University of California, Davis
Department of Economics
Microeconomics

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Time: 5 hours
Reading Time: 20 minutes

PRELIMINARY EXAMINATION FOR THE Ph.D. DEGREE

Please answer **any three** of the following four questions
[If you answer all four questions please indicate which three you want to be graded]

QUESTION 1

There are two customer service workers; worker i ($i \in \{1,2\}$) chooses an effort level $e_i \in [0,\infty)$ to exert in helping a customer. The cost of effort is $c_1(e_1) = \frac{(e_1)^2}{15}$ for worker 1 and $c_2(e_2) = \frac{(e_2)^2}{10}$ for worker 2. The customer's satisfaction level equals the total effort that the workers exert, up to a maximum satisfaction level of 10, that is, it is equal to $\min\{e_1 + e_2, 10\}$. Each worker's payoff is the difference between the customer's satisfaction level and his/her own effort cost.

Consider first the **game *G*** in which the two workers choose their efforts simultaneously.

(a) Write down each worker's payoff function in *G*.

(b) Describe each worker's best response (or reaction) function in *G*.

(c) Draw the two best response functions in the same diagram and find all the pure-strategy Nash equilibria of *G*.

Now consider sequential versions of the interaction.

In **game Γ_2**, first Player 1 (= worker 1) selects an effort level, then Player 2 (= worker 2) − after observing Player 1's choice − chooses her own effort level.

(d) Completely describe all the pure-strategy subgame perfect equilibria of Γ_2.

In **game Γ_3**, the sequence of events starts with those in Γ_2, but after Player 2 moves, Player 1 observes Player 2's choice and decides whether to exert some additional effort $\delta_1 \in [0,\infty)$, so that his own total effort is $e_1 + \delta_1$.

(e) **(e.1)** What is a pure strategy for Player 1 in Γ_3?

(e.2) What is a pure strategy for Player 2?
[You are free to state your answers in words, rather than in mathematical notation; but, in either case, your answers should be explicit and complete.]

(e.3) Give an example of a pure strategy for Player 1 and a pure strategy for Player 2 in the special case where each effort level must be chosen from the set {3,4}, that is, when $e_1, e_2, \delta_1 \in \{3,4\}$.

(f) Completely describe all pure strategy subgame perfect equilibria of Γ_3 when $e_1, e_2 \in [0,10]$ (instead of $[0,\infty)$) and $\delta_1 \in [0, 10 - e_1]$ (instead of $[0,\infty)$).

QUESTION 2

Consider a **simultaneous** two-player **second-price** auction concerning a single indivisible good. The game-frame is as follows: $S_1 = S_2 = B$ where $B = \{p_1, p_2, ..., p_m\}$ is a *finite* subset of $[0,\infty)$ with $p_k < p_{k+1}$ for every $k \in \{1,...,m-1\}$, the set of outcomes is the set of pairs (i, p) where $i \in \{1,2\}$ is the winner of the auction and $p \in B$ is the price that winner has to pay and the outcome function is as follows (b_i denotes the bid of player *i*):

$f(b_1,b_2) = \begin{cases} (1,b_2) & \text{if } b_1 \geq b_2 \\ (2,b_1) & \text{otherwise} \end{cases}$. Let v_i be the value of the object to Player *i* (that is, Player *i* views getting the object as equivalent to getting \$ v_i). We shall consider three kinds of preferences. We state them in terms of Player 1, but the same definitions apply to Player 2. The following apply to all three preferences (this is the "selfish" part):

for every $p < v_1$ and for every p', $(1, p) \succ_1 (2, p')$;

for every p and p', $(1, p) \succ_1 (1, p')$ if and only if $p < p'$.

- Player 1 is *selfish and **uncaring*** if, in addition, his preferences are as follows:
  for every p and p', $(2, p) \sim_1 (2, p')$;
  for every p, $(2, p) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.
- Player 1 is *selfish and **benevolent*** if, in addition, his preferences are as follows:
  for every p and p', $(2, p) \succ_1 (2, p')$ if and only if $p < p'$;
  $(2, p_m) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.
- Player 1 is *selfish and **spiteful*** if his preferences are as follows:
  for every p and p', $(2, p) \succ_1 (2, p')$ if and only if $p > p'$;
  $(2, p_1) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.

In what follows assume that $m > 3$, $v_1, v_2 \in B$, $p_1 < v_1 \leq p_m$ and $p_1 < v_2 \leq p_m$.

(a) Suppose that Player 1 is selfish and ***uncaring***. Does she have a weakly or strictly dominant strategy? If your answer is Yes, state whether it is weak or strict dominance; if your answer is No, prove it.

(b) Suppose that Player 1 is selfish and ***benevolent***. Is bidding v_1 a weakly dominant strategy? Prove your claim.

(c) Suppose that Player 1 is selfish and ***spiteful***. Is bidding v_1 a weakly dominant strategy? Prove your claim.

For questions (d), (e) and (f) assume that $B = \{1,2,3,4,5\}$, $v_1 = 3$ and $v_2 = 5$

(d) Suppose that both players are selfish and ***uncaring***. Find all the pure-strategy Nash equilibria.

(e) Suppose that both players are selfish and ***benevolent***. Find all the pure-strategy Nash equilibria.

(f) Suppose that both players are selfish and ***spiteful***. Find all the pure-strategy Nash equilibria.

Now suppose that we have a situation of incomplete information. As a matter of fact, both players are selfish and benevolent (B), but this is *not* common knowledge. The following is common knowledge:

1. Each player knows her own preferences.
2. Player 1 is either selfish and benevolent (*B*) or selfish and uncaring (*U*). If Player 1 is *B* then she attaches probability 1 to Player 2 being *B*, while if Player 1 is *U* then she attaches probability $\frac{1}{4}$ to Player 2 being *B* and probability $\frac{3}{4}$ to Player 2 being *S* (= selfish and spiteful).
3. Player 2 is either *B* or *S*. If Player 2 is *B* then she attaches probability $\frac{1}{6}$ to Player 1 being *B* and probability $\frac{5}{6}$ to Player 1 being *U*, while if Player 2 is *S* then she attaches probability 1 to Player 1 being *U*.

(g) Use an interactive knowledge-belief structure to represent this situation.

(h) For this question assume that $B = \{p_1, p_2\}$, $v_1 = v_2 = p_2$ and the auction is *not* simultaneous: Player 1 bids first and Player 2 bids second after having been informed of Player 1's bid.

(h.1) Apply the Harsanyi transformation to the incomplete-information situation of part (g), without writing payoffs but writing the outcomes.

(h.2) Suppose that you know that the players have von Neumann-Morgenstern preferences, but all you know is that their von Neumann-Morgenstern preferences induce the ordinal rankings given above. Is it possible to find a pure-strategy weak sequential equilibrium of the game of part (h.1)? If you claim that it is not possible because not enough information is given, state what extra information you would need, otherwise find a pure-strategy weak sequential equilibrium and prove that it is indeed a weak sequential equilibrium.

Question 3

3. Consider an economy with two agents (agents a and b), two goods – labor/leisure and a consumption good – and two firms. Both firms transform labor into the consumption good. Firm 1, with production function $y_1 = f(L_1)$, belongs to agent a, and firm 2, with production function $y_2 = g(L_2)$, belongs to agent b. (L denotes a quantity of labor and y a quantity of consumption good produced). Each agent has 3 units of time which can be used as leisure or sold as labor. Firm 2 uses a technique of production which creates a negative externality z, a chemical which has an adverse effect on the consumers and whose quantity is proportional to the level of production: $z = y_2$. The utility and production functions are as follows

$$u_i(\ell_i, x_i, z) = \sqrt{\ell_i x_i} - \frac{1}{8}z, \quad i = a, b, \quad f(L_1) = L_1, \quad g(L_2) = 2\sqrt{L_2}$$

where ℓ denotes leisure and x the quantity of consumption good.

(a) Derive the competitive equilibrium of this economy, i.e. equilibrium prices, production and consumption plans. [Restrict attention to allocations where both firms produce a positive amount.]

(b) To show that the equilibrium is not Pareto optimal consider the following marginal change from the equilibrium allocation: add $\varepsilon/2$ of leisure to each agent and decrease the labor used by firm 2 by ε. Decrease equally the consumption of the consumption good of the two agents. Do not change anything to firm 1's plan. Compute the marginal change in utility for each agent and show that both agents are better off. Explain. [It may be easier to keep the partial derivative notation for estimating the change in utilities and then evaluate the values of these derivatives at the equilibrium.]

(c) Assume that the government fixes a limit z^* on the amount of pollution emitted by firm 2. Solve for the 'regulated competitive equilibrium' as a function of z^* . [Assume that z^* is smaller than the value found in (a)].

(d) To find the optimal z^* come back to your calculations in (b) and find the value of z^* such that the change considered in that question (or the reverse when $\varepsilon < 0$) cannot improve the utilities of the agents.

Question 4

Ali, the crocodile, lives in the dark water of Putah Creek in the Arboretum just beside the UC Davis campus. To the delight of the ducks there, Ali is a vegetarian. Ali earns income from being employed by UC Davis as a life guard at Putah Creek. His task is to rescue students who fall into the water from time to time. He is especially busy around prelim exams when more students than usual jump into water out of despair. When being rescued by Ali, usually their mood lights up quickly in anticipation of a cool selfie with a crocodile that can be shared on Facebook. By the time they relapse upon realizing that the water damaged their cell phone beyond repair, the authorities have arrived to help (with their cell phones). Anyway, let's not digress further. While we certainly appreciate Ali's work, our academic interest is focused on his consumption behavior. Being a vegetarian, his main diet consists of almonds. They have the inconvenient feature of getting stuck between his 72 teeth. So he is also a rather heavy consumer of toothpicks. Finally there are his frequent gifts he purchases for Mathilda. Mathilda! She is the duck of the Arboretum who waddles most elegantly with her petite legs by delightfully moving well-shaped hips. He is really in love with her; at a platonic level of course so that nobody gets physically hurt.

We write $x_a, x_t, x_g \geq 0$ for the amounts of his consumption of almonds, toothpicks, and gifts, respectively. We assume for simplicity that these goods are infinitesimally divisible. Let $x = (x_a, x_t, x_g)$. We assume that his utility function is given by $u(x_a, x_t, x_g) = x_a^\alpha x_t^\theta x_g^{1-\alpha-\theta}$ with $\alpha, \theta \in (0,1)$ and $\alpha + \theta < 1$. His income or wealth is denoted by $w > 0$. Finally, we denote by $p_a, p_t, p_g > 0$ the prices of almonds, toothpicks, and gifts, respectively, and $p = (p_a, p_t, p_g)$.

a. Use the Kuhn-Tucker approach to derive step-by-step the Walrasian demand function $x(p, w)$. Verify also second-order conditions.

b. Verify that the demand function is homogenous of degree zero and satisfies Walras' Law.

c. To be honest, we do not really know whether Ali has the Cobb-Douglas utility function stated above. Would Ali want to differently substitute a marginal amount of almonds for some toothpicks when having a differentiable utility function different from the one above and optimally demanding positive amounts of all goods? Explain.

d. You would expect that the more almonds Ali eats, the more they get stuck in his teeth and the more toothpicks he purchases. In light of such considerations, does it make sense to assume Ali has the utility function above?

e. Suppose the university would slightly raise Ali's income. (Assuming at most small changes of income is a very realistic assumption at UC Davis.) Can we learn from the Lagrange approach by how much his utility would change?

f. Derive Ali's indirect utility function (denote it by $v(p, w)$). Simplify.

g. Can we also answer part e using Ali's indirect utility function? Briefly explain.

h. Verify that Ali satisfies Roy's identity with respect to almonds.

i. When Professor Schipper interviews Ali about how exactly he arrives at his optimal consumption bundle, Ali expresses ignorance about maximizing utility subject to his budget constraint. Instead, he seems to minimize his expenditure on consumption such that he reaches a certain level of utility. A smart undergraduate student walks by and claims that this is clear evidence against the assumption of utility maximization in economics. Since Professor Schipper hates Cobb-Douglas utility functions and boring calculations, he sends the student to you so that you can show him how expenditure minimization works. Again, use the Kuhn-Tucker approach to derive the Hicksian demand function.

j. Derive the expenditure function. Show that the expenditure function is homogeneous of degree 1 in prices, strictly increasing in $\bar{u}$ as well as nondecreasing and concave in the price of each good.

k. Professor Schipper cannot compute the Hicksian demand using Kuhn-Tucker without a coffee. Unfortunately, Ali has no coffee to offer. Yet, he could offer Professor Schipper his expenditure function. Is there a way to quickly calculate the Hicksian demand from the expenditure function without coffee?

ℓ. Verify the (own price) Slutsky equation for the example of almonds.

m. Which term in the (own price) Slutsky equation refers to the substitution effect? As mentioned previously, we don't really know whether Ali has a Cobb-Douglas utility function. Suppose Ali has a continuous utility function representing locally nonsatiated preferences and that his Hicksian demand function is indeed a function (rather than a correspondence). Would a change in prices still have qualitatively the same effect on Hicksian demand as when he has the above Cobb-Douglas utility function? If yes, provide a short proof. If not, argue why not.

n. Because of the drought, the price of almonds changes from p_a^0 to p_a^1. UC Davis is committed to keep Ali as well off as before the price change. The newly hired Senior Vice Provost for Crocodile Welfare turns to Professor Schipper for advice on the exact amount to be deducted from the budget of the university and paid to Ali as compensation for the price change. Unfortunately, Professor Schipper is so immersed in exciting new research that he is extremely slow in answering his email. Luckily, the Senior Vice Provost for Crocodile Welfare spends most of his time in the hammocks on the quad, where he meets you studying for the prelims. Help him calculate the amount.