Optimal Licensing for Public Intellectual Property:
Theory and Application to Plant Variety Patents

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Abstract

In the United States, public universities may choose to license a plant variety to a limited number of producers (an exclusive license) or to an unlimited number of producers (an open license). This choice has implications for the quantity and distribution of total benefits from the variety. Universities have traditionally released new apple varieties under open licenses, but several universities have now begun exploring or implementing exclusive licensing. In this paper, we consider the choice faced by a public university when licensing a plant variety patent, with a focus on apples. Our work differs from most past studies on patent licensing because we allow licensees to determine the signal of product quality through a trademark and we consider welfare objectives for a public university that differ from simple maximization of patent income. In this context, we compare monopoly licensing and two oligopoly licensing scenarios, and solve for the optimal choice of licensing fees for the university. Using numerical simulations, we find that consumer surplus and social welfare may be higher under exclusive licensing, particularly if consumers are relatively responsive to investment in the trademark but relatively insensitive to price. However, exclusive licenses may create distributional concerns among producers. Furthermore, different objective functions of the university can imply different optimal outcomes for both the number of licensees and the licensing fees. Although we focus on apples, this model and its results could apply in a variety of settings.

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Introduction

The arrangements for funding and managing agricultural R&D in the United States have been evolving to accommodate changes in the economy broadly, in science itself, and in the institutional framework in which science is conducted (e.g., see Alston et al. 2010). Important elements of these changes include: (a) declining public support for using general government revenue to fund public agricultural R&D, (b) the rise of modern biotechnology, (c) the emergence of stronger intellectual property (IP) rights over biological innovations following *Diamond v. Chakrabarty* in 1980, and (d) the Bayh-Dole Act of 1980, which allowed universities to patent results from research financed with federal funds (e.g., Graff et al. 2003). In an ever-tighter research funding environment, the option of using technology patents to augment traditional research funding sources has contributed to shifts in the approaches taken to financing and managing agricultural R&D in public universities.

Along with the new funding opportunities have come some new challenges for universities, producer groups, research scientists, and others, as they engage in public-private partnerships in the production of patented technologies. Leaders in public universities continue to grapple with unresolved issues about the appropriate management of the resulting intellectual property in terms of access to the technology, pricing, and sharing of royalties with researchers. In this paper we explore some of these issues in the context of patented horticultural varieties. The results may be applicable more generally, but some aspects of the model are specific to horticultural crops.

Public universities in the United States have long been key players in producing new horticultural varieties through traditional breeding and, more recently, through genetic engineering. Alston et al. (2010) reported that 34.6% of all U.S. spending on agricultural R&D in 2007 was at universities, the majority of it through state agricultural experiment stations at land grant universities. Examples of horticultural research programs at land grant universities include the apple breeding programs at the University of Minnesota, Washington State University, and Cornell University, the strawberry breeding program at the University of California, Davis, and the blueberry breeding program at North Carolina State University. In this paper, we consider the question faced by

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1 Land grant universities are mostly public. One exception is Cornell University, the land grant university for the state of New York. However, given its land grant status, Cornell shares the public-spirited mission of public universities in the area of agricultural research.

2 Alston and Pardey (2008) documented that about one-sixth of public agricultural research spending in the U.S. is devoted to specialty crops, including horticultural breeding programs. They also reported that over the years 1975–2004, a roughly constant share of total public agricultural research spending was devoted to specialty crops,
public universities in determining how to license new horticultural varieties.

The broad mission of university horticultural breeding programs is to provide benefits to consumers and producers of the varieties they develop, a relatively straightforward purpose in the bygone era that did not involve private-sector partners or IP considerations. This broad mission implies a different set of objectives to consider beyond the simple profit maximization that is often applied in models of private IP management. However, in the current funding environment, other considerations are pertinent in defining the optimal licensing arrangements in addition to the broad payoffs to society. University administrators might seek to recoup the costs of the research program or to earn revenue for use elsewhere at the institution (typically significant imposts are applied to any extramural research funds). Breeders employed by the university might demand a share of any royalties they generate as additional personal income or to help cover the costs of the lab (and in a competitive market for scientific talent, some such sharing is typical). Finally, industry partners who contribute to research support might insist on some say over the pricing of the resulting technology.

This complicated set of factors influencing the university’s choices has implications for licensing arrangements. Land grant universities have traditionally offered open patent licenses. Under this system, anyone who is willing to pay the required patent royalties and/or fees is free to license the variety. However, some universities have moved to or are considering exclusive licensing. With exclusive licensing of a variety, the number of licensees is limited and controlled by the university, creating an oligopoly or monopoly on the production of that variety. Some universities have claimed that moving to exclusive licensing could be beneficial to consumers if restricting the number of licensees incentivizes licensees to produce a higher quality product than they would under open licensing. A 2006 report released by the University of Minnesota said of the SweeTango apple (a trademark name for the Minneiska cultivar): “The University believes exclusive licensing of MN

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3Indeed, in a survey of technology transfer offices (TTOs) conducted by Belenzon and Schankerman (2009), local development objectives were reported to be either “very important” or “relatively important” by 82% of the public universities surveyed. Although this objective is just one of the objectives other than revenue maximization that we discuss, the results of this survey do support our assertion that public universities have a different set of objectives than the standard profit-maximizing licensor.

4Jensen and Thurby (2001) found that the use of royalties is important for inducing inventor effort. Along these same lines, Lach and Schankerman (2008) found that universities offering higher royalty shares to inventors garner more licensing income.

5Usually the variety is licensed to a nursery which in turn licenses openly to growers.
1914 will serve the interests of the consumer, Minnesota apple growers and the University’s breeding program” (UMN 2006). The ability to benefit consumers through an increase in quality rests on producers being able to credibly signal quality and consumers being able to receive this signal. For some horticultural varieties, particularly apples, this signaling is achieved through the use of trademarks and an associated marketing program. Notable examples of branded apple varieties include Pink Lady and Jazz. Assuming consumers associate the trademark with a higher quality product, they will be willing to pay more for this product than for a similar product without a trademark and will benefit from the higher quality they associate with the trademark, while producers of the trademarked variety will benefit from the price premium.

These outcomes would appear to align with public universities seeking to provide benefits to both producers and consumers. However, there has been considerable pushback from excluded growers, and even by some university plant breeders, about the distributional consequences for producers, with these individuals claiming that universities who adopt this policy are not fulfilling their obligation to provide public benefits (Baier 2009, Lehnert 2010, Cahoon 2007). Hence, it remains unclear what is the optimal licensing structure for patented horticultural technologies developed by public universities—any innovation is likely to result in some stakeholders being made worse off, regardless of the intellectual property and pricing arrangements. The harder question is what the university should do in light of its multidimensional objective function, constrained by the realities of mixed public-cum-private elements in financing and managing public agricultural R&D that yields patented technologies.

In this paper we seek a partial answer to this question. We develop a two-stage optimization model that considers the optimization problem faced by the grower/licensee and the optimization problem of the university. Although the results may be applicable more generally, our application is specifically oriented to apples. Apples are unique in that they are identified by varietal names (and sometimes a trademark) throughout the marketing chain. We assume each licensee chooses the (1) investment in the trademark and (2) quantity of apples to produce. The university chooses

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6 We refer to the differentiating characteristic of the trademarked variety as “quality,” but we could just as easily think about some other characteristic of product differentiation for which some consumers are willing to pay a premium—in apples this might be sweetness or crispness.

7 We take a very general approach in considering investment in quality and trademarking (which we will call investment in the trademark from here on). We assume a portion of the investment will go to quality improvement and a portion will go to signaling that quality. We also assume any investment in trademarking or marketing is a credible signal of quality to consumers. In this way our approach is similar to the treating advertising as a
the type of royalty and/or fee (per-unit royalty, fixed fee or both), the number of licensees, and
the monetary measure of benefits to use in its objective function, taking the growers’ optimal
output and quality investment as given. The university can choose to maximize its own revenue,
producer profits, consumer welfare or social welfare. Using simulations, we find that, given our
model, consumers will be better off under exclusive licensing, particularly if they are relatively
sensitive to trademark signals and are not very sensitive to price. We also find that industry
profits (and thus social welfare) will be higher under exclusive licensing. Finally, we find that the
optimal licensee fee and number of licensees depend on the specification of the university’s objective
function and the implicit weights given to the welfare of different groups in society. However, some
distributional impacts that the university may want to consider (e.g. among growers who do or do
not have access to a variety) are not resolved by our model.

Our work differs from previous studies in that we focus on the connection between the number
of licensees and the university’s objectives. Those studies that have looked at the university’s
objectives in patent licensing (e.g., Jensen and Thurby (2001) and Belenzon and Schankerman,
(2008)) have focused on how these objectives relate to internal incentives for researchers, abstracting
from the complexities of the downstream licensees. Furthermore, previous studies in the patent
literature have taken the quality of the variety as given. As the primary claim about the benefits
from exclusive licensing centers on the ability of producers to affect the product quality perceived
by consumers, we assume instead that the perceived quality of the new variety is endogenous and
chosen by the licensee. The licensee affects the perceived quality through investment in quality
and a trademark, which can be seen as analogous to investing in branded product promotion more
generally.

8In the literature, two royalty types are generally considered—per unit and ad valorem. Patent royalties for apples
are generally charged per tree and trademark royalties are generally charged per unit weight of apples. In light of
these practices, either of which can be modeled as per-unit royalties if we make simplifying assumptions about yield,
we have chosen to leave out ad valorem royalties for ease of analysis.

9Examples include Kamien et al. (1988) and Stamatopoulos and Tauman (2008). Kamien et al. (1988) consider
a fixed fee licensing structure and identify circumstances in which an inventor will license to producers of an inferior
incumbent product. The inventor will license to a single licensee if the new product is “drastically” superior in the
sense that the monopoly price of the new product is below the marginal cost of the incumbent product. Stamatopoulos
and Tauman (2008) consider a quality-improving innovation with a logit demand framework and an auction, royalty,
or combined auction/royalty licensing fee structure. They find that when all consumers purchase the new product,
the innovator licenses to a duopoly, because this allows the innovator to extract all of the potential consumer surplus,
which is not possible with a monopoly.
Empirical context

The primary motivation for this paper comes from recent considerations and moves by apple breeding programs at public universities in the United States to change the way they license apple varieties. Unlike most horticultural crops, the names of apple varieties are known to consumers and used in marketing. Empirical evidence indicates that consumers are willing to pay more for certain varieties than others. Consumers are willing to pay a premium for trademarked apples such as Honeycrisp and SweeTango (Yue and Tong 2011).

Several interesting examples in apples highlight the different choices universities face when patenting varieties and the advantages and disadvantages associated with these choices. Cripps Pink apples were developed not at a land grant university, but by a public agency, the Western Australian Department of Agriculture (now the Department of Agriculture and Food, or DAFWA) in 1973 (Cripps 1992). These apples are known to most of us as Pink Lady, their trademarked name. To use the trademark, producers must meet certain quality standards (APAL 2014). The success of the Pink Lady international trademark and associated marketing program in credibly signaling quality to consumers resulted in 72% of the Cripps Pink apples produced around the globe (excluding China) being marketed as Pink Lady in 2012 (Warner 2012). The Pink Lady apple has thus become the poster child for managed varieties.

At the other end of the spectrum we have the Honeycrisp apple. Patented by the University of Minnesota in 1988, this variety was openly licensed by the university in the United States until its patent expired in 2008 (although the university still maintains enforceable patent rights in other regions of the world) (UMN 2014). At the same time, University of Minnesota officials reported being unsure if the name was even trademarked (Olson 2007). U.S. Patent and Trademark Office records show that the university trademarked the variety name in 1991 and abandoned the trademark in the next year (USPTO 2014). Thus, whereas Honeycrisp earned the University of Minnesota a one-time per-tree royalty, DAFWA earned both both a one-time per-tree royalty and a stream of per-unit output royalties from the use of the trademark, which will continue for as long as the output is marketed under the trademark—even after the patent has expired.

Recognizing the possible benefits from both exclusive licensing and trademarking, in light of the success of managed varieties such as Pink Lady, the University of Minnesota took a very
different approach compared with Honeycrisp when it trademarked the SweeTango apple in 2005. The university licensed the Minneiska cultivar exclusively to a single Minnesota grower, which in turn formed a cooperative of a select group of fewer than 50 large growers in the United States and eastern Canada to produce the variety. Although other Minnesota growers were also allowed to grow the variety, they were only permitted to grow limited quantities, sparking considerable controversy among Minnesota growers. Despite this controversy, the University of Minnesota licensed a second variety exclusively in 2014. This as-yet-unnamed variety has been licensed to a grower in Washington state and is expected to hit the market in 2017 (Produce News 2014). The differences in licensing arrangements for these three apple varieties and their marketing highlight the different options available to universities, and the possible benefits from credibly signaling quality to consumers. Both Cripps Pink and Minneiska apples must meet certain quality standards to be marketed under their trademarked names, creating a higher quality, branded product. These cases highlight why allowing the quality signal to be endogenous may provide a different result than the more common assumption of exogenous quality.

Beyond apples, another interesting case to consider is that of the University of California, Davis (UC) strawberry licensing program. Although strawberries are not marketed to consumers under varietal names, the strawberry program highlights several interesting and important aspects that we consider in our analysis. The California Strawberry Commission (CSC) helps to fund the strawberry breeding program in California. The CSC is funded by a mandatory levy (or checkoff) on sales of strawberries, which means all members of the strawberry industry in California contribute in ways that have policy implications. The breeding program has produced a number of successful varieties and has been a highly lucrative program for the university; four strawberry varieties were among the top 25 revenue-generating inventions in the UC system in 2013. Like many other public universities, the university administration splits its revenue with the breeder (UC 2014a).

10 Under the original agreement, Minnesota producers were permitted to grow up to 1,000 trees of the cultivar under the brand name, but they were permitted to sell their output only through direct sales. Minnesota growers challenged this limitation in Minnesota district court, and although the judge in the case deemed the arrangement to be legal in his 2011 ruling, the limit on the number of trees was increased to 2,000 per grower with a state cap of 100,000 trees. The limit will increase to 3,000 trees per grower and 150,000 trees in the state in 2017 (Lehnert 2011).
11 This situation also highlights the importance of determining whose welfare the university is to consider in its objective function. Interestingly, although the University of Minnesota has a major apple breeding program, the state in fact ranks 19th out of the 29 apple growing states in the United States in terms of quantity produced in pounds (NASS 2014). Washington ranks first in terms of production, producing more than 350 times as many pounds of apples as Minnesota in 2014. This suggests that the University of Minnesota is considering the “public” to include more than just Minnesotans.
Since 1997, the policy has been to give the breeder 35% of annual income net of direct expenses of administering the patent (UC 1997). This policy is relevant to our work because private individuals are not bound by the same mission as the university. The actions of the breeder can potentially skew the university’s outcome away from social benefit and toward revenue maximization. In a controversial lawsuit that is still pending, the CSC is suing the UC to ensure the strawberry breeding program will be sustained. This followed an announcement by the current principal breeder that he planned to leave the UC, taking his breeding stock with him (which will also be retained by the UC) and start a private company. The CSC and the UC are currently in negotiations, but this episode highlights the politicization of the university’s decision-making environment.

These situations illustrate the complex optimization problem of the technology transfer office at a land grant university. Although the objective of the university is ostensibly one of public benefit, the university must consider many factors in interpreting that purpose. First, phrases like “public use and benefit,” (from the UC Technology Transfer Office mission statement) are ambiguous (UC 2014b). Consumers, producers and university employees are all part of the “public,” yet their desires may well be at odds, and it is not clear what weights are given to different groups of the public in the same state, let alone different states. Furthermore, technology transfer offices are just one small component of a larger university bureaucracy and must answer to university administrators with potentially different goals. Finally, individual inventors (in this case, breeders) affect the timing and quality of inventions, which affects the timing and magnitude of the flow of benefits to consumers and producers. In our model, we thus consider several interpretations of these university objectives and find the optimal licensing arrangement in each of several cases.

**Model and analytical results**

As highlighted above, the university’s decision-making process entails a complex balance between the often conflicting goals of different stakeholders. One way to represent this complexity is to model the decisions of the university as though it seeks to maximize a weighted welfare function, with different interest groups having different weights. We consider four special cases, where the university maximizes patent revenue, consumer surplus, or producer profits, or the simple sum of

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12 In their survey of TTOs, Jensen and Thurby (2001) found that managers of TTOs found themselves trying to balance the objectives of both inventors and the university administration.
Given these different objectives, we consider the university’s optimal choices for the per-unit royalty rate, fixed fee and number of licensees. In order to optimize, the university must take into account how producers will respond to its choices. First we present models of producers’ profit-maximization problems given an exogenous per-unit royalty rate and fixed fee under three different sets of assumptions about the licensing structure. We then present our model of the university’s optimization problem in these three scenarios.

We assume a single variety of apple patented by a university. The inverse demand for this variety is given by: \( D^{-1} = P(Q, A) \), where \( A \) is the investment in the trademark. This cost could include expenditure on quality improvement, such as precision equipment or improved storage, as well as legal fees, advertising and promotion, grading and packaging or other aspects of marketing and brand creation. It is a lump-sum cost in the sense that it does not depend on \( Q \). We assume complete information—in other words, demand and costs are known.\(^{14}\) We also assume that greater investment in a trademark signals higher quality of the output to all consumers. The university charges a per-unit royalty, \( r \), and a fixed fee, \( F \), for the use of the patent for this variety.\(^{15}\) For the purposes of our simulations, we assume a demand function of the constant elasticity form, \( Q = e^{\beta P^\eta} A^\alpha \), where \( \eta < -1 \), \( 0 < \alpha < 1 \) and \( \beta > 0 \) is a scaling parameter.\(^{16,17}\)

\(^{13}\) In reality, universities implicitly choose some weighted average of these elements of social welfare, reflecting the political, institutional and economic constraints implied by the juxtaposition of the public purpose with private patents. However, looking at these simpler, special cases can provide us with results that yield intuition for other more complex situations.

\(^{14}\) Bousquet et al. (1998) consider the implications of incomplete information.

\(^{15}\) In the context of apples, one might consider the fixed fee to be a minimum per-tree royalty or another type of one-time fee. Although it appears that most apple royalties are charged as per-tree royalties, this is equivalent to a per-apple royalty under the assumption that all trees of the same variety have equal yield and lifespan.

\(^{16}\) Although our setting is one in which we consider a new variety, for simplicity in our model we explicitly consider only the price and quality for that variety and abstract away from other factors that might affect demand for the new variety. This could include prices and qualities of other varieties of apples or other related products such as pears or bananas. In our analysis, all of these other variables are treated as constant, and subsumed in the intercept parameter, \( \beta \). In a more complete model, the prices and qualities of competing products might be jointly endogenous. Some previous studies (e.g. Sen and Tauman 2007) differentiate between “drastic” and “non-drastic” innovations to better understand how a new variety fits into the existing market. While this is an important aspect of understanding the introduction of new varieties, it is not something we consider in this paper.

\(^{17}\) This functional form is common in the agricultural economics literature. It seem appropriate in this case, as it imposes diminishing marginal returns to the investment in quality and the trademark when \( \alpha < 1 \). It also suggests a multiplicative shift of demand as investment in the trademark increases. There is a robust literature that considers the implications of different assumptions about shifts in demand. There are valid criticisms for each assumption; debating these is not within the scope of our paper.
Monopoly

First, consider a profit-maximizing monopolist licensee. The monopolist’s problem is:

\[
\max_{Q_M, A_M} \Pi_M = [P(Q_M, A_M) - c - r]Q_M - A_M - F. \tag{1}
\]

Rearranging, the first-order conditions yields the following conditions:

\[
\frac{P^*_M - c - r}{P^*_M} = -\frac{1}{\eta} \quad \text{and} \quad -\frac{\alpha}{\eta} = \frac{A^*_M}{P^*_M Q^*_M}, \tag{2}
\]

where \(\eta\) is the elasticity of demand with respect to price and \(\alpha\) is the elasticity of demand with respect to investment in the trademark. This is analogous to Dorfman-Steiner’s (1954) result related to advertising, with an additional term, \(r\), representing patent royalties. The fixed fee does not affect the result. Using this result, which holds regardless of functional form, in the constant elasticity demand function, as defined above, we get:

\[
A^*_M = \left[\left(\frac{\alpha}{\eta}e^\beta \left(\frac{\eta(c + r)}{\eta + 1}\right)^{\eta+1}\right)\right]^{\frac{1}{1-\alpha}} \tag{3}
\]

\[
P^*_M = \frac{\eta(c + r)}{\eta + 1} = \frac{c + r}{1 + \frac{r}{\eta}} \tag{4}
\]

\[
Q^*_M = \left[\left(\frac{-\alpha}{\eta}e^\beta \left(\frac{\eta(c + r)}{\eta + 1}\right)^{\eta+1}\right)\right]^{\frac{1}{1-\alpha}} \tag{5}
\]

\[
\Pi^*_M = \left(\frac{1 - \alpha}{\alpha}\right) \left[\left(\frac{-\alpha}{\eta}e^\beta \left(\frac{\eta(c + r)}{\eta + 1}\right)^{\eta+1}\right)\right]^{\frac{1}{1-\alpha}} = \left(\frac{1 - \alpha}{\alpha}\right) A^*_M - F \tag{6}
\]

\[
CS^*_M = -\frac{1}{\eta} \left[\left(\frac{-\alpha}{\eta}e^\beta \left(\frac{\eta(c + r)}{\eta + 1}\right)^{\eta+1}\right)\right]^{\frac{1}{1-\alpha}} = -\frac{1}{\eta} Q^*_M P^*_M \tag{7}
\]

\[
R|_{Q^*_M} = r \left[\left(\frac{-\alpha}{\eta}e^\beta \left(\frac{\eta(c + r)}{\eta + 1}\right)^{\eta+1}\right)\right]^{\frac{1}{1-\alpha}} + F = rQ^*_M + F \tag{8}
\]

\footnote{Dorfman and Steiner (1954) found that in the case of monopoly, the profit-maximizing advertising-to-sales ratio is equal to the elasticity of demand with respect to advertising divided by the absolute value of the elasticity of demand with respect to price. The first equality in equation (2) simply indicates the basic mark-up is equal to the absolute value of the inverse price elasticity—the standard monopoly result. Our work expands on this result in that we consider oligopolies with two different possibilities for trademark investment and then consider the upstream licensor’s actions given the optimal investment in the trademark by the firm.}
To better understand how changes in the university’s choices affect the monopolist licensee’s profit-maximizing choices of investment in the trademark, quantity produced, and other key variables, we consider the partial derivatives of these variables with respect to the royalty rate and the number of licensee firms. We find that when the royalty rate increases, price also increases and investment in the trademark, quantity produced, consumer surplus and profits all decrease. The price increases because the firm passes some of the additional cost on to consumers. Investment in the trademark decreases because this is a variable cost that the firm can reduce to compensate for the increase in other variable costs. Naturally then, since quantity demanded is increasing in the investment in the trademark and decreasing in the price, the optimal quantity produced is now lower. Consumers lose from both the increase and price and the decrease in trademark investment, which they value. Profits are reduced as well because demand is elastic and the loss of revenue cannot be made up for by the decrease in costs. We also find that the Hessian of the profit function is negative semi-definite as long as $|\eta| > 1$ and $|\eta| > \alpha$, confirming that we are indeed looking at a maximum. $R |Q_M^*$ represents the royalty revenue to the university given values for $r$ and $F$, and profit maximization by the monopolist.

**Oligopoly with individual investment in a common trademark**

Now, we can examine the same market, but with a Cournot oligopoly selling the new variety. In this case, all of the producers are still investing in a single trademark, and each firm invests the amount in the trademark that maximizes its profits. We define output and investment in the trademark such that $Q_O = \Sigma_i q^i_O$ and $A_O = \Sigma_i A^i_O$. For the oligopolist we will consider a sequential problem. In the first stage, the oligopolist chooses how much to spend on a trademark, $A^i_O$. In the second stage, the oligopolist chooses the quantity, $q^i_O$, taking $A^i_O$ and $A_O$ as given. Solving by

\[ 19 \text{The fixed fee, } F, \text{ is also chosen by the university, but does not affect the results. It is, however, included in the profit and revenue expressions.} \]

\[ 20 \text{By making investment in the trademark additive, we are assuming that consumers receive trademark signals from all firms and relate this signal to the quality of the output in the industry, not just output by a single firm. In this sense, investments in the trademark are like brand advertising, a collective good for all firms that produce and sell that brand. This seems realistic in a setting like apples, where apple variety names are easily recognized by consumers, but the names of the growers are not.} \]

\[ 21 \text{For the monopolist, the sequential problem and the simultaneous problem yield the same result, hence the simultaneous treatment of that problem.} \]
backwards induction, the oligopolist’s second-stage problem is:

$$\max_{q_O} \Pi_O = P(Q_O, A_O)q_O^i - (c + r)q_O^i - A_O^i - F. \tag{10}$$

Rearranging the first-order condition and assuming identical firms yields:

$$\frac{P_O^* - c - r}{P_O^*} = -\frac{1}{N\eta}, \tag{11}$$

where $N > 1$ is the number of oligopolists. The second-order condition requires $N > 1$ for a maximum. Equation (10) can also be expressed as:

$$q_O^{i*}(A_O) = \frac{1}{N} e^\beta A^\alpha \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^\eta. \tag{12}$$

The oligopolist’s first-stage problem is thus:

$$\max_{A_O} \Pi_O\big|_{q_O^i} = P_O^* q_O^{i*}(A_O) - (c + r)q_O^{i*}(A_O) - A_O^i - F. \tag{13}$$

Rearranging the first-order condition yields:

$$-\frac{\alpha}{N\eta} = \frac{A_O^*}{P_O^* q_O^{i*}}. \tag{14}$$

This second-order condition holds for a maximum if $0 < \alpha < 1$, which we have already assumed. Assuming the same constant elasticity demand function as in the previous problem, and identical
Looking at the partial derivatives, we again have the case that when the royalty rate increases, price increases, while quantity, investment in the trademark, and profits all decrease. These results hold for the same reasons indicated in the previous section. Furthermore, an increase in the number of firms implies decreases in price, firm-level quantity of output produced, firm-level investment in the trademark, and profit, as well as industry investment in the trademark. As more firms enter, market power is reduced, lowering the equilibrium output price. At the same time, this lowers the economic incentive for firms to invest in the trademark to increase demand, so firm-level investment in the trademark decreases. This decrease is significant enough such that industry investment in the trademark also decreases, even though there are now more firms. However, the decrease in expenditures on the trademark is not enough to offset the decrease in revenue, so profits decrease. The results for total quantity produced and consumer surplus depend on parameter values. If the number of firms, $N$, is such that $N < \frac{1}{2\alpha}$, then when the number of firms increases, total quantity produced increases, but if $N > \frac{1}{2\alpha}$, total quantity produced decreases. This result is due to the fact that the decrease in price increases quantity demanded, and the decrease in investment decreases quantity demanded. When consumers are relatively unresponsive to investment in the trademark, the net effect is an increase in total quantity produced for relatively more industry sizes, but when consumers are relatively responsive to price, this positive net effect is found only for the
industry with very few firms, if at all. For $\alpha \geq \frac{1}{2}$, total quantity produced is always decreasing. Similarly, if the number of firms, $N$, is such that $N < \frac{1}{4\alpha}$, then when the number of firms increases, consumer surplus increases, but if $N > \frac{1}{4\alpha}$, consumer surplus decreases. This is due to the same phenomena explained above, although the conditions for consumers are slightly different because of the additional effects of changes in price and investment in the trademark.

**Oligopoly with joint investment in a common trademark**

We can also look at what happens when oligopolistic producers jointly optimize their investment in the trademark (for instance, as part of a marketing order). In this case, solving by backwards induction, the second stage is the same as for the oligopoly case above:

$$\max_{q^*_j} \Pi_J = h(Q_J, A_J)q^*_j - (c + r)q^*_j - A^*_j - F.$$  \hspace{1cm} (21)

Rearranging the first-order condition yields the following condition:

$$\frac{P^*_J - c - r}{P^*_J} = -\frac{1}{N\eta},$$  \hspace{1cm} (22)

where $N$ is the number of oligopolists. An alternative form of equation (21) yields an expression for $q^*_j$ as a function of $A_J$, which we can then use in solving the oligopolists’ first-stage problem. Again, we assume firms are identical in terms of quantity produced. However, now the oligopolists jointly optimize investment in the trademark. The oligopolists’ joint first-stage problem is thus:

$$\max_{A_J} \Pi_J|_{Q_J} = P^*_JQ^*_J(A_J) - (c + r)Q^*_J(A_J) - A_J - NF.$$  \hspace{1cm} (23)

Rearranging the first-order condition yields the following condition:

$$-\frac{\alpha}{N\eta} = \frac{A^*_J}{P^*_JQ^*_J}.$$  \hspace{1cm} (24)
Assuming the same demand function as in the previous problems and identical firms, we get:

\[ A_j^* = \frac{1}{N} \left[ -\frac{\alpha}{N\eta} e^{\beta} \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^{\eta+1} \right]^{\frac{1}{1-\alpha}} \]  \hspace{1cm} (25)

\[ P_j^* = \frac{N\eta(c + r)}{N\eta + 1} = \frac{c + r}{1 + \frac{1}{N\eta}} \]  \hspace{1cm} (26)

\[ Q_j^* = \frac{1}{N} \left[ \left( -\frac{\alpha}{N\eta} \right)^{\alpha} e^{\beta} \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^{\eta+1} \right]^{\frac{1}{1-\alpha}} \]  \hspace{1cm} (27)

\[ \Pi_j^* = \left( \frac{N - \alpha}{N\alpha} \right) \left[ -\frac{\alpha}{N\eta} e^{\beta} \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^{\eta+1} \right]^{\frac{1}{1-\alpha}} = \left( \frac{1 - \alpha}{N\alpha} \right) A_j^* \]  \hspace{1cm} (28)

\[ CS_j^* = -\frac{1}{\eta} \left[ \left( -\frac{\alpha}{N\eta} \right)^{\alpha} e^{\beta} \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^{\eta+1} \right]^{\frac{1}{1-\alpha}} = -\frac{1}{\eta} Q_j^* P_j^* \]  \hspace{1cm} (29)

\[ R|Q_j^* = r \left[ \left( -\frac{\alpha}{N\eta} \right)^{\alpha} e^{\beta} \left( \frac{N\eta(c + r)}{N\eta + 1} \right)^{\eta+1} \right]^{\frac{1}{1-\alpha}} + F = rQ_j^* + F \]  \hspace{1cm} (30)

As in the previous two cases, as the royalty rate and cost of production increase, price increases, and quantity, investment in the trademark, profits and consumer surplus decrease.

The intuition for these results is the same as for the monopoly case. Furthermore, an increase in the number of firms implies decreases in firm-level quantity produced, investment in the trademark, and profits, as well as industry-wide investment in the trademark and profits. If the number of firms, \(N\), is such that \(N < \frac{1}{\alpha}\), then when the number of firms increases, total quantity produced increases, but if \(N > \frac{1}{\alpha}\), total quantity produced decreases. When the number of firms, \(N\), increases, if \(N < \frac{1}{2\alpha}\), then consumer surplus increases, but if \(N > \frac{1}{2\alpha}\), consumer surplus decreases. These two results are similar to those for the individual investment case, although they indicate that in the case of joint investment, total quantity produced and consumer surplus increase in the number of firms for a relatively larger number of industry sizes than in the case of individual investment. This is probably because, in the joint investment case, the investment is relatively large when the number of firms is small. Thus, although investment in the trademark decreases as the number of firms increases, it is still higher given any industry size than in the individual investment case, which means the positive effect on output from the decrease in price outweighs the negative effect of the decreased investment for more industry size scenarios than in the individual investment case.
The university’s objective function

Now that we have determined the producer response given royalty rate, \( r \), and fixed fee, \( F \), we must consider the university’s choices. As previously discussed, the university’s decision problem is potentially very complex and is influenced by a number of factors. This decision could be modeled by assuming the university (acts as though it) maximizes a weighted function of patent revenue, industry profits, and consumer surplus with unequal weights based on political and institutional considerations. To help resolve and characterize this complexity, we consider four possible optimization problems of the university in which the university maximizes patent revenue, producer profits, consumer surplus, or social welfare.

If the university chooses to maximize its patent income, the university’s problem is:

\[
\max_r R = rQ(r) + \Pi(r). \tag{31}
\]

This set-up assumes that the university extracts the maximum possible fixed fee from producers (i.e., total industry profits). The optimal royalty rate under each market structure is then given by:

\[
\begin{align*}
  r^*_M(R_{\text{MAX}}) &= 0, \\
  r^*_O(R_{\text{MAX}}) &= \frac{c[(N\eta(1 - N)(1 - \alpha) - \alpha(1 - N)(\eta + 1)]]}{N^2\eta(\eta + 1) + \alpha(\eta + 1)}, \tag{33} \\
  r^*_J(R_{\text{MAX}}) &= \frac{c[\eta(1 - N)(1 - \alpha)]}{N\eta(\eta + 1) + \alpha(\eta + 1)}. \tag{34}
\end{align*}
\]

In the case of monopoly, the optimal choice for the university is a fixed fee equal to the monopoly rents. In the case of oligopoly, so long as the optimal royalty rate is positive, the optimal choice for the university is a royalty together with a fixed fee equal to oligopoly rents. In the simulations, we explore which scenario yields the highest university revenue under different assumptions about parameter values.

Alternatively, the university could choose to maximize social welfare. The university’s prob-
lem is then:

$$\max_{r,F} SW = CS(r) + \Pi(r,F) + R(r,F). \quad (35)$$

Solving the optimization problem in equation (38), we find the following solutions for optimal royalty rates:

$$r^*_M(SW_{MAX}) = 0 \quad (36)$$
$$r^*_O(SW_{MAX}) = -\frac{c[N^2(1 + \alpha \eta) - \alpha \eta - \alpha + N(\alpha + \eta)]}{N^2(1 - \eta^2) - \alpha \eta - \alpha} \quad (37)$$
$$r^*_J(SW_{MAX}) = -\frac{c[N(1 + \alpha \eta) - \alpha \eta + \eta]}{N(1 - \eta^2) - \alpha \eta - \alpha} \quad (38)$$

Here again we find that the optimal royalty is zero for a monopoly. Since the fixed fee is simply a transfer from producers to the university, changing the fixed fee does not affect social welfare. Thus, any fixed fee the university sets that is less than or equal to monopoly rents will also maximize social welfare. Likewise, in the oligopoly situation, the university will charge the optimal royalty rate to maximize social welfare, and it can then charge any fixed fee it chooses, less than the oligopoly rents, to achieve its income distribution objectives. Again, we explore which scenario yields the highest social welfare under different parameter assumptions in the simulations.

Finally, we consider the case of maximizing either consumer surplus or producer profits. Consumer surplus is maximized where royalties are zero, but the fixed fee can be of any magnitude less than or equal to producer profits since the fixed fee will not affect the quantity of output or investment in the trademark and thus will not affect consumer surplus. Therefore, consumers will always prefer a revenue-equivalent fixed fee to a royalty.

On the other hand, producer profits are affected by both the royalty and the fixed fee. Producers’ profits are maximized when both the royalty and fixed fee are equal to zero. We might expect that producers would prefer a royalty rate to a revenue-equivalent fixed fee, as a portion of the cost will be borne by the consumer in the case of a royalty. However, in our model producers do not always prefer a royalty rate. In some cases, producers prefer a fixed fee. For ease of analysis, we express the royalty rate, \( r \) as a function of the marginal cost, \( c \), such that: \( r = \gamma c \). Then for each of the three cases, the conditions under which producers prefer a revenue-equivalent fixed fee
to a royalty are:

\[
\text{Monopoly: } (1 + \gamma)^{\frac{\eta + \alpha}{1 - \alpha}} < \frac{\alpha - 1}{\alpha - 1 + \gamma(\alpha + \eta)} \quad (39)
\]

\[
\text{Oligopoly (Individual): } (1 + \gamma)^{\frac{\eta + \alpha}{1 - \alpha}} < \frac{\alpha - N}{\alpha - N + \gamma(\alpha + N^2\eta)} \quad (40)
\]

\[
\text{Oligopoly (Joint): } (1 + \gamma)^{\frac{\eta + \alpha}{1 - \alpha}} < \frac{\alpha - 1}{\alpha - 1 + \gamma(\alpha + N\eta)} \quad (41)
\]

As the number of firms, \(N\), increases, the magnitudes of the denominators of the right hand side of equations 43 and 44 increase relative to the magnitudes of the numerators, indicating the inequalities may not hold when the number of firms is large. This result arises from the effect of the royalty rate on investment in the trademark. The royalty rate affects the quantity demanded through price and investment in the trademark. As the royalty rate increases, the optimal price increases and the optimal investment in the trademark decreases. For scenarios in which the number of firms is relatively large, because the trademark is a collective good, the optimal investment in the trademark at a royalty rate of zero is relatively low. Consequently, in the case when the royalty is increased, the distorting effect on investment in the trademark is relatively small and the loss to producers is outweighed by the benefit from sharing the cost of the royalty with consumers. However, when the number of firms is relatively small, the optimal investment in the trademark is relatively high when the royalty rate is zero, and the distorting affect of the royalty is more severe than in the case with many firms. In this case, the negative affect of the royalty on trademark investment and resulting decrease in quantity outweigh the positive benefits of sharing the cost of the royalty with consumers, and firms may prefer a fixed fee.

**Simulations**

To better understand the implications of these analytical results, we ran numerical simulations. All reported simulations assume a scaling parameter of \(\beta = 10\) and marginal cost of production of \(c = 1\). We assume an own-price elasticity of demand of \(\eta \in [-2, -10]\) and elasticity of demand with respect to investment in the trademark of \(\alpha \in [0.05, 0.15]\). These elasticities allow for a fairly wide range of substitutability of the new variety for incumbent apple varieties, and a rate of response to investment in the trademark comparable to observed elasticities of demand response to generic
promotion for agricultural products. For the figures labeled “low,” we assume that $\alpha = 0.05$ and $\eta = -10$. In this case, the effect of investment in the trademark on demand is small, and consumers are sensitive to price changes. For those labeled “high,” we assume $\alpha = 0.15$ and $\eta = -2$. In this case, demand is considerably more affected by the trademark investment and consumers are less sensitive to price. In Figure 1 we assume $r = 0.5$.

We find that investment in the trademark is larger in the case of a monopoly than in either oligopoly case. The trademark investment is also relatively large for oligopolists when the number of firms is small, but drops precipitously as the number of firms increases. This result arises because trademark promotion is a collective good among licensees, and as the number of firms increases, producers are less able to realize the benefits from their own investment and thus have less incentive to invest in the trademark. This can be seen in Panels A and B of Figure 1. As the investment in the trademark goes down, we see the quantity of output follows it, as shown in Panels C and D of Figure 1, because consumers value quality. With the exception of the addition of the first few firms, total output actually decreases as the number of firms increases, as does price and consequently, industry profits, as shown in Panels E and F of Figure 1. As we show in Figures G and H of Figure 1, consumers see a boost in surplus with the addition of the first several firms. In the “low” case ($\alpha = 0.05$ and $\eta = -10$), in which consumers are affected relatively less by investment in the trademark and relatively more by price (Panel H), consumer surplus declines slowly as the number of firms increases. However, in the “high” case ($\alpha = 0.15$ and $\eta = -2$), in which consumers are affected relative more by investment in the trademark and relatively less by price (Panel G), we see that consumer surplus declines quite quickly with increases in the number of firms. These results reflect the fact that consumers benefit from the price decrease that results from the increased competition, but they lose from decrease in quality that results in the more competitive industry. Importantly, we also find that in the case of investment in the trademark by individual firms, consumer surplus is actually greater with a monopoly than with a relatively small oligopoly. Panels I and J of Figure 1 show social welfare, given by adding university revenue to consumer surplus and producer profits. Panel I clearly demonstrates how a monopoly or small oligopoly could be beneficial compared with a more competitive industry under specific demand conditions.

In Figure 1, we assumed $r$ was fixed to better demonstrate the relationship between market
structures. However, the university chooses \( r \). In Figure 2, we show social welfare and university revenue as a function of \( r \). “Joint” refers to the case in which oligopolists jointly optimize their investment in the trademark. “Individual” refers to the case in which they optimize individually. We see that in the “low” case (\( \alpha = 0.05 \) and \( \eta = -10 \)) where consumers are comparatively sensitive to price but not to investments in the trademark, the optimal royalty rate is zero if the university wants to maximize social welfare (Panels B and D of Figure 2); it is positive but small in magnitude if the university wants to maximize revenue for \( N > 1 \) (Panels F and H of Figure 2). The optimal royalty rate is positive and larger in magnitude when consumers are not very responsive to price but are sensitive to investment in the trademark (see Panels A, C, E and G). Furthermore, we see that in all cases, as the number of firms increases, the optimal royalty rate increases or stays constant.

Given the optimal royalty rate in each case, social welfare is higher with an oligopoly that jointly maximizes investment in the trademark than with a monopoly (see Panels A and B). However, for the other four cases under consideration, the monopoly outcome dominates. The optimal royalty rate is higher under revenue maximization than under social welfare maximization regardless of the number of firms.

Finally, in Table 2, we put these results together and present the university’s optimal royalty rate, fixed fee, and number of firms (including the structure of their trademark investment) for a range of plausible parameter values for each objective function. We find that regardless of which welfare measure the university maximizes, it is never optimal for the university to charge a royalty rate. A fixed fee alone is optimal for the university. In addition, we find that when the university maximizes its own revenue or producer profits, it is optimal for the university to license to a monopoly. However, in the revenue maximization case, the university will extract all of the monopolist’s profits, whereas in the profit maximization case, the university will simply give away the technology. In the case of both social welfare maximization and consumer surplus maximization, it is optimal for the university to license to more than one firm. When maximizing social welfare, the university licenses to between 2 and 4 firms, depending on parameter values, and extracts revenue from producer profits, as the magnitude of the fixed fee does not affect social welfare. When maximizing consumer surplus, the university licenses to between 4 and 18 firms, depending on parameter values, and extracts revenue from producer profits, as again, the magnitude of the fixed fee does not affect this welfare measure. These results are interesting in that they tell
us that, given our assumptions, royalties are never optimal for the university. Furthermore, these results suggest that even if the university weights the welfare of consumers very heavily, the optimal number of firms in the market is still relatively small, providing an impetus for exclusive licensing, if demand response to the trademark is significant.

Conclusion

In this paper we have considered two critical aspects of patenting horticultural varieties developed by public universities: the role of investment in a trademark and the complicated optimization problem faced by land-grant institutions. We develop a simple model that abstracts from many of the complexities of the situation. These include but are not limited to: the role of risk and uncertainty in decision-making by the university and firms, the complexities of substitution between the new variety and existing varieties and products if their prices and qualities are jointly endogenous, the inherently dynamic nature of economic problems concerning perennial crops and intellectual property management, and the complex internal and external political influences that affect university TTO managers.

Nevertheless, several important insights stem from our work. First, consumers (and society—in a social welfare sense) could benefit from a university choosing to license fewer firms to produce a new variety. For consumers and society, the optimal size of the industry is smallest when consumers are relatively responsive to investment in the trademark (i.e., if they attach a high value to the perceived quality of the trademarked product) and relatively unresponsive to price. This result helps defend universities against claims that, by using exclusive licensing, they are not fulfilling their public mission. However, with exclusive licensing the producer benefits are shared among a small number of firms; others are excluded. Presumably this distributional issue will be of concern to universities, but we have not addressed it explicitly in our analysis. This was the concern raised by farmers in Minnesota when the SweeTango apple was introduced and has been considered extensively by Cornell University and Washington State University as they develop licenses.

The detail of the university’s objective function matters both in determining the optimal number of licensees and the optimal licensing fee structure. We find that the university will license to a monopoly and charge a fixed fee if maximizing revenue, but will license to more than one firm
and will charge an optimal royalty (and may or may not extract additional producer surplus in the form of a fixed fee) if maximizing social welfare.\footnote{For the parameter values we consider, this optimal royalty is zero; we find no case in which it is optimal for the university to charge a royalty.} In practice, we see university technology transfer offices negotiating with commodity boards, industries, inventors and university administrators to set their fees. Given the many stakeholders, the political complexities, and other constraints, it seems likely that universities are setting their fees and licensing structures in a more complicated context than previous models have allowed—profit-maximization or maximization of social welfare. Our analysis sheds some light on this issue in a comparatively short-run context in which we take as given the amount of research investment, and the quality and quantity of resulting inventions. The structure of the licensing arrangements and fees (and their implications for funds available for university research) are also likely to have implications for the rate of innovation in a longer-run analysis.
Table 1: Analytical Results of University (Licensor) and Firm (Licensee) Optimization

<table>
<thead>
<tr>
<th>Result</th>
<th>Definition</th>
<th>Monopoly</th>
<th>Oligopoly</th>
<th>Oligopoly with Joint Branding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^*$</td>
<td>Output price</td>
<td>$\frac{c+r}{1+\frac{\eta}{N\eta}}$</td>
<td>$\frac{c+r}{1+\frac{\eta}{N\eta}}$</td>
<td>$\frac{c+r}{1+\frac{\eta}{N\eta}}$</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>Industry output</td>
<td>$\left[\frac{-\alpha}{\eta}\right]^{\alpha} e^{\beta} \left(\frac{\eta(c+r)}{\eta+1}\right)^{\eta+\alpha} \frac{1}{1-\alpha}$</td>
<td>$\left[\frac{-\alpha}{N^2\eta}\right]^{\alpha} e^{\beta} \left(\frac{N\eta(c+r)}{N\eta+1}\right)^{\eta+\alpha} \frac{1}{1-\alpha}$</td>
<td>$\left[\frac{-\alpha}{N\eta}\right]^{\alpha} e^{\beta} \left(\frac{N\eta(c+r)}{N\eta+1}\right)^{\eta+1} \frac{1}{1-\alpha}$</td>
</tr>
<tr>
<td>$A^*$</td>
<td>Industry trademark investment</td>
<td>$\left[\frac{-\alpha}{\eta}\right]^{\alpha} e^{\beta} \left(\frac{\eta(c+r)}{\eta+1}\right)^{\eta+1} \frac{1}{1-\alpha}$</td>
<td>$\left[\frac{-\alpha}{N^2\eta}\right]^{\alpha} e^{\beta} \left(\frac{N\eta(c+r)}{N\eta+1}\right)^{\eta+1} \frac{1}{1-\alpha}$</td>
<td>$\left[\frac{-\alpha}{N\eta}\right]^{\alpha} e^{\beta} \left(\frac{N\eta(c+r)}{N\eta+1}\right)^{\eta+1} \frac{1}{1-\alpha}$</td>
</tr>
<tr>
<td>$P^<em>Q^</em>$</td>
<td>Industry revenue</td>
<td>$\frac{-\eta}{\alpha} A^*$</td>
<td>$\frac{-N^2\eta}{\alpha} A^*$</td>
<td>$\frac{-N\eta}{\alpha} A^*$</td>
</tr>
<tr>
<td>$\frac{A^*}{P^<em>Q^</em>}$</td>
<td>Trademark investment ratio</td>
<td>$\frac{-\alpha}{\eta}$</td>
<td>$\frac{-\alpha}{N^2\eta}$</td>
<td>$\frac{-\alpha}{N\eta}$</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>Industry profits</td>
<td>$(\frac{1-\alpha}{\alpha}) A^* - F$</td>
<td>$(\frac{N-\alpha}{\alpha}) A^* - NF$</td>
<td>$(\frac{1-\alpha}{\alpha}) A^* - NF$</td>
</tr>
<tr>
<td>$CS^*$</td>
<td>Consumer surplus</td>
<td>$-\frac{1}{\eta} P^* Q^*$</td>
<td>$-\frac{1}{\eta} P^* Q^*$</td>
<td>$-\frac{1}{\eta} P^* Q^*$</td>
</tr>
<tr>
<td>$SW^*$</td>
<td>Social welfare</td>
<td>$\left(\frac{1-\alpha}{\alpha}\right) A^* + \left(r - \frac{1}{\eta} P^<em>\right) Q^</em>$</td>
<td>$\left(\frac{N-\alpha}{\alpha}\right) A^* + \left(r - \frac{1}{\eta} P^<em>\right) Q^</em>$</td>
<td>$\left(\frac{1-\alpha}{\alpha}\right) A^* + \left(r - \frac{1}{\eta} P^<em>\right) Q^</em>$</td>
</tr>
<tr>
<td>$r^*(R_{MAX})$</td>
<td>Optimal royalty under revenue max.</td>
<td>0</td>
<td>$c\frac{N(N-\eta N)(1-\alpha) + \alpha(N-1)(\eta+1)}{\alpha(\eta+1) + N^2(\eta^2 + \eta)}$</td>
<td>$c\frac{<a href="1-%5Calpha">\eta - \eta N</a>}{\alpha(\eta+1) + N(\eta^2 + \eta)}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c\frac{N^2 + N^2 \eta_\alpha + N \eta - \eta \alpha - \alpha}{N^2 - \eta_\alpha - \alpha - N^2 \eta}$</td>
<td>$c\frac{N \eta_\alpha + N - \eta \alpha + \eta}{N^2 - N - \alpha \eta - \alpha}$</td>
</tr>
<tr>
<td>$r^*(SW_{MAX})$</td>
<td>Optimal royalty under social welfare max.</td>
<td>0</td>
<td>$c\frac{N^2 + N^2 \eta_\alpha + N \eta - \eta \alpha - \alpha}{N^2 - \eta_\alpha - \alpha - N^2 \eta}$</td>
<td>$c\frac{N \eta_\alpha + N - \eta \alpha + \eta}{N^2 - N - \alpha \eta - \alpha}$</td>
</tr>
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</table>
Figure 1. Simulations: \( r = 0.5 \) (High: \( \alpha = 0.15, \eta = -2 \); Low: \( \alpha = 0.05, \eta = -10 \))

Panel 1A: Industry Brand Investment (High)  
Panel 1B: Industry Brand Investment (Low)

Panel 1C: Industry Output (High)  
Panel 1D: Industry Output (Low)

Panel 1E: Industry Profit (High)  
Panel 1F: Industry Profit (Low)

Panel 1G: Consumer Surplus (High)  
Panel 1H: Consumer Surplus (Low)
Figure 2. Simulations: $N \in \{1, 2, 5, 10\}$ (High: $\alpha = 0.15$, $\eta = -2$; Low: $\alpha = 0.05$, $\eta = -10$)

Panel 2A: Social Welfare (Joint/High)

Panel 2B: Social Welfare (Joint/Low)

Panel 2C: Social Welfare (Individual/High)

Panel 2D: Social Welfare (Individual/Low)

Panel 2E: University Revenue (Joint/High)

Panel 2F: University Revenue (Joint/Low)
Table 2: Simulations – Optimal University Licensing Fees and Structure ($\beta = 10$, $c = 1$)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\eta$</th>
<th>$N^*$</th>
<th>$r^*$</th>
<th>$F_i^*$</th>
<th>$P^*$</th>
<th>$N^*$</th>
<th>$r^*$</th>
<th>$F_i^*$</th>
<th>$P^*$</th>
<th>$N^*$</th>
<th>$r^*$</th>
<th>$F_i^*$</th>
<th>$P^*$</th>
<th>$N^*$</th>
<th>$r^*$</th>
<th>$F_i^*$</th>
<th>$P^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2.00</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 15313.14$</td>
<td>2.00</td>
<td>2($J$)</td>
<td>0</td>
<td>$F_i \leq 5458.18$</td>
<td>1.33</td>
<td>4($J$)</td>
<td>0</td>
<td>$F_i \leq 1447.52$</td>
<td>1.14</td>
</tr>
<tr>
<td>0.10</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2.00</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 9992.41$</td>
<td>2.00</td>
<td>2($J$)</td>
<td>0</td>
<td>$F_i \leq 3629.27$</td>
<td>1.33</td>
<td>6($J$)</td>
<td>0</td>
<td>$F_i \leq 446.06$</td>
<td>1.09</td>
</tr>
<tr>
<td>0.05</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2.00</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 7031.07$</td>
<td>2.00</td>
<td>2($J$)</td>
<td>0</td>
<td>$F_i \leq 2597.03$</td>
<td>1.33</td>
<td>11($J$)</td>
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<td>$F_i \leq 101.17$</td>
<td>1.05</td>
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<tr>
<td>0.15</td>
<td>-5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.25</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 4121.03$</td>
<td>1.25</td>
<td>2($J$)</td>
<td>0</td>
<td>$F_i \leq 1586.87$</td>
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<td>$F_i \leq 203.30$</td>
<td>1.08</td>
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<tr>
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<td>-5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.25</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 2892.55$</td>
<td>1.25</td>
<td>3($J$)</td>
<td>0</td>
<td>$F_i \leq 564.37$</td>
<td>1.07</td>
<td>8($J$)</td>
<td>0</td>
<td>$F_i \leq 86.42$</td>
<td>1.03</td>
</tr>
<tr>
<td>0.05</td>
<td>-5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.25</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 2172.55$</td>
<td>1.25</td>
<td>3($J$)</td>
<td>0</td>
<td>$F_i \leq 436.01$</td>
<td>1.07</td>
<td>16($J$)</td>
<td>0</td>
<td>$F_i \leq 17.80$</td>
<td>1.01</td>
</tr>
<tr>
<td>0.15</td>
<td>-10</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>$F_i \leq 1707.69$</td>
<td>1.11</td>
<td>3($J$)</td>
<td>0</td>
<td>$F_i \leq 333.09$</td>
<td>1.03</td>
<td>6($J$)</td>
<td>0</td>
<td>$F_i \leq 88.31$</td>
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<td>1</td>
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<td>0</td>
<td>1.11</td>
<td>1</td>
<td>0</td>
<td>$F_i \leq 1258.75$</td>
<td>1.11</td>
<td>3($J$)</td>
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<tr>
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<td>1</td>
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<td>18($J$)</td>
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<td>$F_i \leq 6.74$</td>
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Note: ($J$) indicates joint investment in the trademark.
References


