Econometrics Preliminary Exam

Agricultural and Resource Economics, UC Davis

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There are **FOUR** questions. Answer each part of each question. All questions are weighted equally. Within each question, each part will receive equal weight in grading.

I. **Applied Probability** You are evaluating employees at a large firm. Each employee i performs a repeatable task multiple times and has a success rate that is determined by his/her skill and work environment. Manager A supervises two-thirds of the employees and manager B supervises the remaining third. The overall success rate for tasks in the firm is 0.95. Tasks supervised by Manager B have a success rate of 0.9.

a. What is the probability that a randomly chosen task is performed successfully by an employee supervised by Manager B?

b. Suppose a task is performed successfully. What is the probability that it was performed by an employee supervised by Manager B?

c. Are the events "supervised by manager B" and "task performed successfully" independent? Explain.

d. Based only on the information given, can you infer whether Manager A is better than Manager B? Explain your reasoning.

e. Define a variable S_{it}, which equals one if employee i completes task t successfully and zero otherwise. You have 100 observations on S_{it} for 120 employees.

   i. Propose an unbiased estimator for the success rate of employee i. Write down the variance of your estimator. State any assumptions you need.

   ii. Consider the estimator

$$\tilde{\pi}_i = 0.5 * \frac{1}{100}\sum_{t=1}^{100} S_{it} + 0.5 * \frac{1}{12,000}\sum_{i=1}^{120}\sum_{t=1}^{100} S_{it}.$$

   Derive the variance of $\tilde{\pi}_i$ and show that it is biased. State any assumptions you need.

   iii. Which estimator out of (i) and (ii) is more efficient (i.e., has smaller variance)? Which estimator do you prefer?

II. **Linear Regression** Consider the following linear regression model

$$\begin{aligned} y_i &= x_i'\beta + \varepsilon_i \\ E[x_i\varepsilon_i] &= 0 \\ E[\varepsilon_i^2|x_i] &= (x_i'\gamma)^2 \end{aligned} \tag{1}$$

where x_i, β, and γ are $k \times 1$ vectors. Assume that $\{y_i, x_i\}$ is independent across i.

a. Show that the OLS estimator of β is unbiased. State any additional assumptions you need.

b. Derive the variance-covariance matrix of the OLS estimator of β. State any additional assumptions you need.

c. Write down a consistent estimator for the variance-covariance matrix of the OLS estimator of β. State any additional assumptions you need.

d. Show that the OLS estimator of β is consistent.

e. Is the OLS estimator efficient? If not, propose an efficient estimator of β.

f. Propose a consistent estimator for γ and prove that it is consistent. State any additional assumptions you need.

III. **Nonlinear Estimation** Consider the general extremum estimation problem, where

$$\theta_0 \equiv \arg\max_{\theta \in \Theta} Q(\theta),$$

where $\theta = (\theta_1, \theta_2, \ldots, \theta_k)'$, i.e. $dim(\theta) = k$.

a. Let $Q_n(\theta)$ be the sample analogue of $Q(\theta)$. Define $\hat{\theta}$, an estimator of θ_0, and give conditions for its consistency. Explain briefly how these conditions imply consistency of $\hat{\theta}$ for θ_0.

b. Consider the problem of estimating $y_i = T(x_i'\theta) + u_i$ using nonlinear least squares, where $T(.)$ is a known function. State the population and the sample objective function, and define the estimand (the parameter to be estimated) and the estimator.

c. Give primitive conditions for the assumptions you provided in (a) for consistency of the nonlinear least squares estimator. Make sure to specify the conditions that $T(.)$ has to satisfy.

d. Now derive an expression of the sampling error, $\sqrt{n}(\hat{\theta} - \theta_0)$, and give conditions for its asymptotic normality. Explain briefly how the conditions you propose imply the asymptotic normality of the sampling error.

e. Suppose $T(z) = \Lambda(z) = e^z/(1 + e^z)$. Does $\Lambda(.)$ satisfy the conditions required for consistency and asymptotic normality you specified above?

f. Using the asymptotic distribution of the NLS estimator you provided in (d), give an estimator of the asymptotic variance, $Avar(\sqrt{n}(\hat{\theta} - \theta_0))$.

g. Propose a Wald test, a likelihood ratio test and a Lagrange Multiplier test for the hypothesis that all elements of θ are equal to each other, i.e. $\theta_1 = \theta_2 = ... = \theta_k$.

IV. **Linear Panel Data Models**.

a. In this problem, we consider the linear fixed effects estimator.

   i. For the linear model, where $y_{it} = x_{it}'\beta + \alpha_i + u_{it}$, define the linear fixed effects (FE) estimator and give conditions under which it is consistent and asymptotically normal. Show how the conditions are sufficient for the results.

   ii. Explain intuitively what type of variation in the data is used to estimate β using FE.

   iii. Show that for $T = 2$, the FE and first-difference estimators are identical.

   iv. Assuming that u_{it} is homoskedastic and serially uncorrelated, write down the asymptotic variance of the FE estimator. Is the FE estimator asymptotically efficient?

b. Consider the following panel of households in different counties in the United States that we observe over time. We observe $i = 1, \ldots, n$ households in each county $c = 1, \ldots, C$ over $t = 1, \ldots, T$ years. Let $y_{ict} = x_{ict}'\beta + \lambda_i + \gamma_t + u_{ict}$, where $dim(x_{ict}) = dim(\beta) = k$. Let $x_{ic} = (x_{ic1}, x_{ic2}, \ldots, x_{icT})$. $E[u_{ict}|x_{ic}, \lambda_i] = 0$, but $E[\lambda_i|x_{ic}] \neq 0$.

   i. Propose a transformation of the above model that allows you to estimate β consistently. Define the estimator in question.

   ii. Compare the variation in the data used in this problem to estimate β relative to your answer in (a.ii).

   iii. Suppose $n \to \infty$, show the consistency of the estimator you propose in (b.i). Specify any condition required for the result.

   iv. Now suppose $y_{ict} = x_{ict}'\beta + \lambda_i + \gamma_{ct} + u_{ict}$, how would your answer in (b.i) change?

ARE 240B Reference Sheet

Notation. θ_0, Θ, y_i, x_i, $s(y_i, x_i; \theta)$ and $H(y_i, x_i; \theta)$ pertain to the objects defined in the 240B lecture notes.

Assumption ULLN 1 $\sup_{\theta\in\Theta} |\sum_{i=1}^n f(y_i, x_i; \theta)/n - E[f(y_i, x_i; \theta)]| \xrightarrow{p} 0$, if the following conditions hold,

(i) *(i.i.d.)* $\{y_i, x_i\}_{i=1}^n$ is an i.i.d. sequence of random variables;

(ii) *(Compactness)* Θ is compact;

(iii) *(Continuity)* $f(y_i, x_i; \theta)$ is continuous in θ for all $(y_i, x_i')'$;

(iv) *(Measurability)* $f(y_i, x_i; \theta)$ is measurable in $(y_i, x_i')'$ for all $\theta \in \Theta$;

(v) *(Dominance)* There exists a dominating function $d(y_i, x_i)$ such that $|f(y_i, x_i; \theta)| \leqslant d(y_i, x_i)$ for all $\theta \in \Theta$ and $E[d(y_i, x_i)] < \infty$.

Assumption ULLN 2 $\sup_{\theta\in\Theta} |\sum_{i=1}^n f(y_i, x_i; \theta)/n - E[f(y_i, x_i; \theta)]| \xrightarrow{p} 0$, if the following conditions hold,

(i) (*Law of Large Numbers*) $\{y_i, x_i\}$ is i.i.d., and $E[m(y_i, x_i; \theta)] < \infty$ for all $\theta \in \Theta$, which implies $\sum_{i=1}^n m(y_i, x_i; \theta)/n \xrightarrow{p} E[f(y_i, x_i; \theta)]$.

(ii) (*Compactness of* Θ) Θ is in a compact subset of $\mathbb{R}^k$.

(iii) *(Measurability in $(y_i, x_i')'$)* $f(y_i, x_i; \theta)$ is measurable in $(y_i, x_i')'$ for all $\theta \in \Theta$.

(iv) (*Lipschitz Continuity*) For all $\theta, \theta' \in \Theta$, there exists $g(y_i, x_i)$, such that $|f(y_i, x_i; \theta) - f(y_i, x_i; \theta')| \leqslant g(y_i, x_i)\|\theta - \theta'\|$, for some norm $\|.\|$, and $E[g(y_i, x_i)] < \infty$.

Formula for the score statistic

$$S \equiv \left(\frac{1}{\sqrt{n}}\sum_{i=1}^n s_i(\hat\theta_R)\right)' A_{nR}^{-1} C_{nR}' \left\{\widehat{Avar}\left(C_{nR} A_{nR}^{-1} \sum_{i=1}^n s_i(\hat\theta_R)/\sqrt{n}\right)\right\}^{-1} C_{nR} A_{nR}^{-1} \left(\frac{1}{\sqrt{n}}\sum_{i=1}^n s_i(\hat\theta_R)\right)$$

GMM Expression for the Sampling Error

$$\sqrt{n}(\hat\theta - \theta_0) = -\left(\mathcal{H}_0' W \frac{1}{n}\sum_{i=1}^n \left.\frac{\partial h(y_i, w_i; \theta)}{\partial \theta}\right|_{\theta=\theta*}\right)^{-1} \mathcal{H}_0' W \frac{1}{\sqrt{n}} \sum_{i=1}^n h(y_i, w_i; \theta_0) + o_p(1)$$

where

$$\mathcal{H}_0 = E\left[\left.\frac{\partial h(y_i, w_i; \theta)}{\partial \theta'}\right|_{\theta=\theta_0}\right]$$

Normal Distribution

Let $Z \sim N(0,1)$, then its density is given by

$$\phi(z) = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{1}{2}z^2\right\} \quad (1)$$

Its cdf is denoted by $\Phi(z)$.

Let $X \sim N(\mu, \sigma)$, then its density is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{1}{2}(x-\mu)^2/\sigma^2\} = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right. \quad (2)$$

and its cdf $F(x) = \Phi((x-\mu)/\sigma)$.