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Sustainable growth with environmental spillovers

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Abstract

In this paper, we apply the approach of Ramsey and Koopmans to the problem of optimal and sustainable growth. Under plausible assumptions, intertemporal neutrality implies that the optimal growth path is sustainable without the contrivance of a sustainability constraint. The model is extended to cases involving environmental disamenities. The solutions equivalently solve the problem of maximizing net national product adjusted for depreciation in natural capital and environmental effects. Green net national product in this framework is constant over time, thus avoiding the paradox of declining sustainable income.

JEL classification: Q56; Q41; Q01

Keywords: Intertemporal neutrality; Sustainable income; Green net national product; Natural capital

> It is relevant to enquire whether sustainable development is necessarily a consequence of growth being optimal.
>
> S. Anand and A.K. Sen

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1. Introduction

In mainstream environmental economics, sustainable growth is modeled as a problem of maximizing an intertemporal utilitarian welfare function, subject to the constraint that the growth of consumption, utility, or social well-being cannot be negative (e.g. Asheim, 1999; Dasgupta, 2001). The constrained-utilitarian approach applies positive discounting of future utility, consistent with Koopmans's (1960) demonstration that there does not exist a utility function defined on all consumption streams that satisfies the usual axioms of rational choice and timing neutrality (i.e. without discounting). In an alternative approach (Beltratti et al., 1993, 1995; Chichilnisky, 1996),[1] social welfare is modeled as a weighted average of conventional growth and a concern for sustainability.

In what follows, we take a different tack, following the question posed by Anand and Sen (2000). Instead of incorporating a sustainability criterion as a side constraint, we incorporate the concern with intergenerational equity into the planner's objective function. We ask whether such a concern, in combination with a specification of interdependency between well-being and the natural environment, will lead to a sustainable consumption path. In particular, we employ the concept of intergenerational neutrality as proposed by Ramsey (1928) and Koopmans (1965). Ramsey (1928, p. 619) warned that the use of a positive utility discount rate is "ethically indefensible" and reveals "a weakness of the imagination". Koopmans himself (1965, p. 240) noted "we welcome equally a unit increase in consumption per worker in any one future decade . . . Mere numbers cannot give one generation an edge over another . . ." Koopmans's (1965) solution to Koopmans's (1960) nonexistence problem relies precisely on the notion of "intergenerational neutrality" for a specific, non-empty subset of feasible consumption paths and is captured in turn by the zero utility discount rate.[2]

The first objective of the present paper is to examine optimal and intertemporally neutral growth with a nonrenewable resource that generates negative externalities such as acid rain and global warming. We suggest that assuming a backstop resource is plausible and that, under these conditions, optimal growth is sustainable, even without the imposition of a sustainability constraint. The second objective is to resolve the inconsistency between the sustainability requirement that consumption growth be non-negative and the finding that, in the standard model, maximizing sustainable income leads to eventually declining consumption. We show that this paradox does not arise under intertemporal neutrality.

In Section 2, we present the conditions for optimal and ethically neutral growth in a model with a non-renewable resource and a backstop technology. The maximum–minimum solution is shown to be a special case of this solution, albeit one which is unlikely to be preferred. In Section 3, we extend the model to include environmental disamenities associated with the use of non-renewable resources. In Section 4, we investigate the relationship between Ramsey–Koopmans optimal growth and sustainable income. Section 5 provides a brief summary and concluding remarks.

[1] See also the discussion of this approach in Heal (1998).

[2] Zero utility discounting has made rare appearances in the sustainability literature. Endress and Roumasset (1994) used it to show that optimal growth can be sustainable in a model with non-renewable resources and a backstop technology. Ayong Le Kama (2001) applied a similar approach in a model with renewable resources and pollution.

2. Intertemporally neutral optimal growth with a non-renewable resource

2.1. Koopmans's impossibility theorem and his solution

An immediate obstacle to maximizing a utilitarian welfare function without discounting is that the value of such a function is infinite for some feasible consumption streams. Koopmans (1960) went further, proving the impossibility of representing intertemporally neutral planner preferences, over all consumption vectors, by any utility function.

Building on Ramsey, Koopmans (1965) argued that one way out of the dilemma of a non-existent utility function capable of ranking all conceivable consumption paths is to identify a subset of all feasible paths on which the planner's utility function can be defined. Ramsey's criterion for eligibility in the subset is a sufficiently rapid approach of the path to a "bliss point". Koopmans's criterion is less restrictive: "We shall find that in the present case of a steady population growth the golden rule path can take the place of Ramsey's state of bliss in defining eligibility" (1965, p. 500). Specifically, consumption must approach the golden rule consumption level with sufficient rapidity that the area of deficit between the "felicity" of consumption and that of golden rule consumption converges to a finite number (i.e., $V(t) < \infty$ where V is the planner's utility, defined as the area given in Eq. (3)).[3] Using this criterion, Koopmans (1965) demonstrates that each eligible path is superior to each path that is ineligible. Moreover, one can rank eligible paths and determine one that is optimal.

2.2. Sustainability without really trying

In order to introduce natural capital into the Ramsey–Koopmans framework, consider an economy that uses a natural resource (R) in addition to capital (K) and labor (L) to produce a single homogeneous good. Assume that the production technology is constant returns to scale so that the production function $Q(K, R, L)$ is homogeneous of degree 1. In order to present the argument in its starkest form, we abstract from population growth and technological change and normalize $L = 1$, such that $Q(K, R, L)$ can be expressed as $F(K, R)$. Following the standard approach, output of production is divided among consumption, gross investment, and the cost of providing the resource as an input to the production process.

Let θ be the unit cost of extracting the natural resource and providing it as an input of production. We assume that this cost is a decreasing function of the resource stock X (e.g. Heal, 1976). Produced capital, K, depreciates at the rate δ. The dynamic equation governing capital accumulation is

$$\dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t. \tag{1}$$

In this section, the natural resource is assumed to be non-renewable, and the dynamic equation governing the resource stock becomes

$$\dot{X}_t = -R_t. \tag{2}$$

[3] Koopmans introduces "felicity" in order to distinguish the planner's utility function from what would otherwise be called the utility of consumption.

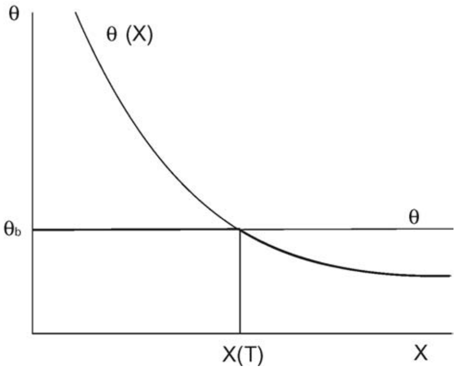

Fig. 1. Marginal extraction cost locus is the lower envelope of $\theta(X)$ and θ_b.

We augment this basic model by incorporating a backstop technology that has a fixed unit extraction cost θ_b. Consider, for example, the case of oil, a non-renewable resource. Oil stocks are drawn down as the economy grows until unit cost, θ, has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source (e.g., coal gasification, nuclear fission/fusion, solar energy). Once the switch has been made, we assume that the backstop technology delivers energy at constant cost, θ_b. Therefore, the locus of unit extraction cost is the lower envelope of curves $\theta(X)$ and θ_b, as shown in Fig. 1. This case contrasts with the conventional Hartwick–Solow model in which extraction costs are constant up to a specific quantity, after which no amount of the resource is obtainable at any cost.[4]

Social welfare over the feasible and eligible consumption paths makes use of the auxiliary "felicity" function, $U(C_t)$.[5] As in Koopmans (1965), we assume $U_C > 0$, $U_{CC} > 0$, and $\lim_{C\to 0} U(C) = -\infty$, such that periods of very low consumption are avoided as much as possible.

Following Koopmans, the social planner's utility function is expressed as[6]

$$V = \int_0^{\infty} [U(C_t) - U(\hat{C})]\,\mathrm{d}t, \tag{3}$$

[4] The assumption of rising extraction costs up to a finite "backstop" limit is considerably more realistic than the inverted L-shaped extraction cost schedule that is usually assumed (see, e.g. Chakravorty et al., 1997 for details). An alternative to the backstop assumption would be to follow Hotelling (1931) and Dasgupta and Heal (1979) wherein resource use is truncated on the demand side. In our framework, however, this would require the complication of multiple consumption goods.

[5] Koopmans' formulation has a deceptive resemblance to classical utilitarianism, but U is the planner's "felicity" and has no necessary connection with consumers' utility. The role of U is to capture the extent to which the planner is averse to intergenerational inequality. While U has quasi-cardinal properties, V is ordinal.

[6] To maximize H with respect to the control variable, R, we implicitly require that $R \geq 0$. Correspondingly, the Kuhn–Tucker condition is $\partial H/\partial R \leq 0$, and complementary-slackness provides that $R(\partial H/\partial R) = 0$. Inasmuch as we can rule out the extreme case of $R = 0$, we postulate that $R > 0$, implying an interior solution.

and the corresponding planner's optimization problem is

$$\begin{aligned} \text{Max} \quad & V \\ \text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \quad K(0) = K_0, \\ & \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\ & X_t \geq 0, \qquad\qquad\qquad\qquad X(0) = X_0, \end{aligned} \tag{4}$$

where $U(\hat{C})$ is felicity at the golden rule level of consumption and T is the endogenous time for which $\theta(X_t) = \theta_b \, \forall t \geq T$.

Since $\dot{K}_t = 0$ in the steady state, the golden rule consumption level, $\hat{C}$, can be found (generalizing from Solow, 1956) by maximizing

$$C = F(K, R) - \delta K - \theta_b R, \tag{5}$$

where R is the amount of the backstop resource consumed in the steady state.

The corresponding first-order conditions, which comprise the golden rule for capital accumulation and resource management, are

$$\frac{\partial C}{\partial K} = F_K - \delta = 0, \tag{6}$$

and

$$\frac{\partial C}{\partial R} = F_R - \theta_b = 0. \tag{7}$$

These conditions yield the golden rule steady-state levels, $\hat{K}$ and $\hat{R}$. $\hat{C}$ is now defined as

$$\hat{C} = F(\hat{K}, \hat{R}) - \delta\hat{K} - \theta_b\hat{R}. \tag{8}$$

The Hamiltonian for this problem is (for simplicity, the subscripts t's are dropped)

$$H = [U(C) - U(\hat{C})] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R]. \tag{9}$$

Incorporating the inequality constraints imposed on the problem, we form the Lagrangian

$$L = H + \phi\{X\} + \tau\{\theta_b - \theta(X)\}, \tag{10}$$

such that the complimentary slackness conditions associated with the inequalities are

$$\tau\frac{\partial L}{\partial \tau} = \tau[\theta_b - \theta(X)] = 0, \qquad \phi\frac{\partial L}{\partial \phi} = \phi X = 0. \tag{11}$$

Application of the maximum principle to this optimal control problem yields the following two efficiency conditions (see Appendix A for details[7]):

$$\eta(C)\frac{\dot{C}}{C} = F_K - \delta, \tag{12}$$

[7] Appendices are available on the JEBO website.

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta}. \tag{13}$$

Condition (12) is the Ramsey condition that governs the optimal path of consumption leading to golden rule steady state. In the analogous approach to the "modified golden rule", $\eta(C)\dot{C}/C + \rho = F_K - \delta$, there are two parameters governing the savings and the rate of capital accumulation. The first is the absolute value of the consumption elasticity of the marginal utility, $\eta(C)$. Lower $\eta(C)$ implies a lower social opportunity cost of savings (greater tolerance for intergenerational inequality), more rapid capital accumulation, lower interest rates, and higher growth rates of consumption. The second parameter is the social rate of time preference ρ, which reflects the social valuation of future felicity in term of today's felicity. However, in our setting, by treating all generations equally, $\rho = 0$.[8]

Condition (13) is a generalization of Hotelling's rule. The LHS is the in situ marginal value of the resource, and the RHS is the marginal user cost.[9] Thus, the optimal consumption trajectory and the optimal motion of the state variables, K and X, are governed by two intuitive conditions: the Ramsey savings rule, with a zero utility-discount rate, and a general-equilibrium Hotelling rule for the case of rising extraction costs.

2.3. *Relationship to other approaches*

Hartwick (1977) and Solow (1974, 1986) have shown that for a Cobb-Douglas production function of capital and a non-renewable resource with a constant extraction cost, extracting the resource according to the Hotelling rule and then saving exactly the resource rents thus generated leads to a consumption path that is sustainable and constant over time. The Hartwick–Solow rule has been justified ex post as being the highest consumption path that is intergenerationally equitable in the sense of delivering equal consumption to all generations.

This maximum–minimum consumption path may be generated as a special case of our basic model. Rearranging Eq. (12), we have

$$\frac{\dot{C}}{C} = \frac{F_K - \delta}{\eta(C)}. \tag{14}$$

As the social aversion to intertemporal inequality, $\eta(C)$, approaches infinity, $\dot{C}/C \to 0$, generating constant consumption for all $t \geq 0$.

As a special case, suppose social planner's preferences are represented by $V = \int_0^\infty [U(C_t) - U(\hat{C})]\,\mathrm{d}t$ where $U(C_t)$ takes the CES form:

$$U(C_t) = -C_t^{-(\eta-1)}, \quad \eta > 1. \tag{15}$$

As η gets larger and larger, the initial level of consumption increases, and the consumption trajectory becomes flatter. This is illustrated in Fig. 2. No matter how high η, the upper bound

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[8] It would be unsound to derive Eq. (12) by setting $\rho = 0$ in the standard Ramsey condition. Nonetheless, the equation that one gets by doing exactly that turns out to be correct.

[9] Endress and Roumasset (1994). For a partial equilibrium market equivalent of Eq. (13), see Hansen's (1980) generalization of the Hotelling rule to the case of rising extraction costs.

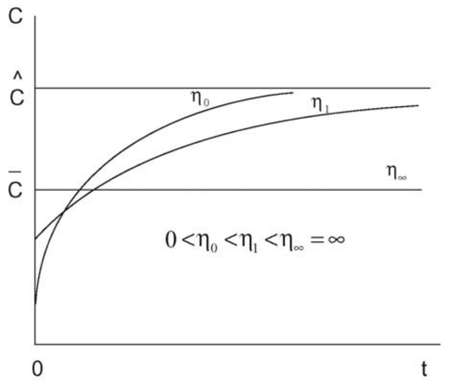

Fig. 2. Social impatience and the optimal consumption trajectory.

of consumption remains at $\hat{C}$ as long as η is not infinite. Once η becomes infinite, however, both the upper and lower bound switch to $\bar{C}$ in Fig. 2, exactly the maximum–minimum level of intertemporal consumption.

An alternative to maxi–min welfare is the concept of constrained utility maximization. The main idea, due to Asheim (1988), is to apply a non-declining utility constraint ($\dot{U}(C) \geq 0$) to the maximization of utilitarian welfare, but as noted by Toman et al. (1995), such an approach does not resolve how the social welfare function should directly reflect concerns about intergenerational equity. Besides the ad hoc nature of the utility constraint, constrained optimization cannot provide a full ranking of alternatives because alternatives that violate the constraint cannot be compared. In the Hartwick–Solow economy, for example, if either the elasticity of substitution between natural capital and produced capital is less than 1 or the output elasticity of natural capital is greater than that of produced capital (with elasticity of substitution equal to 1), the sustainability constraint renders the maximization problem infeasible. In this case, none of the feasible paths can be ranked.[10]

Rather than adding a sustainability constraint or specifying axioms that a “sustainably-correct” social planner’s preferences must satisfy (e.g. Beltratti et al., 1995), our approach follows Ramsey and Koopmans and finds an optimal and intertemporally neutral growth path. In the basic case above and in the cases below wherein the resource generates an environmental amenity or disamenity, we find that the optimal path is sustainable, even though we do not require it to be so.

Inasmuch as Ramsey, Koopmans and others[11] have provided ethical reasons against utility discounting, why does the current generation of mainstream economists continue the

[10] Even if constrained utility maximization is reformulated as a lexicographic (vector-valued) utility function (see Endress, 1994), the model is still characterized by the rejection of tradeoffs (Dasgupta and Mäler, 1995). That is, no negative consumption growth, however close to zero, can be justified, even if it affords higher sustainable consumption in the future.

[11] For example, Harrod (1948, p. 40) remarked that utility discounting is a “polite expression for rapacity and the conquest of reason by passion.”

practice? One possible reason is the misconception that without discounting, the planner's objective function would be infinite for any consumption stream that does not converge to zero sufficiently rapidly and that maximization is therefore impossible. This view simply overlooks the Koopmans transformation (as written in Eq. (3)). A related but more sophisticated objection is that under intergenerational neutrality, a utility function capable of ranking the social planner's preferences over all conceivable consumption streams does not exist. It is curious that economists (e.g. Arrow, 1999; Dasgupta, 2001) cite Koopmans (1960) and Diamond (1965) in support of this view, but fail to note Koopmans' (1965) demonstration that by restricting attention to feasible consumption paths, the planner's utility function exists after all.

A third misconception concerns the application of zero discounting to a model wherein a non-renewable resource is necessary for production and either capital accumulation is neglected (the "cake-eating" economy in Heal, 1993) or the elasticity of substitution between capital and labor is less than 1. In such economies, there is no well-defined maximum (Dasgupta and Heal, 1979). This should hardly be held against the practice of zero discounting, however. One is dealing with necessarily dismal economies, and any feasible consumption path can be improved upon by taking from the present and stretching consumption further into the future.

3. Extensions: environmental effects

3.1. Fund pollution

We now turn to the case wherein use of a non-renewable resource such as petrochemically sourced energy generates pollution. For simplicity, assume that pollution (E_t) is emitted as a constant proportion of resource use (R_t) and that emission units are set such that the proportion is one. Therefore,

$$E_t = R_t, \quad t < T. \tag{16}$$

In the case of fund pollution, emissions are assumed not to accumulate, so emissions, but no stock pollutants, enter into the utility function. Now our maximand becomes

$$\text{Max } V = \int_0^{\infty} [U(C_t, E_t) - U(\hat{C}_t, \hat{E})] \, \mathrm{d}t \tag{17}$$

where $U_C > 0$, $U_E < 0$; and $U_{CC} < 0$, $U_{EE} < 0$, signifying increasing marginal disutility of pollution.

As in the basic case, $\hat{C}$ is associated with a total switch to the backstop technology (e.g. photovoltaics). Use of the primary resource and the corresponding emissions are then both zero so that $\hat{E} = 0$. Golden rule consumption is therefore given by Eq. (8), with $\hat{K}$ and $\hat{R}$ exactly the same as in the basic model.

An optimum trajectory for consumption and capital accumulation satisfies(18)

$$\begin{aligned}
\text{Max} \quad & V = \int_0^\infty [U(C_t, E_t) - U(\hat{C}, 0)]\,\mathrm{d}t \\
\text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \quad K(0) = K_0, \\
& \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\
& X_t \geq 0, \qquad X(0) = X_0,
\end{aligned}$$

Application of the maximum principle to this optimal control problem gives the following efficiency conditions (See Appendix B for details[12]):

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta \tag{19}$$

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} + \frac{1}{F_K - \delta}\left(\frac{\dot{U}_E}{U_C}\right) - \frac{U_E}{U_C}. \tag{20}$$

Eq. (19) appears to be the familiar Ramsey condition. However, since U now has two arguments, the time derivative of U_C will involve a cross-term,

$$-\left(\frac{U_{CC}C}{U_C}\right)\left(\frac{\dot{C}}{C}\right) + \left(\frac{U_{CE}E}{U_C}\right)\left(\frac{\dot{E}}{E}\right) = F_K - \delta. \tag{21}$$

If C and E are separable arguments of the felicity function, Eq. (21) collapses to the conventional Ramsey savings rule.

Turning to Eq. (20), the LHS and the first term on the RHS constitute the generalized Hotelling condition (cf. Eq. (13)). The last term on the RHS is just the marginal damage cost (MDC $= -U_E/U_C$). The last two terms together can be defined as the marginal externality cost, MEC. Inasmuch as corrective taxation in a first-best economy calls for setting the emission tax equal to MEC, it is of some interest to investigate the relationship of MEC and MDC. By differentiating the first-order conditions, we obtain (see Appendix B)

$$\text{MEC/MDC} = \frac{\sigma}{F_K - \delta}\left\|\frac{\dot{E}}{E}\right\|,$$

where σ is the emission elasticity of marginal utility. Since that MEC goes to zero as soon as the backstop resource is utilized, the first-best emission tax would be zero. In the transition to use of the backstop resource, MEC can be greater or less than MDC. Three possible cases are presented in Appendix B, one having to do with MEC/MDC in the neighborhood of, but preceding, the use of the backstop technology. It is plausible that $\dot{E}$ becomes sufficiently small before the backstop resource is employed, such that MEC becomes less than MDC. This occurs for two reinforcing reasons. First, in the case of rising extraction costs, the efficiency price of the resource (F_R) is increasing, albeit at a decreasing rate (Hanson). Second, diminishing marginal rate of technical substitution between the resource and capital

[12] Appendices available on JEBO website.

implies that the smaller resource-price increase begets successively smaller increases in conservation, thus providing an additional reason that $\dot{E}$ becomes small before the backstop price is reached. In this case, the first-best pollution tax would be less than MDC. This result complements the finding that second-best emission taxes may be optimally set below MDC (see e.g. Goulder, 1997).

3.2. Stock pollution

Now consider the case of stock pollution, such as greenhouse gases, wherein emissions contribute to the stock of pollution, M, which depreciates at rate ξ.[13] Our model now becomes

$$\begin{aligned} \text{Max} \quad & V = \int_0^\infty [U(C_t, M_t) - U(\hat{C}, \hat{M})]\, \mathrm{d}t \\ \text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t) R_t - C_t, \quad K(0) = K_0, \\ & \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\ & \dot{M}_t = \begin{cases} R_t - \xi M_t, & t < T \\ -\xi M_t, & t \geq T \end{cases} \\ & X_t \geq 0, \qquad X(0) = X_0, \end{aligned} \tag{22}$$

where $U_C > 0$, $U_M < 0$ and $U_{CC} < 0$, $U_{MM} < 0$, corresponding to the previous section, and where $\hat{M} = 0$ and $\hat{C}$ is the Solow golden rule consumption as before.[14]

Application of the maximum principle leads to a Ramsey condition that is identical to (19), and the expansion of $\dot{U}_C$ will be analogous to Eq. (21). If C and M are separable, we have the conventional Ramsey savings rule once more.

The generalized Hotelling condition for this case is

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} - \left(\frac{1}{F_K - \delta}\right)\left(\frac{U_M - \mu\xi}{U_C}\right), \tag{23}$$

where μ is the shadow price of the pollution stock (see Appendix C). The last term on the RHS is the marginal externality cost for stock pollution. It is smaller than MDC ($-U_M/U_C$) because the shadow price of pollution, μ, is negative.

We now consider the special case of the above for which $\xi = 0$ (i.e. the stock pollutant does not depreciate). In this case, the backstop technology is immediately employed and the primary resource is never used, so that the golden rule stock of pollution is zero ($\hat{M} = 0$). This result provides a case in which the strategy of *strong sustainability* is optimal. In its usual justification, strong sustainability is associated with the preservation of natural capital and is defended as an ecological imperative, not derived (see, e.g. Pearce and Barbier, 2000).

[13] This model is similar to that of Nordhaus (1991) albeit with explicit consideration of resource depletion but without the intervening climate model.

[14] In the golden rule steady state, the economy has switched to the backstop resource and the stock of pollution has depreciated to zero. Analogous to consumption in the Koopmans model, the stock of pollution in the optimal trajectory asymptotically approaches its golden rule level but never actually reaches it.

Strong sustainability critics have derided the strategy as being a "category mistake", that is not derived from more fundamental objectives, and for denying resource-rich economies a major source of savings and capital formation (Dasgupta and Mäler, 1995). The zero pollution result exemplifies a different approach to strong and weak sustainability than is usually found in the literature. Instead of proposing the strategy as both the objective and the means of optimal growth, our approach separates ends and means. But while ecologically oriented proponents of preservation suggest that strong sustainability is especially important when natural capital is essential and irreplaceable, our result suggests that strong sustainability is an optimal strategy when natural capital has an abundant and perfect substitute.

4. Net national product

An alternative to the conventional approaches of optimizing sustainably weighted or sustainably constrained growth is to extend the concept of NNP to include the depreciation of natural capital, ergo the moniker green net national product, GNNP. Maximizing GNNP is roughly equivalent to maximizing intergenerational welfare inasmuch as GNNP can be shown to be a linear approximation of the Hamiltonian of the intergenerational welfare function (albeit without a sustainability constraint).[15] Green national product is also used interchangeably with "sustainable income" (Pearce and Barbier, 2000; World Bank, 1997). Since Weitzman assumes positive discounting, however, this leads to a paradox. With a positive utility discount rate, both GNNP and consumption can eventually fall (Dasgupta and Heal, 1979, Chapter 10; Dasgupta, 2001); thus maximizing sustainable income is consistent with unsustainable income and consumption.[16] In this section, we show that this paradox disappears in the case of timing neutrality.

Consider the model discussed in Section 3.2 with environmental disamenity, M. The Hamiltonian along the optimum trajectory remains constant; that is, $\mathrm{d}H_t/\mathrm{d}t = 0$,[17] where

$$H_t = [U(C_t, M_t) - U(\hat{C}, 0)] + \lambda_t \dot{K}_t + \psi_t \dot{X}_t + \mu_t \dot{M}_t. \tag{24}$$

At the steady state, $\dot{K}_t = \dot{X}_t = \dot{M}_t = 0$ and $U(C_t, M_t) = U(\hat{C}, 0)$, implying a zero value for H_t. Consequently, $H_t = 0$ for all t. Thus,

$$U(\hat{C}, 0) = U(C_t, M_t) + \lambda_t \dot{K}_t + \psi_t \dot{X}_t + \mu_t \dot{M}_t. \tag{25}$$

The golden rule level of utility, $U(\hat{C}, 0)$, is the constant green net national product in utility units for all time periods. Thus maximizing green net national product is equivalent to

[15] Weitzman (1976, 1999).

[16] Total capital stock, consumption, and green national product typically rise and then fall, albeit both capital and green national product fall before consumption. If the utility discount rate is high enough and there is no backstop resource, total capital stock, GNNP, and consumption all fall monotonically in the optimal program (Dasgupta and Heal, 1979, Chapter 10).

[17] See Appendix D (available on JEBO website) for a mathematical derivation. Capital letters with subscript t stands for variables along optimal trajectory. Indeed, this also implies that our solution satisfies the transversality condition for the infinite-horizon, non-discounting problem as examined by Chiang (1992).

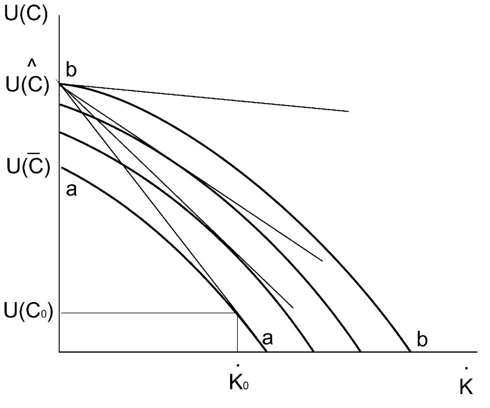

Fig. 3. NNP remains constant at its golden rule level.

optimizing the Ramsey–Koopmans welfare function and results in income and consumption streams that are sustainable, thereby avoiding the paradox.[18]

This proposition is illustrated using a much simpler case where total capital is taken only to include produced capital, K, and instantaneous utility, $U(C)$, is defined solely on consumption. In Fig. 3,[19] curve aa represents the feasibility frontier of the economy at time $t=0$. Consumption level, $\bar{C}$, is the maximum attainable level of consumption at time $t=0$ if no investment were to take place, and $U(\bar{C})$ is the associated level of utility. The utility–investment pair $(U(C_0), \dot{K}_0)$ lies on the optimal trajectory to the steady state. As capital is accumulated, the feasibility frontier moves outward and towards the right until maximum attainable consumption reaches the golden rule level, $\hat{C}$, and $\dot{K}_t = 0$, as depicted by curve bb. The shadow price of capital, illustrated by the slope of the tangency line, decreases monotonically. For the general case considered above with three types of capital, NNP still transitions to $U(\hat{C})$, the shadow price of capital decreases monotonically, and the other shadow prices transition to constant values.

5. Summary and concluding remarks

The literature on sustainable growth has foundered on the question of whether to represent sustainability as an ad hoc constraint on the objective function or by restricting the social planner's preferences. Instead of searching for what is optimal and sustainable, we follow

[18] While NNP remains constant, "felicity" is ever rising to the golden rule steady-state level. We do, however, lose the interpretation of NNP as wealth times the utility discount rate, because wealth is infinite. The Koopmans transformation is a mechanical device to solve for the optimal consumption path. The device does not change the reality that wealth is infinite under intertemporal neutrality. Accordingly, wealth is an inappropriate indicator of intergenerational welfare in this case.

[19] Fig. 3 generalizes Weitzman's (1976) illustration of the hypothetical stationary equivalent consumption of NNP by illustrating the transition to the steady state. Unlike the Weitzman case, however, NNP remains constant under timing neutrality and equal to the steady-state consumption level. We also illustrate NNP in utility units to avoid the linear approximation problem.

the canonical approach of Ramsey, Koopmans, and Diamond and solve for what is optimal and intertemporally neutral. This frees us to explore the conditions under which such a program results in sustainable utility.

We find that optimal and intertemporally neutral growth is sustainable, even in the presence of non-renewable resources. Adding a constraint that restricts growth of consumption or utility to be nonnegative would be not only ad hoc but also redundant under the specified conditions.[20] The necessary conditions for optimal growth require that the economy save at the rate given by the familiar Ramsey condition and that resource use and conservation conform to a generalized Hotelling condition. The constraint of weak sustainability, which requires that the depletion of natural capital not exceed the accumulation of produced capital, is similarly redundant. Total capital increases along the optimal growth path, albeit at a declining rate.

The model is extended to accommodate environmental disamenities, resulting in modifications of the Ramsey and Hotelling conditions. In the cases of fund and stock pollution, the Ramsey condition is expanded to include a disamenity term. The Hotelling condition contains an additional term, the “marginal externality cost”, which is, however, less than the marginal damage cost for the stock pollution case and ambiguously so for the fund pollution case. This means that the optimal pollution tax may be less than its Pigouvian level even without second-best considerations of public finance.[21]

Another interesting result concerns the case wherein the stock of pollution does not depreciate. In this case, the optimal strategy turns out to be not to touch the non-renewable resource and immediately exploit the more costly, but non-polluting backstop. This result shows that the strategy of strong sustainability, which is often advocated on the grounds that natural capital is essential and irreplaceable, turns out to be correct in the opposite case where natural capital has a perfect substitute.

The Ramsey–Koopmans approach also resolves the inconsistency between optimizing growth with intergenerational equity and using GNNP as a measure of sustainable income. When intergenerational equity is taken to mean Ramsey–Koopmans intergenerational neutrality, GNNP retains its status as an appropriate indicator of intergenerational well-being and moreover is sustainable and constant in the optimal program.

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[20] Under more pessimistic conditions, as in models without capital accumulation, a sustainability constraint renders the set of feasible consumption paths empty (see the discussion at the end of Section 2).

[21] See, e.g. Bovenberg and Goulder (1996) for a discussion of second-best emission taxes and the “double-dividend” debate.

Appendix A

The problem is to minimize the difference between actual consumption and optimal consumption, subject to dynamic constraints on capital accumulation and resource use:

$$\begin{aligned}
\text{Max} \quad & V = \int_0^\infty [U(C_t) - U(\hat{C})]\,\mathrm{d}t \\
\text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \quad K(0) = K_0, \\
& \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\
& X_t \geq 0, \qquad X(0) = X_0.
\end{aligned}$$

The Hamiltonian expression for this problem is (for simplicity, the subscripts *t* is dropped)

$$H = [U(C) - U(\hat{C})] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R].$$

The standard first-order conditions for this optimal control problem are

$$\frac{\partial H}{\partial C} = U_C - \lambda = 0, \tag{A.1}$$

$$\frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi = 0, \tag{A.2}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta], \tag{A.3}$$

$$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R. \tag{A.4}$$

From the first and the third conditions,

$$\dot{\lambda} = \dot{U}_C, \tag{A.5}$$

and

$$\dot{\lambda} = -\lambda[F_K - \delta]. \tag{A.6}$$

Equating expressions for $\dot{\lambda}$ and rearranging yields

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta, \tag{A.7}$$

or

$$\eta(C)\frac{\dot{C}}{C} = F_K - \delta, \quad \text{where } \eta(C) = -\frac{U_{CC}C}{U_C} > 0.$$

From the second necessary condition,

$$\dot{\psi} = \dot{\lambda}[F_R - \theta(X)] + \lambda[\dot{F}_R - \theta_X \dot{X}] = -\lambda(F_K - \delta)[F_R - \theta(X)] + \lambda[\dot{F}_R + \theta_X R]. \tag{A.8}$$

From the fourth necessary condition,

$$\dot{\psi} = \lambda\theta_X R. \tag{A.9}$$

Equating expressions for $\dot{\psi}$ and rearranging yields

$$-(F_K - \delta)[F_R - \theta(X)] + \dot{F}_R = 0, \tag{A.10}$$

or

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta}. \tag{A.11}$$

Appendix B

The problem is

$$\begin{aligned}
\text{Max} \quad & V = \int_0^\infty [U(C_t, E_t) - U(\hat{C}, 0)]\,\mathrm{d}t \\
\text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \quad K(0) = K_0, \\
& \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\
& X_t \geq 0, \qquad X(0) = X_0.
\end{aligned}$$

The Hamiltonian for this problem is (for simplicity, the subscript t is dropped)

$$H = [U(C, E) - U(\hat{C}, 0)] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R].$$

The standard necessary first-order conditions for this optimal control problem are

$$\frac{\partial H}{\partial C} = U_C - \lambda = 0, \tag{B.1}$$

$$\frac{\partial H}{\partial R} = U_E + \lambda[F_R - \theta(X)] - \psi = 0, \tag{B.2}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda(F_K - \delta), \tag{B.3}$$

$$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R. \tag{B.4}$$

From the first equation,

$$\dot{\lambda} = \dot{U}_C, \tag{B.5}$$

and from (B.3),

$$\dot{\lambda} = -U_C[F_K - \delta]. \tag{B.6}$$

Equating these two expressions for $\dot{\lambda}$ and rearranging gives

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta. \tag{B.7}$$

Differentiating Eq. (B.2) with respect to time t, we get

$$\dot{\psi} = \dot{U}_E + \dot{U}_C[F_R - \theta(X)] + U_C[\dot{F}_R + \theta_X R]. \tag{B.8}$$

Equating with (B.4) to obtain

$$\frac{\dot{U}_E}{U_C} - (F_K - \delta)[F_R - \theta(X)] + \dot{F}_R = 0. \tag{B.9}$$

Meanwhile,

$$\frac{\dot{U}_E}{U_C} = \left(\frac{\dot{U}_E}{U_C}\right) + \frac{U_E}{U_C}\frac{\dot{U}_C}{U_C} = \left(\frac{\dot{U}_E}{U_C}\right) - \frac{U_E}{U_C}(F_K - \delta). \tag{B.10}$$

Plugging back into (B.9),

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} + \frac{1}{F_K - \delta}\left(\frac{\dot{U}_E}{U_C}\right) - \frac{U_E}{U_C}. \tag{B.11}$$

Consider marginal damage cost, $\mathrm{MDC} = -U_E/U_C$.

$$\begin{aligned} \dot{\mathrm{MDC}} &= \frac{\partial(\mathrm{MDC})\dot{E}}{\partial E} + \frac{\partial(\mathrm{MDC})\dot{C}}{\partial C} = \left[-\frac{U_{EE}}{U_C}\right]\dot{E} + \left[\frac{U_E U_{CC}}{U_C^2}\right]\dot{C} \\ &= \left[-\frac{U_{EE}E}{U_E}\frac{U_E}{U_C}\right]\frac{\dot{E}}{E} + \left[\frac{U_E}{U_C}\frac{U_{CC}C}{U_C}\right]\frac{\dot{C}}{C} = \sigma\,\mathrm{MDC}\frac{\dot{C}}{E} + \eta\,\mathrm{MDC}\frac{\dot{C}}{C}, \end{aligned} \tag{B.12}$$

where $\sigma = U_{EE}E/U_E \geq 0$ and $\eta = -U_{CC}C/U_C \geq 0$.

From (B.7), $\eta\dot{C}/C = F_K - \delta$. So,

$$\dot{\mathrm{MDC}} = \mathrm{MDC}\left[(F_K - \delta) + \sigma\frac{\dot{E}}{E}\right]. \tag{B.13}$$

The second two terms on the right-hand side of Eq. (B.11) represent marginal externality cost, which we write as

$$\mathrm{MEC} = -\frac{1}{F_K-\delta}\dot{\mathrm{MDC}} + \mathrm{MDC} = \mathrm{MDC} - \left[\frac{(F_K-\delta)+\sigma\dot{E}/E}{F_K-\delta}\right]\mathrm{MDC}$$
$$= \mathrm{MDC}\left[1-\frac{(F_K-\delta)+\sigma\dot{E}/E}{F_K-\delta}\right] = -\,\mathrm{MDC}\frac{\sigma}{F_K-\delta}\frac{\dot{E}}{E} = \mathrm{MDC}\frac{\sigma}{F_K-\delta}\left\|\frac{\dot{E}}{E}\right\|.$$

We assume that $(F_k-\delta)$ is bounded away from zero before transition to the backstop.

$$F_K-\delta \geq B > 0 \quad \text{for}\, t \leq T.$$

Consider three cases:

Case 1: $\sigma = 0$ (flat), then MEC = 0;
Case 2: $\sigma/(F_K-\delta)(\dot{E}/E) = 1$, then MEC = MDC;
Case 3: $||\dot{E}/E|| < (F_K-\delta)/\sigma$, which would hold if $(F_k-\delta)$ is bounded away from zero and σ is small. Then MEC < MDC.

Appendix C

The problem is

$$\text{Max } V = \int_0^\infty [U(C_t, M_t) - U(\hat{C}, 0)]\,\mathrm{d}t$$
$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \quad K(0) = K_0,$$
$$\dot{X}_t = \begin{cases} -R_t, & t < T, \\ 0, & t \geq T, \end{cases} \qquad X_t \geq 0, \quad X(0) = X_0.$$
$$\dot{M}_t = \begin{cases} R_t - \xi M_t, & t < T, \\ -\xi M_t, & t \geq T, \end{cases}$$

The Hamiltonian for this problem is (for simplicity, the subscript t is dropped)

$$H = [U(C, M) - U(\hat{C}, 0)] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R] + \mu(R - \xi M).$$

The standard necessary conditions for this optimal control problem are

$$\frac{\partial H}{\partial C} = U_C - \lambda = 0, \tag{C.1}$$

$$\frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi + \mu = 0, \tag{C.2}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta], \tag{C.3}$$

$$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda \theta_X R, \tag{C.4}$$

$$\frac{\partial H}{\partial M} = -\dot{\mu} = U_M - \mu \xi. \tag{C.5}$$

From the first and the third conditions

$$\dot{\lambda} = \dot{U}_C, \tag{C.6}$$

and

$$\dot{\lambda} = -\lambda [F_K - \delta]. \tag{C.7}$$

Equating expressions for $\dot{\lambda}$ and rearranging yields

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta. \tag{C.8}$$

From the second necessary condition

$$\dot{\psi} - \dot{\mu} = \dot{\lambda}[F_R - \theta(X)] + \lambda[\dot{F}_R + \theta_X R]. \tag{C.9}$$

From the fourth and fifth necessary conditions

$$\dot{\psi} - \dot{\mu} = \lambda \theta_X R + U_M - \mu \xi. \tag{C.10}$$

Equating expressions on the RHS of (C.9) and (C.10), plugging in the expression for $\dot{\lambda}$, and rearranging yields

$$F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} - \frac{U_M - \mu \xi}{U_C} \frac{1}{F_K - \delta}. \tag{C.11}$$

Appendix D

For the case that applies to the model of Section 3.2, consider the following problem:

$$\begin{aligned}
\text{Max} \quad & V = \int_0^{\infty} [U(C_t, M_t) - U(\hat{C}, 0)]\, \mathrm{d}t \\
\text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t) R_t - C_t, \quad K(0) = K_0, \\
& \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\
& \dot{M}_t = \begin{cases} R_t - \xi M_t, & t < T \\ -\xi M_t, & t \geq T \end{cases} \\
& X_t \geq 0, \qquad X(0) = X_0.
\end{aligned}$$

The Hamiltonian is (for simplicity, the subscript t is dropped)

$$H = [U(C, M) - U(\hat{C}, 0)] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R] + \mu(R - \xi M).$$

The standard necessary conditions for this optimal control problem are

$$\frac{\partial H}{\partial C} = U_C - \lambda = 0, \tag{D.1}$$

$$\frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi + \mu = 0, \tag{D.2}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta], \tag{D.3}$$

$$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R, \tag{D.4}$$

$$\frac{\partial H}{\partial M} = -\dot{\mu} = U_M - \mu\xi. \tag{D.5}$$

Now, on the optimal trajectory leading to steady state, consider

$$H_t = [U(C_t, M_t) - U(\hat{C}, 0)] + \lambda_t\dot{K}_t + \psi_t\dot{X}_t + \mu_t\dot{M}_t. \tag{D.6}$$

Differentiation with respect to time:

$$\frac{dH_t}{dt} = U_C\dot{C} + U_M\dot{M} + \dot{\lambda}\dot{K} + \lambda[F_K\dot{K} + F_R\dot{R} - \delta\dot{K} - \theta_X R\dot{X} - \theta(X)\dot{R} - \dot{C}] + \dot{\psi}\dot{X} + \psi(-\dot{R}) + \dot{\mu}\dot{M} + \mu[\dot{R} - \xi\dot{M}] = [U_C - \lambda]\dot{C} + [U_M + \dot{\mu} - \mu\xi]\dot{M} + [\dot{\lambda} + \lambda(F_K - \delta)]\dot{K} + [-\lambda\theta_X R + \dot{\psi}]\dot{X} + \{\lambda[F_R - \theta(X)] - \psi + \mu\}\dot{R} = 0,$$

where the last step utilizes the five first-order conditions above.

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