Grazing Fees versus Stewardship on Federal Lands

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Abstract:
Livestock grazing on public lands continues to be a source of intense conflict and debate. We analyze this problem using a dynamic game. Low grazing fees let ranchers capture more rent from grazing. This increases the incentive to comply with federally mandated regulations. Optimal grazing contracts therefore include grazing fees that are lower than competitive private rates. The optimal policy also includes random monitoring to prevent strategic learning by cheating ranchers and avoid wasteful efforts to disguise noncompliant behavior. Finally, an optimal policy includes a penalty for cheating beyond terminating the lease. This penalty must be large enough that the rancher who would profit the most from cheating experiences a negative expected net return.

Keywords: Renewable resources, public lands grazing policy, optimal contracts

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1. Introduction

Fishing without a license; poaching big game; hiking and off-road vehicle use in fragile or protected areas; taking petrified wood, fossils, or Native American artifacts from public lands; cutting and hauling firewood in unapproved areas; and overgrazing by privately owned livestock are examples of a common resource management problem. Each of these activities reduces the quality of a public resource for other users. The economic problem is that individual users face incentives that are not socially optimal. To help finance the cost of protecting, improving, and policing public lands, users are charged access fees. But higher fees produce greater incentives to engage in unlawful activities.

This paper concentrates on livestock (privately owned cattle, sheep, and horses) grazing on federal lands to analyze and better understand the nature of, and solutions to, this type of public resource use problem. Livestock grazing on federal land has been hotly contested for more than a century. Livestock currently grazes on over 260 million acres of federal land, 167 million acres administered by the Bureau of Land Management (BLM) and 95 million acres administered by the Forest Service (USFS) land (USDI, 2003; USDA, 2003) – a total land area larger than the Eastern seaboard plus Vermont, Pennsylvania and West Virginia. Nearly 28,000 livestock producers hold permits to graze their animals on federal lands, roughly 3% of all livestock producers in the United States but about 22% of the livestock producers in the eleven Western contiguous States (USDI-BLM, USDA-USFS, 1995). The forage grazed on federal land accounts for approximately 2% of all feed consumed by beef cattle in the United States (USDI 1992).
One focus of the debate over public grazing is the argument that public lands ranchers are being subsidized relative to grazing fees on private lands. Figure 1 illustrates the extent of this discrepancy in eleven western states during the period 1964-2005, in constant 2005 dollars, using the implicit price deflator for gross domestic product to adjust for inflation.

Environmentalists, and some economists, have argued that higher grazing fees are linked to the quality of the environment on public lands. This view is best articulated by President Clinton’s Council of Economic Advisors:

“The controversy over rangeland reform shows the importance of integrating pricing with regulation to use the Nation’s resources more efficiently and strike a better balance between economic and environmental objectives. A central point of contention involves the fees that the federal Government charges ranchers to graze animals on federal land. These fees should reflect both the value of the forage used by an additional animal and the external environmental costs of grazing an additional animal ... Charging ranchers the marginal value of forage ... encourages efficient use of the range. By preventing overgrazing, it protects the condition of the range for future uses. It also promotes long-run efficiency in the industry ... Promoting efficiency thus means both increasing grazing fees and ensuring that federal grazing fees change from year to year in accordance with changes in rent on private grazing land.” Economic Report of the President, 1994:182-83.

But this argument ignores several issues that apply to public lands. First, the BLM and USFS deal with a large number of grazing permits and an even larger land area. A typical BLM ranger is responsible for nearly 400,000 acres of rangeland and many are responsible for over a million acres. With limited manpower and budgets, the cost of
continuously monitoring all grazing allotments is high. In contrast, a typical private landowner tends to lease grazing privileges to a small number of tenants on a small number of parcels. Private landowners also capture all of the benefits from monitoring and enforcing their grazing leases. Employees of the BLS and USFS personally can capture little, if any, of the benefits from monitoring and enforcing public lands grazing leases.

Second, the BLM and the USFS determine the allowable number of animals (the stocking rate) on each allotment. The annual payment by a public lands rancher is the grazing fee times the allowed stocking rate. Thus, federal grazing fee payments are fixed costs.\(^1\) This implies that an increase in the grazing fee increases the cost of compliance for public lands ranchers.

Third, public grazing land is a renewable resource. As a result, public agencies and public lands ranchers play a dynamic economic game. In this game there is a conflict of interest between society at large and ranchers because ranchers can not directly capture the benefits to non-grazing users.

We develop an economic model of this game. In the first stage of this game, the

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\(^1\) Public lands ranchers can apply for non-use status on a periodic basis and gain some relief from grazing fee payments. However, non-use for more than two years can result in a permanent reduction in the allowable stocking rate. Johnson and Watts (1989) estimated the long-run elasticity of non-use due to an increase in the federal grazing fee on BLM land to be less than 0.2. A 1992 GAO review of the BLM’s monitoring practices indicated that nearly 45% of all public rangeland reclaimed by that agency was land ranchers failed to graze.
government chooses the administrative rules (the stocking rate and the dynamic path of forage extraction), grazing fees, penalties for failing to comply with grazing regulations, and a monitoring strategy. These are all announced publicly and the government commits to this policy regime for all time. In each later stage, each rancher chooses a stocking rate and the government chooses its monitoring actions. We assume that all parties are risk neutral and form rational expectations, and focus on a subgame perfect Nash equilibrium for the dynamic resource use game.

In this environment, an increase in the grazing fee does not lead to a decrease in the actual stocking rate. The economic intuition for this result is fairly clear. Because grazing fee payments are fixed costs, keeping fees low lets ranchers capture more of the rent from grazing on public lands. This increases the incentive to comply with the grazing regulations. Ranchers are less likely to be out of compliance with more valuable grazing permits. Thus, we find that optimal public grazing contracts include grazing fees that are lower than competitive rental rates.

Second, the optimal monitoring strategy must be random and statistically independent across both space and time. Independent, stationary Poisson processes for monitoring all allotments is a feasible monitoring rule. An intuitive explanation of this result has two parts. First, randomness and statistical independence prevents strategic learning by ranchers about the location of current monitoring activity. Second, a stationary Poisson process produces an exponential distribution for the waiting time until the next monitor.

As a result, the rancher is faced with an autonomous decision problem. The optimal decision regarding whether to cheat or comply with the grazing lease in the first stage of
the rancher’s play is a subgame perfect Nash equilibrium strategy for all subsequent stages of the game. This permanently separates compliant and non-compliant ranchers on all grazing allotments. The government is then able to discover each rancher’s entire extraction path at every monitoring date for each allotment.

Third, the optimal policy includes penalties beyond terminating the lease. Once again the economic intuition is straightforward. Incentive compatibility for ranchers requires penalties large enough to cause the present value of a compliant strategy to exceed the present value of a non-compliant strategy regardless of the rancher’s unobservable characteristics. The optimal penalty makes the rancher who would gain the most face an expected loss from cheating.

The next section develops an economic model of the dynamic game between public lands ranchers and the government. The third section analyzes the relationship between grazing fees, monitoring and enforcement activity, and rancher compliance. In the fourth section, we derive the optimal policy. The fifth section summarizes and concludes.

2. A Model of Forage Exploitation

In this section, we develop a dynamic economic model of the incentives and conflicts between a regulatory agency and public lands ranchers. Let $x(t)$ be the stock of forage and let $s(t)$ be the stocking rate, which determines forage harvest. Let $A$ denote the set of grazing allotments and $I$ the set of rancher types. For each $(a,i) \in A \times I$ the net return from grazing is $v(s(t),x(t),a,i)$ and the net benefit to non-grazing users is $b(s(t),x(t),a)$. We assume that $v(\cdot,a,i)$ is increasing in $(x,s)$, $b(\cdot,a)$ is increasing in $x$
and decreasing in $s$, and $v(\cdot, a, i)$ and $b(\cdot, a)$ are twice continuously differentiable and jointly concave in $(x, s)$. Non-grazing benefits do not depend on the characteristics of the rancher. The agency cannot choose or affect the rancher’s type.

The equation of motion for the forage resource is

$$\dot{x}(t) = f(x(t), a) - s(t), \quad x(0) = x_0(a) \text{ fixed},$$

(2.1)

where $f(x,a)$ is twice continuously differentiable in $x$, $f(0,a) = 0$, $\partial f(0,a)/\partial x > 0$, and $\partial^2 f(x,a)/\partial x^2 < 0 \quad \forall \quad x \geq 0$. We assume that a unique maximum sustainable forage level, $x^{msy}(a) > 0$, satisfying $\partial f(x^{msy}(a), a)/\partial x = 0$ exists for each $a \in A$ (Stoddard, Smith, and Box, p. 273; Libecap, p. 67).

Suppose that rancher $i \in I$ maximizes the discounted present value of profits from grazing on allotment $a \in A$,

$$\max_{\{x(t), s(t)\}, x(0) = x_0(a)} \int_0^\infty e^{-rt} v(s(t), x(t), a, i) dt$$

(2.2)

subject to (2.1), where $r > 0$ is the real discount rate. The rancher’s privately optimal wealth-maximizing forage use path satisfies (2.1) and the following differential equation for the stocking rate,$^2$

$$\dot{s} = (r - f_x)v_s - v_x - v_{ss}(f - s)$$

(2.3)

The long-run steady state satisfies $\dot{s} = \dot{x} = 0$, so that $s^0(a, i) = f(x^0(a, i), a)$, and the private value of the marginal product condition,

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$^2$ Subscripts denote partial derivatives.
\[ F(x^0(a,i),a,i) = v_x(f(x^0(a,i),a),x^0(a,i),a,i) + \\
\quad v_x(f(x^0(a,i),a),x^0(a,i),a,i) \cdot \left[ f_x(x^0(a,i),a) - r \right] = 0 . \] 

(2.4)

We assume that \( \partial F(x,a,i) / \partial x < 0 \ \forall \ x \geq 0 \). This is a sufficient condition for a unique, globally stable saddle point equilibrium.

Now consider the socially optimal decision rule, which includes both the rancher’s and non-grazing benefits. The socially optimal path is the solution to

\[
\max_{s(t),x(t)} \int_0^\infty e^{-rt} \left[ v(s(t),x(t),a,i) + b(s(t),x(t),a) \right] dt ,
\]

subject to (2.1). This path satisfies (2.1) and the following differential equation for the stocking rate:

\[
\dot{s} = \frac{(r - f_x)(v_x + b_x) - (v_x + b_x) - (v_{xx} + b_{xx})(f - s)}{v_{xx}} .
\]

(2.6)

The steady state now satisfies \( s^1(a,i) = f(x^1(a,i),a) \) and the social value of the marginal product condition,

\[
G(x^1(a,i),a,i) = v_x(f(x^1(a,i),a),x^1(a,i),a,i) + b_x(f(x^1(a,i),a),x^1(a,i),a) \\
\quad + \left[ v_x(f(x^1(a,i),a),x^1(a,i),a,i) + b_x(f(x^1(a,i),a),x^1(a,i),a) \right] \cdot \left[ f_x(x^1(a,i),a) - r \right] = 0 .
\]

(2.7)

We assume that \( \partial G(x,a,i) / \partial x < 0 \ \forall \ x \geq 0 \). This is a sufficient condition for the social
It is straightforward to show that $\forall (a,i) \in A \times I, x^1 > x^0$. It follows that the privately optimal stocking rate is initially higher and the long-run equilibrium forage stock is lower than is socially optimal. This is illustrated in Figure 2. The intuition is as follows. Because the stocking rate has a negative marginal value to non-grazing users, the value of the marginal product for $s$ is lower for society than for the rancher. Similarly, because forage has a positive marginal value to non-grazing users, society’s value of the marginal product for $x$ is higher than for the rancher. Both effects work together, producing incentives for the rancher to graze more intensively and harvest more forage than is socially optimal.

3. Optimal Public Grazing Leases

Given the above conflict between society’s goals and those of a public lands rancher, consider livestock grazing in the presence of imperfect monitoring and enforcement. If the agency does not monitor and enforce a federal grazing lease, there is no penalty for pursuing a privately optimal grazing plan. However, monitoring and enforcement are costly. Let $c_m$ be the agency’s marginal cost per permit of inspecting the range, let $N$ be the total number of leases under the agency’s management, and let $B_m$ denote the

\[3\] The first-order conditions for the private and the social optima also are sufficient given concavity of $v(.,a,i), b(.,a)$, and $f(.,a)$.
exogenously determined agency budget available for monitoring activities. Then $M = B_m/c_m \ll N$ is the largest number of grazing permits that can be monitored in any period, and a limited budget precludes monitoring all allotments in all periods.

Assume that the distribution of rancher types, $\Psi : I \rightarrow [0,1]$, is known to the agency and is time invariant. Each rancher with a public grazing lease is considered by the agency to be a random draw from this distribution. The agency is unable to select $i$ for any allotment. The agency also is unable to learn $i$ regardless of the resources committed to seeking this information.

The agency maximizes the expected discounted net benefits on each allotment,

$$\max_{\{s(t), x(t)\}} \int_0^\infty e^{-rt} \left[ \bar{v}(s(t), x(t), a) + b(s(t), x(t), a) \right] dt,$$

subject to (2.1), with the expectation taken over the distribution of rancher types,

$$\bar{v}(s(t), x(t), a) = \int_{i \in I} v(s(t), x(t), a, i) d\Psi(i).$$

Now the long-run steady state satisfies $s^2(a) = f(x^2(a), a)$ and the value of the marginal product condition,

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4 This portion of the BLM and USFS budgets is independent of grazing fees collected. Approximately 50% of fees collected go to state legislatures to be distributed to counties as “Payment in Lieu of Taxes,” 25-50% are earmarked for range improvement, and less than 25% return to the federal treasury. The General Accounting Office (1991) and the BLM estimate that the cost of monitoring the range greatly outweighs total grazing fees collected.
\[ H(x^2(a),a) = \bar{v}_x(x^2(a),a, x^2(a),a) + b_x(f(x^2(a),a),x^2(a),a) \]
\[ + \left[ \bar{v}_s(f(x^2(a),a),x^2(a),a,i) + b_s(f(x^2(a),a),x^2(a),a) \right] \left[ f_x(x^2(a),a) - r \right] = 0. \] (3.3)

As before, we assume that \( H_x(x,a) < 0 \) \( \forall x \geq 0 \), so that there is a unique, globally stable saddle point equilibrium.

The rancher’s choices for \( x(t) \) and \( s(t) \) are observed by the agency if, and when, the grazing lease is monitored. Let \( \mu(a) \) denote the (constant) hazard rate for inspection times.\(^5\) Then the rational expectation of the distribution of agency monitoring times is determined by the exponential probability density function, \( \phi(t,a) = \mu(a)e^{-\mu(a)t} \). Once the agency monitors the allotment, it has complete information.\(^6\) If the agency observes a forage stock that is below, or a stocking rate that is above, the socially optimal level, then it concludes that the permit has been violated. In that case, the government will permanently terminate the lease and impose an additional penalty.\(^7\)

In the next two sub-sections we turn to two issues. First, does the grazing fee affect the stocking rates of compliant or non-compliant ranchers? Second, does the grazing fee

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\(^5\) Independent and stochastic monitoring when the regulator is unable to differentiate among agents was first discussed in Viscusi and Zeckhauser (1979).

\(^6\) Perfect detection of violations once an agent is monitored is a common property in the regulatory enforcement literature (e.g., Viscusi and Zeckhauser).

\(^7\) Costly monitoring and limited budgets have frequently been argued to lead to optimal enforcement strategies with random detection and penalties for violations (e.g., Becker 1968, Stigler 1970, and Polinsky and Shavell 1979).
affect the compliance choice?

3.1 Ranchers’ Decisions in a Regulated Environment

The Taylor Grazing Act of 1934 set much of grazing policy still in effect today. Public grazing fees have always been lower than private grazing fees (LaFrance and Watts). Real public grazing fees have been at or below their current level since the Forest Service was created in 1905, with the exception of the late 1970’s and early 1980’s when cattle prices were unusually high. This suggests that sufficient time has past for compliant ranchers to have reached a long-run sustainable equilibrium. Therefore, assume that $x_0(a) = x^2(a)$. Then the optimal compliant strategy is the sustained yield stocking rate $s(t,i) \equiv s^2(a) \forall t \geq 0$, and the wealth of a compliant rancher of type $i$ on allotment $a$ is

$$W_c(a,i) = \int_0^\infty \left[ v(s^2(a),x^2(a),a,i) - p_g s^2(a) \right] e^{-rt} dt$$

$$= \frac{1}{r} \left[ v(s^2(a),x^2(a),a,i) - p_g s^2(a) \right], \quad \text{(3.4)}$$

where $p_g$ is the grazing fee.

On the other hand, the expected wealth of a noncompliant rancher is partially determined by the frequency and timing of monitoring.\(^8\) The first time that the agency monitors an allotment, any cheating is detected. To mask their cheating, noncompliant

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\(^8\) Becker (1968) and Stigler (1970) argue that individuals compare the expected benefits and costs of compliance with laws and regulations. Viscusi and Zeckhauser (1979) extend this to the regulation of profit-maximizing firms.
ranchers will pay $p_g s^2(a)$. Consequently, grazing fee payments are fixed costs to cheating ranchers, and the expected wealth for a noncompliant rancher is\(^9\)

$$\bar{W}_n(a,i) = \int_0^\infty \varphi(t) \left\{ \int_0^t [v(x(t; \tau, a, i), s(\tau; a, i), a, i) - p_g s^2(a)] e^{-\tau t} d\tau \right\} dt. \quad (3.5)$$

Integration by parts using

$$u(t) = \int_0^t [v(x(\tau; a, i), s(\tau; a, i), a, i) - p_g s^2(a)] e^{-\tau t} d\tau \quad (3.6)$$

and $\Phi'(t) = \varphi(t)$ lets us rewrite this as (e.g., Kamien and Schwartz 1991, pp. 61-62),

$$\bar{W}_n(a,i) = \int_0^\infty [v(s(t; a, i), x(t; a, i), a, i) - p_g s^2(a)] e^{-(r + \mu(a)) t} dt. \quad (3.7)$$

A noncompliant rancher’s optimal control path satisfies (2.1) and the following differential equation for the stocking rate,

$$\dot{s} = \frac{[r + \mu(a) - f_x] v_s - v_x - v_{ss} (f - s)}{v_{ss}}. \quad (3.8)$$

Note that the numerator in (3.8) is positive when there is no monitoring. This follows from $x^0 < x^1$ and the monotonicity of optimal paths in autonomous control problems. Therefore, $\mu(a) > 0$ increases the incentive for cheating ranchers to overstock the range.

The long-run steady state for a cheating rancher satisfies $s^3(a,i) = f(x^3(a,i), a)$ and

\(^9\) Sharon (1967) and Srinivisan (1973) argue that tax evaders will hide their actions to avoid suspicion by regulators.
\[ v_\ast (s^3(a,i), x^3(a,i), a, i) + v_\ast (s^3(a,i), x^3(a,i), a, i) \cdot [\mu (x^3(a,i), a) - (r + \mu (a))] = 0. \] (3.9)

It follows that the equilibrium stocking rate and forage level are both independent of the grazing fee. It also follows that \( \partial x^3(a,i)/\partial \mu < 0 \). Thus, the long-run equilibrium forage stock is a decreasing function of the hazard rate, the initial stocking rate is an increasing function of the hazard rate, and the grazing fee plays no role in an optimal cheating strategy.

### 3.2 Grazing Fees versus Compliance

At this point, we have established that the grazing fee plays no role in the livestock stocking rate choices of both compliant and non-compliant ranchers. It is equally clear that the wealth of both types of ranchers decreases with increased grazing fees. We turn now to whether changes in grazing fees affect decisions to comply with Federal grazing regulations. We find that increasing the grazing fee increases the incentive to cheat.

The rancher’s decision to cheat or comply hinges on the expected net benefit from cheating, \( R \equiv W_n - W_c \). The optimal decision rule is to comply if \( R \leq 0 \) and to cheat if \( R > 0 \). We analyze the qualitative properties of this decision rule by appealing to the dynamic envelope theorem and curvature results of LaFrance and Barney (1991). This gives the following results, which we will make use of below:

\[ \partial W_n / \partial \mu < 0; \] (3.10)

\[ \partial^2 W_n / \partial \mu^2 > 0; \] (3.11)

\[ \partial W_n / \partial p_g = -s^2(a) / [r + \mu (a)] < 0; \] (3.12)
\[ \frac{\partial^2 \tilde{W}_n}{\partial \tilde{p}_g \partial \mu} = s^2(a) \left[ r + \mu(a) \right]^2 > 0; \]  
(3.13)

\[ \frac{\partial^2 \tilde{W}_n}{\partial \tilde{p}_g^2} = 0 \]  
(3.14)

\[ \frac{\partial W_c}{\partial \tilde{p}_g} = -s^2(a)/r < 0; \]  
(3.15)

and

\[ \frac{\partial W_c}{\partial \mu} = \frac{\partial^2 W_c}{\partial \tilde{p}_g \partial \mu} = \frac{\partial^2 W_c}{\partial \tilde{p}_g^2} = 0. \]  
(3.16)

If \( \mu(a) = 0 \), then the optimal strategy is to cheat for any \( p_g \geq 0 \) since there is no penalty for doing so. It also follows from (3.10)–(3.16) that \( R \) is strictly increasing in the grazing fee,

\[ \frac{\partial R}{\partial \tilde{p}_g} = \mu(a)s^2(a)/r \left[ r + \mu(a) \right] > 0, \]  
(3.17)

and strictly decreasing in the hazard rate,

\[ \frac{\partial R}{\partial \mu} = \frac{\partial \tilde{W}_n}{\partial \mu} < 0. \]  
(3.18)

As a result, on each allotment, for any \( \mu(a) > 0 \), there is a unique grazing fee (which may be negative), \( p_g(\mu(a), a, i) \), such that \( R = 0 \), so that rancher \( i \) is indifferent between complying and cheating. Differences across both ranchers and allotments imply different \( (\mu(a), p_g) \) pairs for which a given rancher is indifferent between compliance and cheating on a given allotment. In the absence of any additional penalty imposed when cheating is detected, we are likely to find some ranchers cheating and others complying.

It also follows from (3.10)–(3.16) that the monitoring rate must increase with the grazing fee to maintain a constant incentive for compliance on any given allotment,

\[ \frac{\partial \mu}{\partial \tilde{p}_g} \bigg|_{R=R^o} = -\frac{\partial R}{\partial \tilde{p}_g} \frac{\partial \tilde{W}_n}{\partial R} > 0. \]  
(3.19)
Moreover, the monitoring rate must increase at an increasing rate, since

\[
\frac{\partial^2 \mu}{\partial p_g^2} \bigg|_{R=R^o} = \frac{2}{\partial^2 R} \left( \frac{\partial R}{\partial \mu} \right) \left( \frac{\partial R}{\partial p_g} \right) - \frac{\partial^2 R}{\partial \mu^2} \left( \frac{\partial R}{\partial p_g} \right)^2 \left( \frac{\partial R}{\partial \mu} \right)^3 > 0. \tag{3.20}
\]

Thus, a constant compliance rate with higher grazing fees requires greater expenditures on monitoring and the monitoring cost is strictly convex in the grazing fee.

4. Optimal Penalties

We now consider socially optimal grazing fees and penalties to discourage cheating. If at date \( t \) the agency observes that rancher \( i \) on allotment \( a \) has been cheating, the lease is terminated and the penalty \( P(s(t), x(t), a) \) is imposed.\(^{10} \) Because the hazard rate \( \mu(a) \) and penalty function are independent of time, the rancher’s problem remains autonomous.

The expected wealth of a cheating rancher now becomes

\[
\bar{W}_n = \int_0^\infty \phi(t) \left\{ \int_0^t \left[ v(x(\tau), s(\tau), a, i) - p_g s^2(a) \right] e^{-r\tau} d\tau + P(s(t), x(t), a) e^{-rt} \right\} dt. \tag{3.21}
\]

Integrating by parts similar to the previous case, this can be rewritten as

\[
\bar{W}_n = \int_0^\infty e^{-(r+\mu(a))t} \left[ v(s(t), x(t), a, i) - p_g s^2(a) - \mu(a) P(s(t), x(t), a) \right] dt. \tag{3.22}
\]

\(^{10} \) Section 43 CFR 4150.3 provides for penalties for willful unauthorized grazing on federal lands to include the value of forage consumed – defined in 43 CFR 4150.3(c) as three times the private grazing lease rate in the state where the violation occurs, the value of damage to public land and other Federal property, the cost of detecting, investigating, and enforcing violations, and the cost of impounding the trespassing livestock.
The optimal penalty will discourage any rancher – regardless of type – from cheating. Therefore, incentive compatibility requires
\[
\sup_{i \in I} (\bar{W}_n - W_c) \leq 0. \tag{3.23}
\]
This places a lower bound on the penalty. For each allotment; it must equal or exceed the largest possible net benefit for any cheating rancher. This in turn implies that no rancher on any allotment will optimally choose to cheat.

On the other hand, for any allotment that is actively grazed, we must have
\[
\min_{i \in I} W_c \geq 0. \tag{3.24}
\]
This sets an upper bound on that grazing fee,
\[
p_g \leq \min_{i \in I} \left\{ \psi(s^2(a), x^2(a), a, i) / [r \cdot s^2(a)] \right\}. \tag{3.25}
\]
Hence, the grazing fee must be strictly less than the minimum average value product of \( s \), and nearly all ranchers will receive what amounts to a subsidy relative to the private market for grazing privileges.

4. Conclusions

Livestock grazing on public lands continues to be a source of intense conflict and debate. A primary source of this conflict is diverse groups with incompatible interests that differ over the use and management of public lands. Property rights and use rights are not well defined and it is unlikely that this will change in foreseeable future. There also is ample evidence that public lands ranchers are being subsidized. We analyze this problem using a dynamic game.
Because grazing fee payments are, for all practical purposes, a fixed cost, low fees let ranchers capture more of the rent from grazing public lands. This increases the incentive to comply with federally mandated stocking rates and other regulations. Optimal grazing contracts therefore include grazing fees that are lower than competitive rates.

An optimal grazing contract must include random monitoring. Randomness and independence prevent strategic learning by a cheating rancher. This avoids wasteful efforts to disguise noncompliant behavior.

The optimal policy also includes penalties for cheating beyond terminating the lease. These penalties must be large enough that the rancher who would profit the most from cheating has a negative expected return from doing so.
References


Figure 1. Federal and Private Grazing Fees in the Eleven Contiguous Western States.

West Coast

Mountain States

Desert States
Figure 2. Private versus Socially Optimal Grazing Paths.
Let \( x(t) \) be the stock of forage in the allotment at time \( t \), \( s(t) \) the stocking rate, \( z(t) \) a vector of variable inputs other than the stocking rate, \( k \) a vector of fixed capital structures such as fences, corrals, buildings, water wells and delivery systems, and so forth, essential to public lands ranching, and \( q(t) \) the livestock’s net rate of weight gain from grazing the public range. Assume that the production process for the public lands rancher is given by

\[
q(t) = g(x(t), s(t), z(t), k, a, i)
\]  
(A.1)

where \( g(\cdot) \) is twice continuously differentiable, increasing, and concave in \((x, s, z, k)\).

The net return over non-livestock variable inputs is

\[
v(x, s, a, i) = \max_{z \in \mathbb{R}^n} \left\{ pq - w'z : q = g(x, s, z, k, a, i) \right\}, \quad (A.2)
\]

where \( p \) is the price per unit weight of livestock, \( w \) is the vector of prices for variable inputs, and the arguments \((p, w, k)\) are omitted to simplify the notation. We have:

(a) \( \partial v/\partial x \equiv p \partial g/\partial x > 0 \);

(b) \( \partial v/\partial s \equiv p \partial g/\partial s > 0 \); and

(c) \( v(\cdot) \) is jointly concave in \((x, s)\).

The equation of motion for the grazing resource is given by

\[
\dot{x} = f(x, a) - s, \quad x(0) = x_0(a), \ \text{fixed.} \quad (A.3)
\]

Let subscripts denote partial derivatives. The function \( f(x, a) \) is assumed to be twice continuously differential and to satisfy \( f(0, a) = 0, \ f_x(0, a) > 0, \ f_{xx}(x, a) < 0 \), and a unique
$x^{msy}(a) > 0$ exists such that $f_x(x^{msy}(a),a) = 0$ is the maximum sustainable yield level of the grazing resource on allotment $a$. The rancher is assumed to maximize the discounted present value of expected profits from grazing the public range.

**Private Optimum**

The unregulated rancher solves

$$\max_0^\infty \int_0^\infty v(x,s,a,i)e^{-rt} dt$$  \hspace{1cm} (A.4)

subject to (A.3). The current value Hamiltonian is

$$H = v(x,s,a,i) + \lambda[f(x,a) - s].$$  \hspace{1cm} (A.5)

The first-order necessary and sufficient conditions for an optimal path are:

$$H_s = v_s - \lambda = 0;$$

$$H_x = v_x + \lambda f_x = r\lambda - \dot{\lambda}, \lim_{t\to\infty} e^{-rt}\lambda = 0;$$

$$H_\lambda = f - s = \dot{x}, x(0) = x_0(a), \text{ given.}$$  \hspace{1cm} (A.6)

The unregulated steady state satisfies $f(x^0(a,i),a) = s^0(a,i)$ and the marginal condition:

$$v_x(f(x^0(a,i),a),x^0(a,i),a,i)$$

$$+ v_s(f(x^0(a,i),a),x^0(a,i),a,i)[f_x(x^0(a,i),a) - r] = 0.$$  \hspace{1cm} (A.7)

The necessary and sufficient condition for a unique, globally stable equilibrium is

$$v_{xx} + v_{xs}(2f_x - r) + v_{ss}f_x(f_x - r) + v_s f_{xx} < 0, \forall x \geq 0.$$  \hspace{1cm} (A.8)

We assume this is satisfied. The optimal path is monotonic for all $x_0(a)$ (Hartl 1987).

**Social Optimum**

Let $b(x,s,a)$ denote the flow of net benefits for uses and amenities other than livestock
grazing. Assume $b$ is twice continuously differentiable and jointly concave in $(x, s)$, $b_s < 0$, and $b_x > 0$. The socially optimal decision rule solves

$$\max_{0}^{\infty} \left[ v(x,s,a,i) + b(x,s,a) \right] e^{-rt} dt \quad (A.9)$$

subject to (A.3). The current value Hamiltonian is

$$H = v(x,s,a,i) + b(x,s,a) + \lambda[f(x,a) - s]. \quad (A.10)$$

The first-order necessary and sufficient conditions for an optimal path are:

$$H_s = v_s + b_s - \dot{\lambda} = 0;$$
$$H_x = v_x + b_x + h_f x = r\dot{\lambda} - \lambda, \quad \lim_{t \to \infty} e^{-rt}\lambda = 0;$$
$$H_\lambda = f - s = \dot{x}, \quad x(0) = x_0(a), \text{ given.} \quad (A.11)$$

The socially optimal steady state satisfies $f(x^1(a,i), a) = s^1(a,i)$ and the condition:

$$v_x(f(x^1(a,i), a), x^1(a,i), a, i) + b_x(f(x^1(a,i), a), x^1(a,i), a)$$
$$+ [v_s(f(x^1(a,i), a), x^1(a,i), a, i) + b_s(f(x^1(a,i), a), x^1(a,i), a)] [f_s(x^1(a,i), a) - r]$$
$$= 0. \quad (A.12)$$

Assume the necessary and sufficient condition for a unique, globally stable steady state:

$$v_{xx} + v_{xs}(2f_x - r) + v_{ss}f_x(f_x - r) + v_s f$$
$$+ b_{xx} + 2b_{xs}f_x + b_{ss}f_x^2 - r(b_{xs} + b_{ss}f_x) < 0, \quad \forall \ x \geq 0. \quad (A.13)$$

The following is an immediate consequence of these developments.

**Lemma 1.** $x^1(a,i) > x^0(a,i) \ \forall \ (a,i) \in A \times I$.

**Optimal Regulation**

The Hamiltonian for the noncompliant rancher is

$$H = (1 - \Phi(t,a)) \left[ v(x(t), s(t), a, i) - p_s s^2(a) \right] e^{-rt} + \lambda(t,a) \cdot [f(x(t), a) - s(t)]. \quad (A.14)$$
Define the current value shadow price, \( \theta \), by

\[
\theta(t,a) \equiv \lambda(t,a) \left( e^{rt} / [1 - \Phi(t,a)] \right).
\]  

(A.15)

This implies

\[
\dot{\theta} = (r + \eta)\theta + \lambda \left( e^{rt} / (1 - \Phi) \right),
\]

(A.16)

where \( \eta = d \ln \Phi / dt \). The generalized current value Hamiltonian is

\[
\tilde{H} = v(x(t), s(t), a, i) - p_g s^2(a) + \theta(t,a) \left[ f(x(t), a) - s(t) \right].
\]  

(A.17)

The first-order necessary and sufficient conditions for an interior optimal path are

\[
\begin{align*}
\tilde{H}_x &= v_x - \theta = 0; \\
\tilde{H}_s &= v_x + \theta f_x = (r + \eta)\theta - \dot{\theta}; \\
\tilde{H}_f &= f - s = \dot{x}, \ x(0) = x_0(a). 
\end{align*}
\]  

(A.18)

Differentiate with respect to time, solve for \( \dot{\theta} \), eliminate \( \theta \), and solve for \( \dot{s} \) to obtain

\[
\dot{s} = \frac{(r + \eta - f_x)v_x - v_s - (f - s)v_{ss}}{v_{ss}}.
\]  

(A.19)

The numerator is positive if \( \eta = 0 \) since \( x^0(a,i) < x^1(a,i) \) for all \( (a,i) \in A \times I \) and by the monotonicity of optimal paths in autonomous control problems. If \( \eta(t) > 0 \), then there is an increased incentive to overstock the range to capture higher short-run profits prior to capture and eviction from the grazing lease.

Define \( \mu(a) = \lim_{{t \to \infty}} \eta(t,a) \). The steady state is defined by

\[
\begin{align*}
v_x - \theta &= 0; \\
v_x + \theta[f_x - (r + \mu)] &= 0; \\
f &= s.
\end{align*}
\]  

(A.20)
We have:
\[
\begin{bmatrix}
  v_{ss} & v_{sx} & -1 \\
  v_{sx} & v_{xx} + \theta f_{xx} & f_x - (r + \mu) \\
  -1 & f_x & 0 \\
\end{bmatrix}
\begin{bmatrix}
  s_r \\
  x_r \\
  \theta \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
  \theta \\
\end{bmatrix}.
\] (A.21)

The necessary and sufficient condition for a unique, globally stable steady state is a positive determinant of the 3×3 Hessian for the generalized current value Hamiltonian,
\[
\Delta \equiv -v_{ss} f_x [f_x - r] - [v_{sx} + \theta f_{xx}] - v_{xx} [2 f_x - (r + \mu)] > 0.
\] (A.22)

Assume this condition holds. Then we have:
\[
s_r = s_\mu = -\frac{\theta f_x}{\Delta}; \\
x_r = x_\mu = -\frac{\theta}{\Delta} < 0.
\] (A.23)

**Grazing Fees versus Compliance**

Assume that \( \eta(t,a) = \mu(a) \forall t \geq 0 \), so that the waiting time for first monitoring has an exponential distribution, \( \Phi(t,a) = 1 - e^{-\mu(a)t} \), and that there is no additional penalty other than termination of the lease if a cheating rancher is first monitored. The present value of a compliant strategy is
\[
W_c(a,i) = \frac{1}{r} \left[ v(x^3(a),s^3(a),a,i) - p_g s^3(a) \right].
\] (A.24)

The expected present value of a noncompliant is
\[
\overline{W}_n(a,i) = \int_{0}^{\infty} \left[ v(s(t;a,i),x(t;a,i),a,i) - p_g s^3(a) \right] e^{-(r + \mu(a))t} dt.
\] (A.25)

The envelope theorem and convexity results of LaFrance and Barney (1991) imply
\[
\frac{\partial W_n}{\partial \mu} = -\int_0^\infty t^2 \left[ v(x, s, a, i) - p_g s^3(a) \right] e^{-(r+\mu)t} dt < 0;
\]

\[
\frac{\partial^2 W_n}{\partial \mu^2} = \int_0^\infty t^2 \left[ v(x, s, a, i) - p_g s^3(a) \right] e^{-(r+\mu)t} dt - \int_0^\infty t \cdot \left( v x x_{\mu} + v s s_{\mu} \right) e^{-(r+\mu)t} dt > 0;
\]

\[
\frac{\partial W_n}{\partial p_g} = -s^3(a)/(r+\mu) < 0;
\]

\[
\frac{\partial^2 W_n}{\partial p_g^2} = s^3(a)/(r+\mu)^2 > 0;
\]

\[
\frac{\partial^2 W_n}{\partial p_g \partial \mu} = 0;
\]

\[
\frac{\partial W_c}{\partial p_g} = -s^3(a)/r < 0;
\]

\[
\frac{\partial W_c}{\partial \mu} = \frac{\partial^2 W_c}{\partial p_g \partial \mu} = \frac{\partial^2 W_c}{\partial p_g^2} = 0.
\]

Define \( R = W_n - W_c \). The compliance decision is:

\[
\text{If } R \begin{cases} 
\leq 0, \text{ comply,} \\
> 0, \text{ cheat.}
\end{cases}
\]

It follows from (A.26) that expenditures on monitoring and enforcement must increase with the grazing fee to maintain a constant incentive for rancher compliance,

\[
\left. \frac{\partial \mu}{\partial p_g} \right|_{R = R^o} = -R_{p_g}/R_{\mu} > 0,
\]

and that these expenditures must increase at an increasing rate,

\[
\left. \frac{\partial^2 \mu}{\partial p_g^2} \right|_{R = R^o} = \frac{2R_{\mu p_g} R_{\mu p_g} - R_{\mu p_g}^2 R_{\mu}^2}{R_{\mu}^3} > 0.
\]