters, which are likely imperfectly measured. More accurate measurement of the first two parameters is feasible but beyond the scope of this study. The gains to such an endeavor are likely to be large, however. Increasing the growth rate of demand for water to 2% per year brings the need for desalination 42 years closer to the present. Clearly, optimizing with a wrongly assumed growth value can lead to large welfare losses; underestimating growth would result in under-pricing, and hence necessitate the implementation of the costly backstop resource even sooner.

The baseline value for demand elasticity is based on existing studies of water demand. Recent work (Olmstead, Hanemann, and Stavins 2007), however, suggests that elasticity under increasing block-pricing structures may be larger than previous estimates suggest. Increasing the demand elasticity to -0.5 delays the need for desalination by 12 years. A higher elasticity means that the price increase required to achieve a target level of demand-side conservation is lower. Small changes in elasticity have a large impact on the optimal paths.

The third parameter of interest, natural infiltration to the aquifer or natural recharge, is typically assumed constant over the entire planning horizon in groundwater economic studies. This may have been a good approximation for the past century, but with climate change looming on the horizon and threats of invasive species on the rise, historic measurements of recharge are likely no longer accurate approximations of future recharge. Global warming will certainly have an impact on recharge via changes in quantities and patterns of precipitation, evapotranspiration, and runoff, and increases in invasive species populations will affect runoff and evapotranspiration through changes in land cover. Ideally, a hydrologic-economic groundwater optimization model would incorporate all factors that influence recharge, but integrating regional climate and watershed models into a resource economics framework remains a challenge. In this section, we consider several recharge scenarios that could result from climate change or degradation of the watershed.

The numerical simulations are re-run assuming a 10%, 20%, and 30% reduction in recharge. The reduction could be due to a decline in precipitation, increased runoff due to more extreme rainfall events or damage to the watershed from invasive species, or any combination of those factors. It is intuitive that lower recharge results in the need for desalination sooner (17 years in the 10% reduction case); the growth of the aquifer is slower, so the resource is depleted more rapidly. The $432.1 million present value of lost recharge is calculated as the difference between the PV of the baseline scenario and the 90%-recharge scenario. Although in reality the recharge would be declining gradually over time, the 10% reduction is meant to be illustrative. Clearly, the PV cost of doing nothing is not trivial. The lost PV is even larger with higher reductions in recharge. Preventing climate change on a local scale is not feasible, but investment in natural capital within the watershed is one means of adaptation.

Conclusions

Although many natural resource problems involve the use of multiple sources, most renewable resource economic models focus on a single source and a single demand. When more than one source is available, optimal management involves how much to extract from each, including the possibility of extracting nothing from one or more of the sources. The optimal sequencing of resource extraction cannot be determined by extraction costs alone, as is the case when multiple non-renewable resources supply a single demand. Rather, the endogenously determined marginal user costs allow the ordering of the scarcity values (marginal opportunity costs) to deviate from the ordering of the extraction costs. Optimality requires extracting from the source with the least MOC first. Instead of a separate $p = c + MUC$ equation for both sources, however, the single efficiency price is characterized in a given period by the marginal opportunity cost of the resource(s) in use. Therefore, the efficiency price path for the multiple, renewable-resource problem can kink, even though there is only one aggregate demand for the resource.

The optimal steady state for the case of growing and unbounded demand (e.g. 1% per year), will involve both aquifers being maintained at their MSY head levels (if the extraction cost curves are sufficiently flat), both aquifers being maintained above their MSY head levels, or one aquifer being maintained at its MSY head level and the other at an internal optimum. In the case that both aquifers start below their respective optimal steady state head levels, the "backstop" resource is used initially, allowing the aquifers to build toward their optimum long-run equilibrium levels. Desalinated water is used in every period, and groundwater is only extracted in the steady state. In this scenario, desalination is *a resource for all seasons.*

Whereas Chakravorty and Krulce's (1994) least-price-first rule for non-renewables collapses to the Herfindahl least-cost-first rule for a single demand, optimal extraction from multiple renewable resources such as aquifers is governed by the more general least-price-first rule, even when there is only one demand. Also contrary to the nonrenewable case (Gaudet et al. 2001), a single demand can optimally be supplied by more than one renewable resource in any given period. In the steady state, all sources are used simultaneously, and stages of simultaneous use prior to the steady state are also feasible.

While optimal extraction accords with “least-price-first,” those prices (which refer to marginal opportunity costs along the optimal path) are endogenous. Which resource has the lower initial marginal opportunity costs is not generally determinable without actually solving the optimization problem at hand. There are two countervailing forces; extraction decreases leakage (increases stock growth) by lowering the head level, but at the same time it increases extraction costs. If unit extraction costs are approximately constant, optimality requires initially extracting exclusively from the aquifer for which the value of net gained recharge (including movement along the leakage function) exceeds the value of lost recharge from the unused aquifer. We find a sufficiency condition that not only determines the order of extraction but also ensures complete drawdown of the first resource to its MSY head level before switching to the second resource.

When the model is applied to the two aquifers supplying groundwater to South O‘ahu, we find that extraction optimally occurs in three stages. First, Pearl Harbor aquifer is used exclusively, allowing the smaller Honolulu aquifer to accumulate water. This is followed by a stage of simultaneous use, and finally a steady state wherein MSY extraction from each aquifer is supplemented by desalination. Rather than withdrawing groundwater from each resource according to extraction costs or in proportion to the size of the aquifer, the resource manager should completely specialize in extraction from Pearl Harbor aquifer for an initial period. The larger Pearl Harbor aquifer is drawn down first because its resulting decrease in leakage (increase in net recharge) is greater than the loss in net recharge from the accumulating Honolulu aquifer. Even the relatively low rate of demand growth assumed (1%) is sufficient to overcome the effect of rising efficiency prices, such that total consumption increases monotonically up to the steady state. The extraction paths described are largely different from those obtained by optimizing the aquifers independently, and the resulting welfare gains are substantial. Switching from independent to joint management increases welfare by $4.7 billion.

Incentivizing water users to consume in accordance with the optimal profile of groundwater extraction requires only that they face the efficiency price of water at the margin. That is, charging a different price for units prior to the last unit consumed does not distort incentives. An increasing block pricing structure can preserve efficient incentives without making consumers worse off as long as the effective block is set at the marginal opportunity cost. Moreover, inasmuch as water utilities are typically required to maintain a balanced budget, block pricing can be used to return to water consumers the excess revenue generated from the optimal pricing schedule (Pitafi and Roumasset 2009).

There are many possible research extensions to the current work. The one-demand two-source scenario is really a special case of a multiple-demand multiple-source framework. Thus, this analysis serves as an intermediate step in extending Chakravorty and Krulce's (1994) 2x2 least-price rule for nonrenewable resources to renewables. Second, the multiple-aquifer hydrologic-economic framework should integrate watershed and regional climate models. As scientists develop and improve such models, the integration envisioned will become a more feasible undertaking. Third, there may be cases in which multiple aquifers/resource-sites are available to serve multiple consumption districts with non-zero transportation costs between districts. In that case, independent management may be optimal until scarcity increases enough to justify costly transportation of water between districts. The endogenous boundary between locales would adjust over time along the optimal trajectory. Finally, many natural resource stocks are directly linked, and adjacent aquifers are no exception. While inter-aquifer flow is relatively small in the South O‘ahu case, the model can be generalized to include interdependent stocks in the equations of motion for each stock (e.g. for pelagic fisheries and predator-prey situations). It remains to be seen whether such extensions will permit analytical results regarding the sequencing of resource extraction.

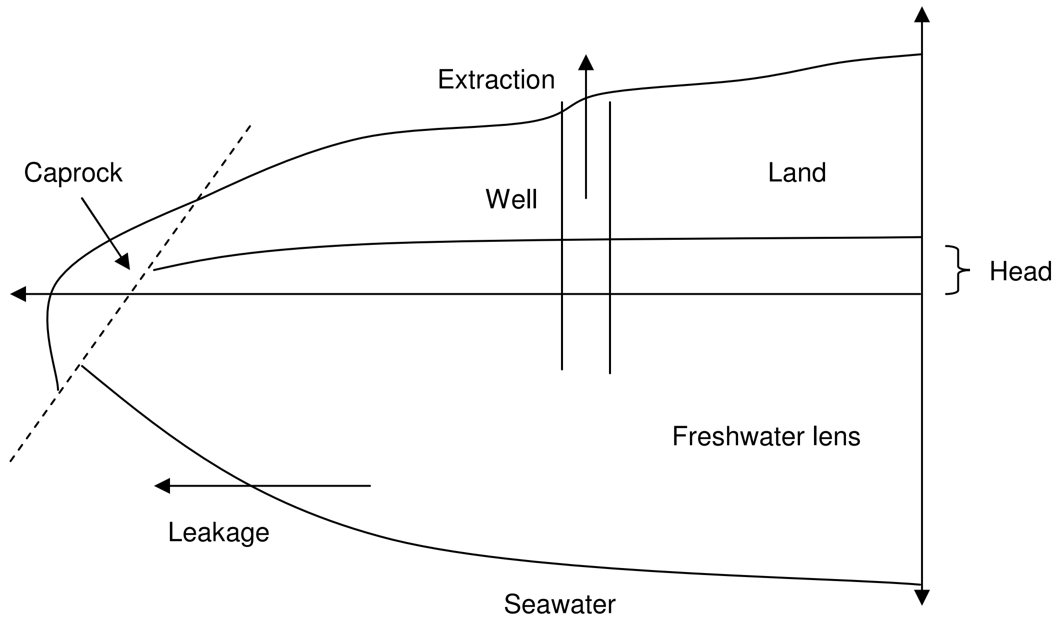

Figure 1. Coastal aquifer cross-section, adopted from Mink (1980)

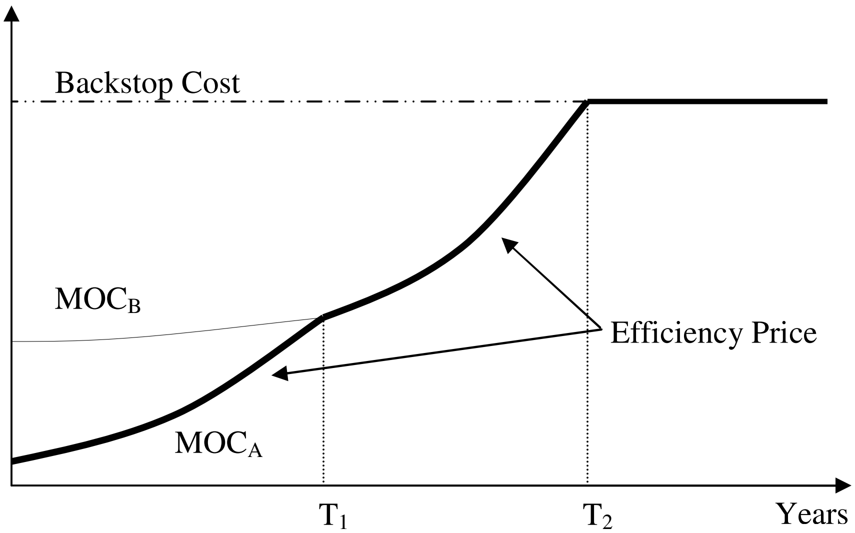

Figure 2. Hypothetical efficiency price path for scenario 1 is the lower envelope of the MOC paths

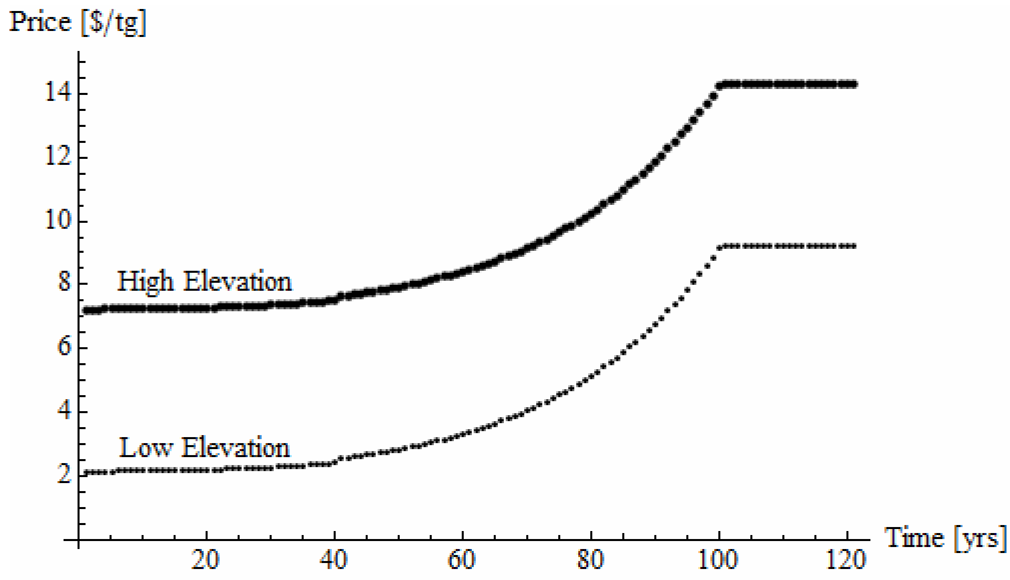

Figure 3. Efficiency price paths

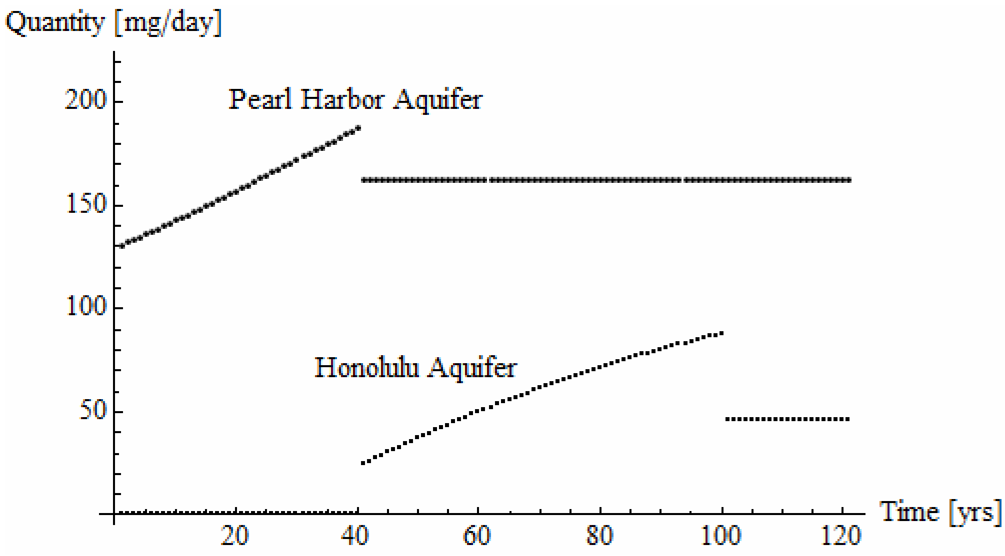

Figure 4. Extraction paths

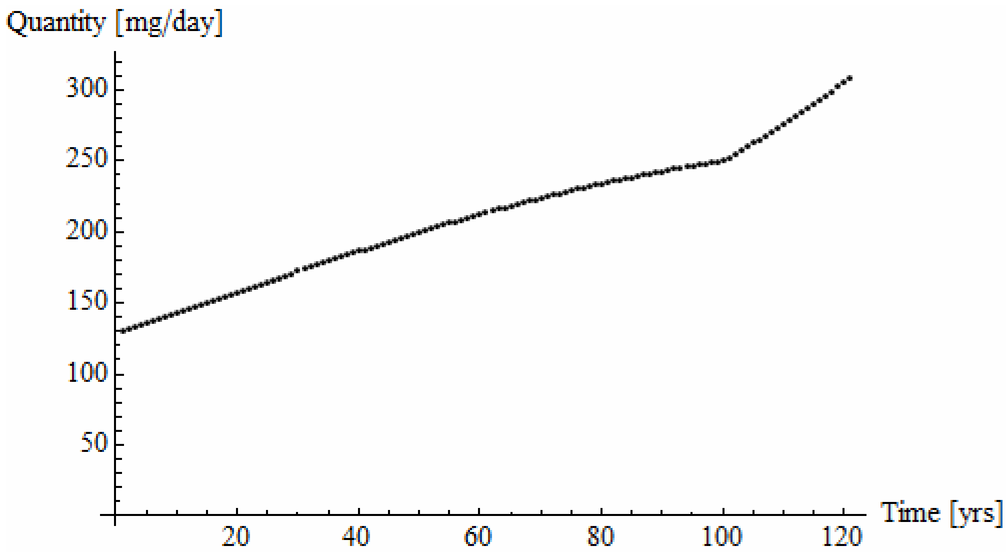

Figure 5. Aggregate consumption path

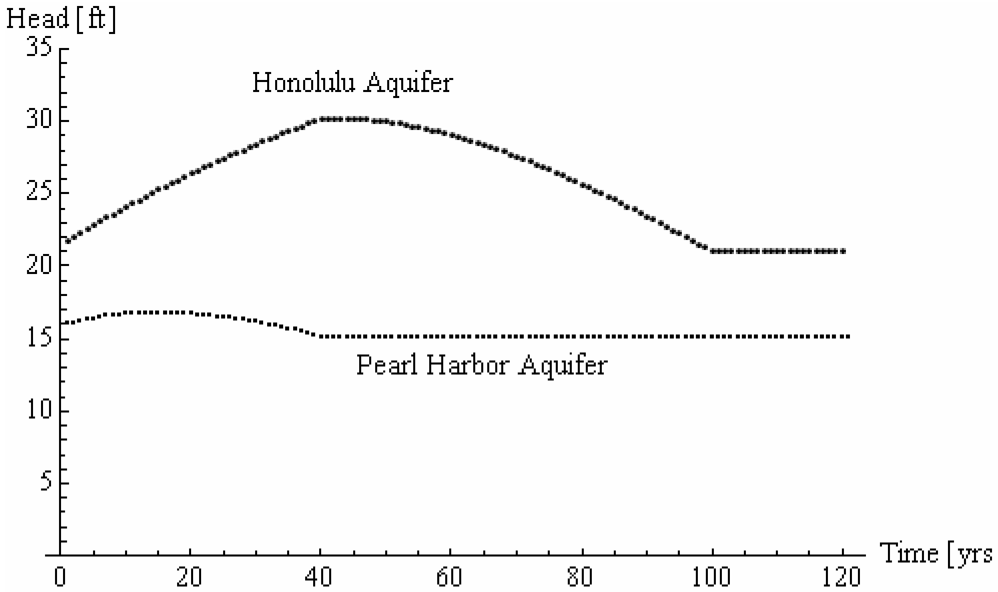

Figure 6. Head level paths for both aquifers

Table 1. Order of Extraction in the Case of Two Aquifers

	Stage 1	Stage 2	Stage 3	Stage 4
Scenario 1	Extract from A	Extract only MSY from A Extract from B	Steady State	
	$p_t = \pi_t^A < \pi_t^B$	$p_t = \pi_t^A = \pi_t^B$	$p_t = \pi_t^A = \pi_t^B = c_b$	
Scenario 2	Extract from A	Extract from A Extract from B	Extract only MSY from A Extract from B	Steady State
	$p_t = \pi_t^A < \pi_t^B$	$p_t = \pi_t^A = \pi_t^B$	$p_t = \pi_t^A = \pi_t^B$	$p_t = \pi_t^A = \pi_t^B = c_b$
Scenario 3	Extract from A	Extract from A Extract from B	Extract from A Extract only MSY from B	Steady State
	$p_t = \pi_t^A < \pi_t^B$	$p_t = \pi_t^A = \pi_t^B$	$p_t = \pi_t^A = \pi_t^B$	$p_t = \pi_t^A = \pi_t^B = c_b$

Note: We use "extract" to mean any removal of groundwater from an aquifer, even when the extraction rate does not exceed net recharge, i.e. an aquifer's head level may be rising while extraction is occurring.

Table 2. Parameter Values for the Honolulu and Pearl Harbor Aquifers

Parameter	Honolulu	Pearl Harbor
e_i [ft]	50	272
ξ_i [$/(tg*ft)]	0.00786	0.00121
$c_i(h_0^i)$ [$/tg]	0.22	0.31
h_0^i [ft]	21.5	16
g	0.01	0.01
η	0.3	0.3
r	0.03	0.03
$h_{\min}^i$ [ft]	21	15.125

Table 3. Demand Coefficients and Distribution Costs

Category	Elevation (ft)	Distribution Cost ($/tg)	Qty (mgd)	Coefficient (α_j)
1	0	$1.81	70.34	89.48
2	500	$2.35	6.21	7.9
3	789	$3.21	1.17	1.49
4	1039	$4.37	0.65	0.83
5	1086	$5.62	0.17	0.21
6	1345	$6.90	0.12	0.15
7	0	$1.86	47.43	60.34
8	552	$2.37	3.7	4.7
9	887	$2.95	1.18	1.51

Note: Categories 1-6 represent the Honolulu consumption district and 7-9 the Pearl Harbor district.

Source: Unpublished Honolulu Board of Water Supply Data (2006)

Table 4. Sensitivity Analysis

Scenario	τ_1 (yrs)	T (yrs)	PV (million $)	PV lost recharge (million $)
Baseline	40	100	$12,047.80	-
$g = 2\%$	29	58	$15,912.90	-
90% R	24	83	$11,615.70	$432.10
80% R	12	63	$11,492.70	$555.10
70% R	8	46	$10,942.70	$1,105.10
$g = 2\%$ & 90% R	20	49	$16,713.70	$686.90
$\eta = 0.5$	40	112	$10,457.80	-

Appendix A. Current Value Hamiltonian and Pontryagin Conditions

The corresponding discrete-time current value Hamiltonian (Conrad and Clark 1987) is:

$$\text{(A.1)} \quad H = \sum_j \left(\int_0^{\sum_i q_t^{ij} + b_t^j} D_j^{-1}(x,t)dx - \sum_i \left(q_t^{ij}[c_i(h_t^i) + c_d^j] \right) - b_t^j [c_b + c_d^j] \right)$$

$$+ \rho \sum_i \left(\gamma_i^{-1} \lambda_{t+1}^i [f_i(h_t^i) - \sum_j q_t^{ij}] \right) + \sum_i \left({}_t^i [h_t^i - h_{\min}^i] \right)$$

and the Maximum Principle requires that $\forall i, j$:

$$\text{(A.2)} \quad \frac{\partial H}{\partial q_t^{ij}} = D_j^{-1}(\sum_i q_t^{ij} + b_t^j, t) - c_i(h_t^i) - c_d^j - \rho \gamma_i^{-1} \lambda_{t+1}^i \leq 0, \qquad q_t^{ij} \geq 0, \; q_t^{ij} \frac{\partial H}{\partial q_t^{ij}} = 0$$

$$\text{(A.3)} \quad \frac{\partial H}{\partial b_t^j} = D_j^{-1}(\sum_i q_t^{ij} + b_t^j, t) - c_b - c_d^j \leq 0, \qquad b_t^j \geq 0, \; b_t^j \frac{\partial H}{\partial b_t^j} = 0$$

$$\text{(A.4)} \quad \rho \lambda_{t+1}^i - \lambda_t^i = -\frac{\partial H}{\partial h_t^i} = \sum_j q_t^{ij} c_i'(h_t^i) - \rho \gamma_i^{-1} \lambda_{t+1}^i f_i'(h_t^i) - {}_t^i$$

$$\text{(A.5)} \quad \frac{\partial H}{\partial \, {}_t^i} = h_t^i - h_{\min}^i \geq 0, \qquad {}_t^i \geq 0, \quad {}_t^i \frac{\partial H}{\partial \, {}_t^i} = 0$$

$$\text{(A.6)} \quad \gamma_i [h_{t+1}^i - h_t^i] = \frac{\partial H}{\partial (\rho \lambda_{t+1}^i)} = f_i(h_t^i) - \sum_j q_t^{ij}$$

Appendix B. Kinks in the Price Path

More formally, recall that the MOC is defined as $\pi_t^B \equiv c(h_t^B) + \gamma^{-1}\lambda_t^B$. That an upward "kink" exists at T_1 implies that the time derivative of π_t^B, $c'(h_t^B)\dot{h}_t^B + \gamma^{-1}\dot{\lambda}_t^B$, is greater in the limit when approached from the right than when approached from the left, i.e. $\lim_{t \to T_1^+} \dot{\pi}_t^B > \lim_{t \to T_1^-} \dot{\pi}_t^B$. The adjoint equation governing the time path of the shadow price remains unchanged whether the resource is used or not. Thus, the kink arises from the change in the first term in $\dot{\pi}_t^B$, as extraction becomes positive.

The wholesale efficiency price, which is equivalent to the MOC of aquifer A over the entire time horizon, gets flatter at T_1. The first term in $\dot{\pi}_t^A$ is positive (product of two negative numbers) before T_1, but becomes zero after T_1 because $\dot{h}_t^A = 0$ for $t > T_1$, i.e. $\lim_{t \to T_1^+} \dot{\pi}_t^A < \lim_{t \to T_1^-} \dot{\pi}_t^A$.

Appendix C. Sufficiency Condition

Proposition 1: When

$$c_1(h^1_{\min}) + \frac{[p_t - c_1(h^1_{\min})]f_1'(h^1_{\min})\gamma_1^{-1}}{r} - \frac{\gamma_1^{-1}c_1'(h^1_{\min})f_1(h^1_{\min})}{r} <$$
$$c_2(h^2_{\max}) + \frac{[p_t - c_2(h^2_{\max})]f_2'(h^2_{\max})\gamma_2^{-1}}{r} - \frac{\gamma_2^{-1}c_2'(h^2_{\max})f_2(h^2_{\max})}{r}$$

for all $p_t \in [\min\{c_1(h^1_{\min}), c_2(h^2_{\max})\}, c_b]$, it is optimal to draw aquifer 1 down to its MSY level before extracting from aquifer 2.

Proof: Since we do not know the price and head paths before solving the entire optimization problem, the proposition considers the most extreme head values, and hence is a sufficient but not a necessary condition. It is straightforward to derive the above expression for MOC using the Pontriyagin conditions for the optimal control problem in appendix A.

Since the terms on the left and right hand side of the inequality above are the marginal opportunity costs for aquifer 1 and 2 respectively, we will show first that the left hand side is the largest feasible value. Since $c_1 > 0$ and $c_1' < 0$, $h^1_{\min}$ results in the largest possible value for the first term. By definition, $f_1' < 0$ and $f_1'' < 0$, so $h^1_{\min}$ ensures that the second term is at its highest feasible value. Finally, since c_1' is constant inasmuch as $c_1'' = 0$, $h^1_{\min}$ yields the largest possible value for f_1 and hence the largest possible value for the third term. By an analogous argument, the RHS yields the lowest feasible value for the marginal opportunity cost of aquifer 2. If the smallest feasible marginal opportunity cost for aquifer 2 still exceeds the largest feasible marginal opportunity cost for aquifer 1, then by the least-price-first rule, aquifer 1 should be drawn down exclusively to its MSY level in the first stage of extraction.□

Appendix D. Desalination Cost

The desalination cost estimate is based on reverse osmosis membrane technology. The following simple amortization formula (Zhou and Tol 2005) is used:

$$C_K = \frac{P \cdot i \cdot (1+i)^t}{(1+i)^t - 1}$$

where C_K is the amortized cost, P is the original start up investment, i is the bond rate, and t is the expected plant life. Adding the amortized start up cost to the operation and maintenance costs yields the total annual cost of desalination. Dividing that cost by total output per year then gives the unit cost of desalination. Values for plant capacity (5 mgd), set up cost ($40 million) and operation and maintenance costs ($5.75 million) are obtained from a study by Oceanit (2003) and adjusted for inflation (USDL 2008) and increases in energy costs (US EIA 2008). Amortization rates and estimated average plant life varies among desalination studies, but for the current study i=7.5% and t=25 years. Finally, it is assumed that buildings and equipment have no salvage value and capital replacement costs are already incorporated in operation and maintenance costs. Thus the unit cost of desalination is $7.43/tg.

The unit cost of desalination is an important factor in the optimal allocation of groundwater. The current values of parameters that determine the desalination cost estimate may change over time or may need adjustment. Sensitivity analysis can be performed to address this issue.

Appendix E. Coincidence of Terminal Head and Price Conditions

Proposition 2: If $p_{T_1} = c_b$ and $h_{T_2} = h_{\min}$ in the optimal steady state, it must be $T_1 = T_2$.

Proof: Suppose $p_{T_1} = c_b$, $h_{T_2} = h_{\min}$, and $T_1 < T_2$. Then for $t \in (T_1, T_2)$, $p_t = c_b$, $h_t > h_{\min}$, and the head level must be drawn down further to satisfy its terminal condition. However, drawing the head down would raise the marginal opportunity cost of groundwater above c_b. If this were true, then the backstop would optimally be used to satisfy all of the quantity demanded at the optimal price over the interval. But if extraction is zero, then the head level rises and $h_T = h_{\min}$ is never achieved, which is a contradiction.

Suppose instead that $h_{T_1} = h_{\min}$, $p_{T_2} = c_b$, and $T_1 < T_2$. Then for $t \in (T_1, T_2)$, $p_t < c_b$ and $h_t = h_{\min}$. Once the head constraint is binding at T_1, extraction is limited to recharge and the price must be equal to c_b, since desalination must be used to satisfy any consumption in excess of natural recharge. But $p_{T_1} < c_b$, which is a contradiction. Thus, if $p_{T_1} = c_b$ and $h_{T_2} = h_{\min}$, it must be that $T_1 = T_2$. □

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